¶ The Grounde of Artes: teaching the perfecte vvorke and practiſe of Arithmetike, both in whole nũbers and fractions, after a more eaſie and exact ſort, than hitherto hath bene ſet forth.

Ll. 20. 51

Made by M. Robert Recorde, D. in Phyſick,
and afterwards augmented by M. Iohn Dee.

And now lately diligently corrected, & beautified with ſome newe Rules and neceſſarie Additions:
And further endowed with a thirde part, of Rules of
Practize, abridged into a briefer methode than
hitherto hath bene publiſhed: with diuerſe
ſuch neceſſary Rules, as are incident
to the trade of Merchandize.

Whervnto are alſo added diuers Tables & inſtructions
that will bring great profite and delight vnto Merchants, Gentlemen, and others, as by the contents of this treatiſe ſhal appeare.

By *Iohn Mellis* of *Southwark*, Scholemaſter.

CERVA · CHARISSIMA · ET · GRATISSIMVS · HINNVLVS · PRO. 5.

Imprinted by I. Hariſon, and H. Bynneman.
Anno Dom. 1582.

To the Reader.

THat which my friend hath well begon
For verie loue to common weale,
Of goodwill hath bene ouer runne
directing ech part euerie deale.
And bewtified for thy behoofe
With breifer wayes for practiſe lore,
As by the triall and the proofe
Was neuer yet in print before.
Of numbers vſe the endleſſe might
No wit nor language can expreſſe,
Apply,and trie,both day and night
And then this truth thou wilt confeſſe.

The Bookes Verdicte.

To pleaſe or diſpleaſe ſure I am,
But not of one ſort to euerie man.
To pleaſe the beſt ſort would I faine,
The froward diſpleaſe ſhal I certaine.
Yet wiſh I wil,though not with hope,
All eares and mouthes to pleaſe or ſtoppe.

UNIVERSITY LIBRARY
13 JUL 86
CAMBRIDGE

86...439

To the Right worſhipfull M. ROBERT FORTH Doctor of Law, and *one of the Maiſters of the Queenes Maieſties high Court of Chauncerie.*

BIAS the wiſe Philoſopher of *Greece*, when his Country was ſpoyled, and the people caried their goods away, being demaunded why he did not the like? anſwered, that he caried all that he had with him, meaning his *vertue* and *learning:* So I (my Right worſhipful & ſingular good Maſter) hauing nothing of value in ſubſtance to beſtow, doe here preſent you with ſuch treaſure as the Lord hath vouchſafed vpon me, which according to *Bias* opinion and mine owne likewiſe, is the greateſt *Iewell* I haue, not doubting but you wil accept the giuers mind more than the gift, which I confeſſe is ſmall. But being throughly acquainted with the great fauor your Worſhip bereth to ſuch as delight in any good exerciſe, it hath imboldned me to put forth this ſimple Addition vnder your Worſhips defence. The entyre loue & exerciſe of this excellēt Arte, with *drawing of proportions, Mappes, Cardes, Buildings, Plattes*, &c. were the only ſtudies whereunto I euermore haue bene enclined. Touching Drawing it was onely *Dei beneficio*, naturally giuen me from my youth, without inſtruction of any man, more than *Loue* thereof, delectation, deſire, and practiſe. In this Art alſo hauing great delight, I had no other inſtruction at my firſt beginning but onely this good Authors Booke, but afterwards I greatly encreaſed the ſame during the time I ſerued your Worſhip in *Cambridge*, in going to the *Arithmeticke* Lecture at the common Schoole: And more furthered ſince the time that I left your Worſhips ſeruice, which is about 18.yeares paſt, by continuall exerciſe therein (the mother and nurſe of Science,) during which time my onely vocation hath bene (thinking it a meete exerciſe for a common welth) in training vp of youth to write and draw, with teaching of them the infallable principles and briefe practiſes of this worthie Science, hauing (I praiſe God for it) brought vp a number to become faithfull and ſeruiceable to their maſters in great affaires, and many of them good members of a common wealth, which is no ſmall comfort to me in Chriſt. Amongſt which number, a countriman of mine hath oftentimes bene verie inportunate with me, to do a deede of Charitie vpon the ground of Arts, vz. to peruſe and amende it of the imperfections and faultes, that haue crept into it through negligence of often printing. Which erneſt requeſt of his, bred two ſtrifes in me: The one was, I was loth to do it, knowing my ſelfe inferior to a great number that might better do it than I. And yet conſidering it is a Booke hath done many a thouſand good, which when a young beginner commeth to a confuſed or miſtaken figure, it bringeth him into a wonderful diſcoragement and maze: which thing conſidered, for meere loue to a common wealth, and to the Booke, being my firſt Author, I willingly graunted to do my goodwill. And paſſing vnder the file of correction, I heere and there encreaſt it with ſuch neceſſarie Additions as I knewe might encorage a yong learner, and more woulde haue done, but for feare the Booke would riſe too thicke or grow too dere. And being thus entered into the vaine thereof, and knowing that this Author was the onely light and the chiefeſt Lodeſtone vnto the vulgar ſort of Engliſh men in this worthy ſcience, that euer writ in our naturall tong, I haue (according to my ſimple knowledge) yelded againe ſome part of my receiued talent with aduantage : and endowed him to the further encreaſe of his memorie, with Rules of breuitie and practize, abridged into a briefer method than hitherto hath bene publiſhed in our Engliſh tongue, with other right neceſſarie Additions, Rules and Tables, whiche I truſte will doe my countrie good, and be right commodious to all ſorts of men : all which I commit to the fauorable cenſure of your worſhip, and al ſuch as loue knowledge, deſiring their fauorable correction herein if ought be amiſſe. The which with greater affection than I am able to vtter, I Dedicate to your Worſhip, as a meete Patrone, both for lerning, godlines, and loue of the ſame, which comming from your worſhip into the hands of many (ſhall I doubt not) do many good, as heretofore it hath alreadie done. So ſhall you (as the beſt benefactor of theſe labors) be partaker of all their prayers that ſhall reape profite or knowledge by this worthy Art, in Commendation whereof if I ſhould write, I ſhould rather blemiſh thā adorne it. For the Authors Epiſtle vnto that famous Prince of worthy memorie *K. Edward* the ſixt, and his Preface to the Reader, are ſufficient.

Thus crauing fauourable acceptation of this my homely and dutifull preſent, I humblie leaue you to the conduction of the Almightie, whom I beſeech long to preſerue you in continuall health, with dayly encreaſe of worſhip, to the glorie of his name, and to the ioy of all ſuch as loue you.

At my houſe in Southwarke this 12. of Iune,
Your Worſhips moſt bounden to command,
IOHN MELLIS.

TO THE MOSTE mightie Prince Edwarde the ſixt, by the grace of God *King of Englande, Fraunce, and Irelande. &c.*

THe excellencie of mans nature, being ſuch, as it is by Gods diuine fauoure (moſt mightie Prince) not onely created in highneſſe of Degree farre aboue all other corporall things, but by perfection of reaſon and ſearch of wit, much approching toward the image of God, as not onely the holy Scriptures doe teſtifie, but alſo thoſe naturall Philoſophers, which exactly did conſider the nature of man, and namely the farre reach and infinite compaſſe of the workes of the minde, were inforced to confeſſe, that man ſcarcely was able to knowe himſelfe. And if he would duly ponder the nature of himſelfe, he woulde finde it ſo ſtrange, that it might ſeeme vnto him a verie miracle. And thereof ſprang that ſaying: *magnum miraculum eſt homo, maximum mariculum ſapiens homo.*

For vndoubtedly as man is one of the greateſt miracles that euer God wrought, ſo a wiſe man is plainely the greateſt.

And therefore was it that ſome did account the heade of a man the greateſt miracle in the worlde, becauſe not onely of the ſtraunge workemanſhippe that is in it, but much more of the efficacie of reaſon, witte, memorie, imagination, and ſuch other powers and workes of the minde, which can more eaſily conceiue any thing in a manner, than vnderſtande it ſelfe. And amongſt all the creatures of God it findeth none more difficult to be perceiued than theſe ſame powers of it ſelfe, whereby it doth conceiue and iudge as it may be well coniectured by the diuerſitie of opinions, that the wiſeſt Philoſophers did vtter touching the ſpirite of man and the ſubſtance of it: whereof at this preſent I intende to make no rehearſall, but who ſo liſteth to reade thereof, may finde it largely ſet forth not onely in Ariſtotle his bookes De anima; but alſo in Galene his booke called Hiſtoria philoſophica, & againe in Plutarch his worke, De philoſophorum placitis, whoſe wordes are alſo repeated of Euſebius in the xv. booke, τῆς ἐυαγγελικῆς προπαρασκευῆς, vnto whom I remit them that haue deſired to vnderſtande the intricate difficultie of knowing our owne ſelues, as touching our beſt part, and that part whereby we deſerue to beare the name of men.

This matter ſeemed ſo obſcure and difficulte, in knowledge, that Galen, who for his excellent wiſedome and iudgement in naturall workes, is called of many men a Miracle in nature, yet in ſearching the nature and ſubſtance of the ſpirite of man, he not onely confeſſeth himſelfe ignorant, but counteth it plaine temeritie to attempt to finde it: ſo farre aboue the hope of mans knowledge is that parte, whereby man doth knowe and iudge of things. And although the ignorant ſort (which hate all things that they knowe not) doe little eſteeme the profoundneſſe of mans ſpirit and of reaſon, the chiefe power and facultie of it, yet as there is a kind of feare and obedience of vnreaſonable beaſtes vnto man by the working power of God, ſo is there in thoſe ſmall reaſoned perſons a certaine kinde of reuerence towarde wiſedome and reaſons, which they doe ſhewe oftentimes, and by power of perſuaſion are inforced to obey reaſon, will they nil they.

And hereby came it to paſſe, that yͤ rudeneſſe of the firſt age of man was brought vnto ſome more ciuill trade, as it is well declared by Cicero in the beginning of his firſt booke *De inuentione Rethorica*, where he ſaith thus:

Nam fuit quoddam tempus quum in agris homines paſſim beſtiarum more vagabantur, & ſibi victu ferino vitam propagabant, nec ratione animi quicquam, ſed pleraq; viribus corporis adminiſtrabant. Nondum diuinæ religionis, non humani officij ratio colebatur: Nemo legitimas viderat nuptias, non certos quiſquam inſpexerat liberos, non ius æquabile quid vtilitatis haberet, acceperat: ita propter errorem atque inſcitiam, cæca ac temeraria dominatrix animi cupiditas, ad ſe explendam viribus corporis abutebatur, perniciosiſſimis ſatellitibus.

Quo tempore quidam, magnus videlicet vir & ſapiens, cognouit quæ materia eſſet, & quanta ad maximas res oportunitas in animis ineſſet hominum, ſi quis eam poſſet elicere, & præcipiendo meliorem reddere: Qui diſperſos homines in agris, & in tectis ſylueſtribus abditos ratione quadam compulit in vnum locum, & congregauit: & eos in vnamquamque rem inducens vtilē atque honeſtam primo propter inſolentiam reclamantes, deinde propter rationem atque orationem ſtudioſius audientes, ex feris & immanibus mites reddidit, & manſuetos.

This long repetition of Tullies wordes will ſeeme te-
dious

dious to them which loue but little, and care much leſſe for the knowledge of reaſon, but vnto your Maieſtie (I dare ſay) it is a delectable remembrance, and vnto mee it ſeemed ſo pleaſant, that I coulde ſcarſe ſtaye my penne from writing all that mine eyes did ſo greedily reade.

This ſentence of Cicero am I lothe to tranſlate into Engliſhe, partly for that vnto your Maieſtie it needeth no tranſlation, but eſpecially knowing howe farre that grace of Tullies eloquence doth excell any Engliſh mans tongue, and much more exceedeth the baſeneſſe of my barbarous ſtyle, yet for the fruite of the ſentence, I had rather vnto my meere Engliſh Countrymen vtter the rudeneſſe of my tranſlation, than to defraud them of the benefite of ſo good a leſſon, truſting they will alſo gladly and greedily imbrace all good ſciences that may helpe to the iuſt furniture of the ſame, when they conſider that informed reaſon was the onely Inſtrument, or at leaſt yͤ chiefe meane to bring men vnto Ciuile regiment, from barbarous manners and beaſtly conditions.

For the time was (ſaith Tullie) that men wandered abroad in the fields vp and downe like beaſts, and vſed no better order in feeding than they, ſo that by reaſons rule they wrought nothing, but moſte of their doing did they atchiue by force of ſtrẽgth. At this time there was no iuſt regarde of religion towarde God, nor of duetie towarde man. No man had ſeene right vſe of mariage, neither did any man knowe their owne children from other, nor no man had felt the commodity of iuſt lawes: ſo that through errour and ignorance, wilfull luſt, like a blinde and heady ruler, abuſed bodily ſtrength as a moſt mortall miniſter for the ſatiſfying of his deſire. At that time was there one, which not onely in power, but alſo in wiſedome was great: and he conſidered, howe that in the mindes of men was bothe apte inſtrumeutes, and greate occaſion to the due accompliſhment of moſt weightie affaires, if a man coulde applie them to vſe, and by teaching of rules frame them to better trade. This man with perſuaſion of reaſon, gathered into one place the people that were wandring about the fieldes, and lay lurking in wilde cotages and woods: And bringing them into one common ſocietie, did trade them to all ſuch things, as either were profitable or honeſt, although not without repyning at ẏ firſt, by reaſon that they had not beene ſo accuſtomed before. Yet at length through reaſon and perſuaſion of wordes they obeyed him more diligently, and ſo of a wilde and cruell people, he made them curteous and gentle.

Thus hath Tullie ſet forth the efficacie of reaſon and perſuaſion, howe it was able to conuert wilde people to a mildeneſſe, and to change their furious crueleneſſe into gentle nature. Were it not nowe a great reproch in this our time (when knowledge raigneth ſo large) that men ſhould ſhewe themſelues leſſe obſequious to reaſon? vnleſſe it may be thought, that now euery man hauing ſufficient knowledge of himſelfe, needeth not to hearken to the perſuaſion of others.

Indeede he that thinketh himſelf wiſe, will not eſteeme the reaſon of any other, be he neuer ſo wiſe, ſo that of ſuch one it may well be ſaide, He that thinketh himſelfe wiſer than he is, may iuſtly be counted a double foole: wherefore ſuch men are not to bee permitted in open audience to talke, but muſt be put to ſilence, and made to giue eare to reaſon, which reaſon conſiſteth not in a multitude of wordes, heaped raſhly together, and applyed for one purpoſe, but reaſon is the expreſſing of a iuſt matter with wittie perſuaſions, furniſhed with learned knowledge. Such knowledge had Moſes being expert in all learning of the Ægyptians, as the Scriptures declare, and therefore was able to perſuade the ſtubborne people of the

Iewes, although not without great paine. Such knowledge and ſuch reaſons did Druys ſhewe, which was the firſt lawe maker of all the Weſt partes of Europe. Like reaſon and wiſedome did Xamolxis vſe amongeſt the Gothes: Lycurgus vnto the Lacedemonians, Zeleucus to the Locrians, Solon to the Athenienſes, and Donwallo Molmitius two thouſande yeares paſt, amongſt the olde Britanes of this realme. And therby ti came to paſſe, that their lawes continued long, till more perfite reaſon altered many of them, and wilfull power oppreſſed moſte of them.

Druis was ſonne to K. Sarron, & ſucceeded him in his kingdome.

At the beginning when theſe wiſe men perceiued howe harde it was to bring the rude people to vnderſtande reaſon, they iudged the beſt meanes to attaine this honeſt purpoſe to depende of learning in euerie kind, for by learning, as Ouid ſaith: *Pectora molleſcunt, aſperitaſq́ fugit:* Stoute ſtomackes doe waxe milde, and ſharpe fierceneſſe is exiled. Therefore as Beroſus doth teſtifie, Sarron that was the thirde King ouer all this Weſt parte of Europe, for to bring the people from beaſtly rage to manly reaſon, did erect Schooles of liberall Artes, which tooke ſo good ſucceſſe, that his name continued in that ſort famous aboue two thouſand yeares after: for Diodorus Siculus, which was in the time of Iulius Cæſar maketh mẽtion of the learned men of the Gothes, and named them Sarronides, that is to ſay, Sarron his Scholers and followers.

Among theſe Artes that then were taught, ſome did informe the tongue, and make men able both to vtter aptly their minde, and alſo to perſuade, as Grammer, Logike, and Rhethorike, although not ſo curiouſly as in this time: ſome other did appertaine to the iuſt order of partition of Landes, the true vſing of waights, meaſures, and reckenings in all ſorts of bargaines, and for order of building and ſundrie other vſes, thoſe were Arithmetike and Geometrie. Againe, to incourage men to the honour of God, they taught Aſtronomie, whereby the wonderfull workes of God were ſo manifeſtly ſet forth, that no mans tongue nor pen can in like ſort expreſſe his infinite power, his vnſpeakable wiſedome, and his exceeding goodneſſe toward man, wherby he doth bountifully prouide for man all neceſſaries, not onely to liue, but alſo to liue pleaſantly. And ſo was their confidence in Gods prouidence ſtrongly ſtayed, knowing his goodneſſe to be ſuch, that he woulde helpe man as he coulde, and his power to bee ſo great, that he could doe what he woulde: and thirdly his wiſedome to be ſo pure, that he would doe nothing, but that was beſt. Beſide theſe Sciences they taught alſo Muſike, which moſt commonly they did apply partely to religious Sciences, to drawe men to delight therein, & partly to ſongs made of the maners of men in praiſe of Vertue, and diſcommendation of Vice, whereby it came to paſſe, that no man would diſpleaſe them, nor do any thing euill that might come to their hearing: for their Songs did make euill men more abhorred in that time, than any excommunication doeth in this time. The poſteritie of theſe Muſitians continue yet both in Wales and Ireland called Bardes vnto this day, by the auncient name of Bardus, their firſt founder.

This Bardus Druidius the v. king of the Celtes, raigned 69. yeres, and died 1832. yeares before Chriſt.

And as theſe Sciences did encreaſe, ſo did vertue increaſe thereby. Againe, as theſe ſciences did decaye, ſo vertue loſt hir eſtimation, and conſequently was little in vſe: whereof to make a ful declaration, were a thing meet for a Prince to heare, but it woulde require a peculiar treatiſe. Wherefore at this preſent I count it ſufficient lightly to haue touched this matter in generall wordes, and to ſay no more of the particularitie thereof, but onely touching one of thoſe ſciences, that is, Arithmetike, by which not onely iuſt partition of landes was made, but alſo touching buying an ſelling, all aſſiſes, weights, and meaſures were deuiſed, and all reckenings and accounts driuen: yea by proportion of it, were the true orders of Iuſtice limited, as (Ariſtotle in his Ethikes doth declare) & the degrees of eſtates in the common wealth eſtabliſhed. Although that proportion be called Geometricall, and not Arithmeticall, yet doth that proportion appertaine to the Art of Arithmetike: & in Arithmetike is taught the Progreſſion of ſuch proportions, and all things thereto belonging. Wherefore I may well ſay, that ſeeing Arithmetike is ſo manv wayes needefull vnto the firſt planting of a common wealth, it muſt needes be as much required to the preſeruation of it alſo: for by the ſame meanes is any common wealth continued, by which it was erected and eſtabliſhed. And if I ſhall in ſmal matters in appearance, but in deede verie waightie, put one example or two, what ſhall we ſay for the ſtatutes of this realme, which be the onely ſtaie of good order in maner nowe? As touching the meſuring of ground by length and bredth, there is a good and an auncient ſtatute made by Arte of Arithmetike, and now it ſhalbe to little vſe, if by the ſame Art it be not practiſed and tried. For the aſſyſe of breade and drinke, the two moſt common and moſt neceſſarie thinges for the ſuſtentation of man, there was a goodly ordinance in the Lawe made, which by ignorance hath ſo growen out of knowledge and vſe, that fewe men doe vnderſtand it, and therefore the ſtatute bookes wonderfully corrupted, & the commons cruelly oppreſſed: notwithſtanding ſome mē haue written, that it is too doubtful a matter to execute thoſe Aſſiſes by thoſe ſtatutes, by reaſon they depend of the ſtanderd of the coine, whiche is muche chaunged from the ſtate of that time when thoſe ſtatutes were made. Thus ſhall euerie man reade that liſteth in the Abridgement of Statutes in the title of Waightes and Meaſures, in the ſeauenth number of the Engliſhe Booke, where hee ſhoulde haue tranſlated a good ordinance, which is ſet forth in the French booke: but no maruaile if the Abridgement doth omit it, ſeeing the great booke of Statutes doth omitte the ſame Statute, as it hath done diuers other verie good lawes. And this is the fruite of ignorance, to reiect all that it vnderſtandeth not, although they vſe ſome clokes for it: but ſuch clokes, as being allowed, might ſerue to repell all good lawes, which God forbid.

Againe there is an auncient order for aſſiſe of fyre wood, and coales, which was renued not many yeares paſt, and nowe howe Auarice and Ignorance doth canuas the ſtatute, it is too pitifull to talke of, and more miſerable to feele.

Furthermore, for the Statute of Coinage, and the ſtanderd thereof, if the people vnderſtoode rightly the ſtatute, they ſhoulde not, nor woulde not (as they often doe) gather an excuſe for their follie therby, but as I ſaid, theſe ſtatutes by wiſedome and good knowledge of Arithmetike were made, & by the ſame muſt they be continued. And let Ignorance no more meddle with the vſe of them, than it did with the making of them. Oh in how miſerable caſe is that Realme, where the Miniſters and interpreters of the Lawes are deſtitute of all good ſciences, which be the keyes of the laws? How can they either make good Laws, or maintaine them, that lack that true knowledge wherby to iudge them? And happie may that realme be accounted where the Prince himſelfe is ſtudious of learning, and deſireth to vnderſtande equitie in all lawes. Therefore moſt happy are we the louing ſubiectes of your Maieſtie which may ſee in your highneſſe not onely ſuch towardneſſe, but alſo ſuch knowledge of diuers Artes as ſeldome hath bene ſeene in any Prince of ſuch yeares, whereby we are inforced to conceiue this hope: Certainly, that he which in thoſe yeares ſeeketh knowledge when knowledge is leaſt eſteemed, and of ſuch an age can diſcerne them, to be enimes both to his royall perſon, and to his realme, which labour to withdrawe him from knowledge to exceſſiue paſtime, and from reaſonable ſtudie to ydle or noyſome pleaſures, he muſt needes, when he commeth to more mature yeares, be a moſt prudent Prince, a moſt iuſt Gouernour, and a right Iudge, not onely of his ſubiectes commonly, but alſo of the miniſters of his lawes, yea, and of the Lawes themſelues. And to be able to conceiue the true equitie and exact vnderſtanding of al his Lawes and Statutes, to the comfort of his good ſubiectes, and the confuſion and reproch of them which labour to obſcure or peruerte the equitie of the ſame lawes and ſtatntes. Howe ſome of thoſe ſtatutes may bee applied to vſe, as well in our time, as in any other time, I haue particularly declared in this Booke, & ſome other I haue omitted for iuſt conſiderations, till I may offer them firſt vnto your Maieſtie, to weigh them, as to your highneſſe ſhall ſeeme good: for many things in them are not to be publiſhed without your highneſſe knowledge and approbation, namely becauſe in them is declared all the rates of alloyes for all ſtanderds from one ounce vpwarde, with other myſteries of Mynte matters, and alſo moſt part of the varieties of coynes, that haue beene currant in this your Maieſties realme by the ſpace of ſixe hundred yeares laſt paſt, and many of them that were currant in the time that the Romaines ruled here.

All which, with the auncient deſcription of Englande and Irelande and my ſimple cenſure of the ſame, I haue almoſt compleated to be exhibited to your Highneſſe.

In the meane ſeaſon moſt humbly beſeeching your Maieſtie,

ieſtie, to accept this ſimple treatiſe, not worthy to be preſented to ſo high a Prince, but that my lowly requeſt to your maieſtie is, that this amongſt other of my bookes may paſſe vnder the protection of your highneſſe, whom I beſeeche God moſt earneſtly and dayly, according to my duetie, to aduaunce in all honour and princely regalitie, and to increaſe in all knowledge, iuſtice, and godly policie.

AMEN.

Your Maieſties moſt obedient ſubiect
and ſeruant, ROBERT
RECORDE.

TO THE LOVING Reader.

The Preface of Maiſter Ro: Record.

SOre oftetimes haue I lamented with my ſelfe the infortunate cōdition of *England*, ſeeing ſo many great Clerks to ariſe in ſundrie other partes of the world, and ſo few to appeare in this our nation: whereas for pregnancie of naturall witte (I thinke) few nations doe excell Engliſh men: But I cannot impute the cauſe to any other thing than to the contēpt or miſſeregard of learning. For as Engliſhmen are inferior to no men in mother witte, ſo they paſſe all men in vaine pleaſures, to which they may attaine with great paine or labour: and are as ſlacke to any, neuer ſo great commoditie, if there hang of it any paineful ſtudie or trauelſome labour.

Howbeit, yet all men are not of that ſort, though the moſt part be, the more pitie it is: but of them that are ſo glad, not onlie with paineful ſtudie and ſtudious paine to attaine learning, but alſo with as great ſtudie & paine to communicate their learning to others, and make all England, if it might be, partakers of the ſame, the moſte part are ſuche, that vnneth they can ſupport their owne neceſſarie charges, ſo that they are not able to beare any charges in doing of that good, that elſe they deſire to do.

But a greater cauſe of lamentation is this: that when learned men haue taken paines to do things for the aide of the vnlearned, ſcarce they ſhall be allowed for their wel doing, but derided and ſcorned, and ſo vtterlye diſcouraged to take in hand any like enterprice againe. So that if any be founde (as there are ſome) that do fauour learning and learned wittes, and can bee content to further knowledge, yea only with their word, ſuch perſons, though they bee rare, yet ſhall they encourage learned men to enterpriſe ſome things, at the leaſt, that Englād may reioice of. And I haue good hope that England wil (after ſhe hath taken ſome ſure taſte of learning) not only bring forth more fauorers of it, but alſo ſuch learned men, that ſhe ſhall bee able to compare with anie realme in the world. But in the meane ſeaſon, where ſo few regarders of learning are, how greatly they are to be eſteemed that doe fauour and further it, my penne wyll not ſuffice at ful to declare.

Therefore, gentle reader, where as I doe vppon moſte iuſt occaſion iudge, yea and know aſſuredlie, that there be ſome men in this realme, which both loue, and alſo much deſire to further good learning, and yet am not well able to write their condigne praiſe for the ſame, I thinke it better with ſilence to ouerpaſſe it, then eyther to ſay too little of it, or to prouoke againſte them the malice of ſuche other, which do nothing themſelues that is praiſe worthie, and therfore can not abide to heare the praiſe of any other mans good deede.

And conſidering their greate fauour vnto learning, though I my ſelfe be not worthie to be reckoned in the number of great learned men, yet am I bolde to put my ſelfe in preace with ſuch abilitie, as God hath lent mee, thoughe not with ſo greate cunning as manye men,yet with as great affection as any man,to help my countrymen,and wil not ceaſe dayly,(as much as my ſmall abilitie wil ſuffer me)to endite ſome ſuch thing, that ſhall be to the inſtruction,though not of learned men. yet at the leaſt of the vulgare ſort, whoſe argument alwayes ſhall be ſuch,that it ſhall delight al learned wits, though they do not learne any great things out of it.

But to ſpeake of this preſent Booke of Arithmetick, I dare not nor wil not ſet it foorth with any words, but remit it to the iudgement of all gentle readers,& namelie ſuch as loue good learning, beſeeching them ſo to eſteeme it,as it doth ſeeme worthie. And ſo either to accept the thing for it ſelfe, either at the leaſte to allowe my good endeauour.But I perceiue I neede not vſe anye perſuaſions vnto them, whoſe gentle nature and fauourable minde is readie to receiue thankefullye, and interpreate to the beſt, of al ſuche enterprices attempted for ſo good an ende,though the thing do not alwayes ſatisfie mens expectation.

This conſidered, did bolden me to publiſh abroade this little Booke of the Arte of numbring,which if you ſhal receiue fauourablie, you ſhal encourage me to gratifie you hereafter with ſome greater thing.

And as I iudge ſome menne of ſo louing a minde to their natiue countrey that they woulde much reioyce to ſee it to proſper in good learning and wittie Artes, ſo I hope well of all the reſt of Engliſhmen, that they wyl not be vnmindeful of his due praiſe, by whoſe meanes they are helped and furthered in anye thing. Neither ought to eſteeme this thing of ſo little value, as manye men of little diſcretion oftentimes do: For who ſo ſetteth ſmall price by the wittie deuiſe and knowlege of numbring, he little conſidereth it to be the chiefe point (in manner)wherby men differ from all bruite beaſtes:

for as in al other things , (almoſt) beaſtes are partakers with vs, ſo in numbring we differ cleane from them, and in manner peculiarlie, ſith that in manie things they excel vs againe.

The Fox in craftie witte exceedeth moſt men,
A dogge in ſmelling hath no man his peere,
To foreſight of weather if you looke then,
Many beaſtes excel man, this is cleere.
The wittineſſe of Elephants doth letters attaine,
But what cunning doth there in the Bee remaine?
The Emmet foreſeeing the hardeneſſe of winter,
Prouideth vitailes in the time of Sommer.
The Nightingale, the Linet, the Thruſh, the Larke,
In Muſical harmonie paſſe manie a Clarke.
The Hedgehog of Aſtronomie ſeemeth to knowe
And ſtoppeth his caue, where the wind doth blowe
The Spider in weauing ſuch art doth ſhow,
No man can him mende, nor follow I trow.
When a houſe wil fall, the Mice right quicke,
Flee thence before, can man do the like?

Many things elſe of the wittineſſe of beaſts & byrdes might I heere ſaye, ſaue that another time I entende to write wherein they excel in manner all men, as it is daylie ſeene: but in number was there neuer beaſt found ſo cunning, that coulde know or diſcerne one thing from manye, as by daylie experience you may well conſider, when a Bitch hath manie whelpes, or a Hen many Chickens: and likewiſe of other whatſoeuer they be, take frō them al their yong, ſauing onlie one, and you ſhall perceiue plainly, that they miſſe none, though they wil reſiſt you in taking them away, and wil ſeeke them again if they may know where they bee, but elſe they wil neuer miſſe hem truelye, but take awaie that one that is left, and then wil they crie and complaine: and reſtore to them that one, then are they pleaſed againe : ſo that of nũber this may I iuſtlie ſay, It is the only thing (almoſt) that ſeparateth man from beaſtes. He therefore that ſhal contemne number, he declareth himſelfe as brutiſh as a beaſt, and vnworthy to be counted in felowſhip of men. But I truſt there is no man ſo foule ouerſeene, thoughe manie right ſmallie do it regarde.

Therefore wil I now ſtay to write againſt ſuche, and returne againe to this booke, whiche I haue written in the forme of a Dialogue, bicauſe I iudge that to bee the eaſieſt way of inſtruction, when the Scholer may aske euerie doubt orderlie, and the maiſter may anſwer to his queſtion plainelie.

Howbeit I thinke not the contrary, but as it is eaſier to blame an other mans worke than to make the lyke, ſo there wil be ſome that wil finde fault, bicauſe I write in a Dialogue: but as I coniecture, thoſe ſhal be ſuche, as doe not, cannot, either will not perceiue the reaſon of right teaching, and therefore are vnmeete to be aunſwered vnto, for ſuch men with no reaſon wil be ſatiſfied.

And if any man obiect that other bookes haue bene written of Arithmetike alreadie ſo ſufficientlie, that I needed not now to put pen to the booke, except I will condemn other mens writings: to them I aunſwer. That as I condemne no mans diligence, ſo I know that no one mã can ſatiſfie euerie man, and therefore like as manye do eſteme greatly other Bookes, ſo I doubt not but ſome wil like this my Booke aboue any other Engliſh Arithmetike hitherto wrtiten, and namely ſuch as ſhall lacke inſtructers, for whoſe ſake I haue ſo plainely ſet foorthe the examples, as no Booke (that I haue ſeene) hath done hitherto : whyche thing ſhall bee greate eaſe to the rude readers.

Therefore gentle reader, though this booke can bee ſmall aide to the learned ſorte, yet vnto the ſimple ignorant(which needeth moſt help)it may be a good furtherance and meane vnto knowledge. And though vnto the King his Maieſtie priuatelye I doe it dedicate, yet I doubt not(ſuche is his clemencie) but that hee can bee content, yea and much deſirous, that all his louing ſubiects ſhal take the vſe of it, and employ the ſame to their moſt profit: Which thing if I perceiue that they thankefullie do, and receiue with as good will as it was written, then wil I ſhortly with no leſſe kindneſſe ſet forth ſuche introductions in to Geometry and Coſmography, as I haue at other times promiſed, and as hitherto in Engliſh hath not bene enterpriſed, wherwith I dare ſay al honeſt heartes wil be pleaſed, and all ſtudious wittes greatlye delighted.

I wil ſay no more, but let euerie man iudge as he ſhal ſee cauſe. And thus for this time I will ſtaye my penne, committing you all to that true fountaine of perfect number, which wrought the whole world by number and meaſure: he is Trinitie in Vnitie, and Vnitie in Trinitie: To whom be all praiſe, honor and glorie.

AMEN.

Here foloweth a Table of al the Contents of this Booke.

The Contents of the firſte Dialogue containeth the Declaration of the profite of Arithmetike.

Numeration with an eaſie & large Table.
Addition.
Subtraction.
Multiplicatiõ.
Diuiſion.
with diuers Exãples, and all their proofs, and ſome newe formes of workings, &c.
Reduction, with diuers declarations of Coines Waights and Meaſures of ſundrie formes newly added, with a newe Table, containing moſt part of the golde Coines throughout Chriſtendome, with the true waight and valuation of them in currant money Engliſh, &c.

Progreſſion both Arithmeticall and Geometricall, with diuers ſundrie queſtions touching the ſame.

The Golden Rule of thrée : and the Backer Rule of thrée: with diuers queſtions thereunto belonging, newly added & augmẽted.

The double Rule of Proportion.

The Rule of thrée compoſed of 5 numbers.

The Rule of Felowſhip, both with time, and without time.

Unto all theſe are added their proofes.

THE CONTENTS.

The ſecond Dialogue containeth

The firſt 5 kindes of Arithmetike wroughte by Counters.
The common kindes of caſting of accomptes after the Merchants faſhiō, & Auditors alſo.
Numbring by the hande newly added.

The Contents of the ſecond parte, touching Fractions.

What a Fraction is.
Numeration in Fractiōs.
The order of working fractions.
Multiplication.
Diuiſion.
with diuers familiar queſtiōs proponed for ỹ perfit vnderſtanding, & proof of ech of thē.

Reduction of
- diuers fractions into one denomination in 3 varieties.
- Fractions of Fractions.
- Improper Fractions.
- Fractions to the ſmalleſt denomination, with eaſie rules how to conuert thē therunto.
- Fractions in other partes of things, with a Table demōſtratiue of their proportiōs.
- Fraction, and how it may bée turned into any other Fraction, or into what Denomination you liſte.

Againe of
- Multiplication
- Duplation
- Diuiſion
- Mediation
- Addition
- Subtraction.

The Golden Rule with diuers queſtions, and their proofes.
The Backer Rule.
A queſtion of Loane.
The ſtatute of Aſſiſe of Breade and Ale recogniſed and applied to this time, with newe tables therevnto annexed.
The Statute of Meaſuring of ground, with a table thereof faithfully calculated and corrected.
Queſtions of Societie, with the reaſon of the Rules and proofes of their workes.
To finde thrée numbers in any proportion.
The Rule of Alligation, with diuers queſtions and the proofes of their workes, with many varieties of ſuch ſolutions.
The rule of Falſhode, or falſe Poſition, with diuers queſtions, and their proofes.

The Contents of the third Addition to this Booke.

The firſt Chapter entreteth of Rules of Breuity and Practiſe, after a briefer Methode than euer yet was publiſhed in the Engliſh tong.

The ſecond Chapter treateth of yͤ briefer Reduction of diuers Meaſures, as Elles, Yeards, Braces, &c. by Rules of Practiſe.

The third Chapter entreateth of the Rule of thrée in Broken numbers after the trade of Merchaunts, ſomthing differing from Maſter Records order, which is comprehended in 3 Rules.

The fourth Chapiter entreateth of Loſſe and Gaine in the trade of Merchandize.

The fifth Chapter entreateth of Loſſe & Gain in the trade of Merchandize vppon time, &c. with neceſſarie queſtions therein wrought by the double Rule of thrée, or the Rule of 3 compoſed.

The ſixth Chapter entreateth of Rules of payment, and of the neceſſarieſt Rules that appertaineth to buying and ſelling, &c.

The ſeauenth Chapter entreateth of Buying and Selling in the Trade of Merchandize, wherein is taken part ready mony, and diuers dayes of payments giuen for the reſt, and what is won or loſte in the 100 ℔ forbearance for 12 moneths, &c.

The eight Chapter entreateth of Tares and alowances in the trade of Merchandize ſold by waight, and of their Loſſes and Gaines therein, &c.

The ninth Chapter entreteth of Lengths and Breadths of Arras, and other Clothes, with diuers queſtions incident therevnto.

The tenth Chapter entreateth of reducing of Pawnes of *Geanes* into Engliſh yeards.

The eleauenth Chapter entreateth of Rules of Loane and Intereſt with diuers queſtions incident therevnto.

The twelfth Chapiter entreateth of the making of Factors.

The thirteenth Chapiter entreateth of Rules of Barter or Exchange of Merchandize, wherein is taken parte ware, & part readie money with their proofes, and diuers other neceſſarie queſtions therevnto belonging.

The fourteenth Chapter entreateth of exchanging of mony from one place to an other, with diuers neceſſarie queſtions incident there-

therevnto.

The fiftéeth Chapter entreateth of ſixe ſundrie formes of practiſe for the Reduction of Engliſh, Flemiſh, and French money, and howe eche of them may eaſily be broughte to others money ſterling.

The ſixtéenth Chapter containeth a brief note of the ordinarie Coines of moſte places of Chriſtendome for traffique, and the manner of their exchaunging from one Citie or towne to an other, which knowen the Italians call Pary: whereby they finde the gaine or loſſe vpon the Exchange.

The ſeauentéenth Chapiter containeth alſo a Declaration of yͤ diuerſitie of the waights and meaſures of moſte places of Chriſtendome for traffique, at the ende wherof are two Tables, the one for waight, and the other for meaſure, proportionated to an equalitie vnto our Engliſhe meaſure and waight, wherby the ingenious practitioner may eaſily reduce the waight and meaſure of eche Countrey into other.

The eightéenth Chapiter entreateth of diuers Sportes and Paſtimes, done by Number.

FINIS.

A Collection of ſuche Tables as are contained in this Treatiſe.

A large Table of Numeration.

A Table of Multiplication.

A Table of Diuiſion.

A Table of the money currant in this Realm when the Author firſt publiſhed this booke.

A Table of all the vſuall ſiluer Coines nowe currant in this Realm, newly added.

A Table of all the golde Coines in this realm with all the moſt vſuall Golde Coines thoroughout Chriſtendome, with their ſeuerall waights of Pence and Graines, and what they are worth in currant mony Engliſhe.

Certaine Tables or Notes of the contentes of Ale, Beere, Wine, Butter, Sope, Salmō, Eeles, &c. both what ſuche veſſels ought to containe by the Statute, and what thoſe veſſelles emptie ought to wey.

A Table of the quantitie of drie meaſures, as Peckes, Buſhels, Quarters, Weyes, &c.

A Table of the proportion of meaſure, touching Lengths or breadths: to wit, from the inche to the foote, and ſo to the yeard, the Ell,

Ell with their partes : the perch, the rōd, the furlong, the myle, &c.

A Table made by Progreſſion Arithmetical, whyche contayneth a double table of Multiplication.

A Table of the Arte of Numbring by the hande.

A Table or demonſtration of a figure or meaſure for the perfect vnderſtanding of Fractions of Fractions.

A Table of the contents of the Statute for the aſſiſe of the waight of bread. From 1 ß the quarter to 20 ß faithfully corrected, and amended.

A neceſſarie Table of the Statute of meaſuring of grounde, vpon the breadth giuen, what length it ought to containe: faithfully corrected according to the equitie of the ſtatute: wherein the Author declareth how neceſſarie this worthie Art of Arithmetike is vnto Gentlemen Students of the lawe, and ſuche other as are deſirous of infallible trueth.

Briefe Tables of the ready reducing of Engliſhe, French, and Flemiſh money, eache into others common currant monies.

A briefe Table or collection of the common &

vſuall monies of moſte places of Chriſtendome for traffique, the maner of their paymentes or exchaunging from one Citie or towne, to an other : right neceſſarie for Merchants, and other Occupiers, Traueluellers, &c.

Tables of the Waightes, Meaſures, and Cuſtomes of moſte places of *Europe* for traffique.

2 Tables, the one for waight, the other for meaſure, reduced to an equalitie, vnto our Meaſures and waight here at *London*, with the helpe of whiche 2 Tables, and the aide of the rule of 3, the ingenious may eaſilye reduce our meaſure to the perfect valuation of other Countries meaſure or waight, and likewiſe theirs to ours.

Laſtly, a Table demonſtrating the true ſolution of thrée diuers things hidden of 3 ſeuerall perſons in paſtime.

A dialogue betwene THE MAYSTER and the Scholer, teaching the Arte and vſe of Arithmetike with penne.

THE SCHOLER SPEAKETH.

SYR, SVCH is your authoritie in myne eſtimation, yͭ I am content to conſent to your ſaying, and to receyue it as truthe, thoughe I ſee none other reaſon that doth leade me therevnto: wheras elſe in mine owne conceite it appeareth but vaine, to beſtowe any time priuately in learning of that thing, that euery childe maye and doeth learne at all times and houres, when he doeth anye thyng hymſelfe alone, and

C. much

much moꝛe when he talketh oꝛ reaſoneth wͭ other.

Maiſter. Lo this is the faſhion and chance of all them that ſéeke to defende their blynde ignoꝛaunce: that when they thinke they haue made ſtrong reaſon foꝛ themſelfe, than haue they pꝛoued quite cõtrary. Foꝛ if nũbꝛing be ſo cõmon (as you grant it to be) that no man can do any thing alone, and much leſſe talke oꝛ bargayne with other, but he ſhall ſtil haue to do with Number: this pꝛoueth not Number to be contemptible and vile, but rather right excellent and of high reputation, ſith it is the grounde of all mens affayꝛes, ſo that without it no tale can be tolde, no communication wythout it can be long continued, no bargayning without it can duely bée ended, oꝛ no buſyneſſe that man hathe, iuſtlye completed. Theſe commodities (if there wer none other) are ſufficiente to appꝛoue the woꝛthyneſſe of Number. But there are other vnnumerable farre paſſing all theſe, whyche declare Number to excéede al pꝛaiſe. Wherefoꝛe, in all great woꝛkes are Clerkes ſo muche deſired? Wherefoꝛe are Auditours ſo richely fedde? What cauſeth Geometrians ſo highlye to be enhaunced? Why are Aſtronomers ſo greately aduaunced? Bycauſe that by Number ſuche things they doe finde, whiche elſe ſhoulde far excel mans minde.

Scholer. Verilye Syr if it be ſo, that theſe men, by numbring their cunning do attaine, at whoſe great workes moſt men doe wonder, then I ſee well I was much deceyued, and numbring is a more cunning thing then I toke it to be.

Maiſter. If Number were ſo vile a thing as you did eſtéeme it, then néede it not to bée vſed ſo muche in mennes communication. Exclude Number and aunſweare to thys queſtion. Howe manye yeares olde are you?

Scholer. Mum.

Maiſter. Howe manye dayes in a wéeke? howe many wéekes in a yeare? What lands hath your father? Howe manye men doth hée kéepe? How long is it ſith you came frõ hym to me?

Scholer. Mum.

Maiſter. So that if Number want, you aunſweare all by Mummes: How many myle to London?

Schole. A poke ful of Plummes.

Maiſter. Why, thus you may ſée, what rule Number beareth, and that if Number be lacking, it maketh men dumbe, ſo that to moſte queſtions, they muſte aunſwere Mum.

Scholer. This is the cauſe ſir, that I iudged it ſo vile, bycauſe it is ſo common in talking euery while: For plentie is not deintie, as the common ſaying is.

Maiſter. No, nor Store is no ſore: perceiue you this? The more common that a thing is, being néedefully required, the better is the thyng, and the more to be deſired. But in Numbring as ſome of it is light and playn, ſo the moſt parte is difficulte, and not eaſye to attaine. The eaſyer parte ſerueth al men in commune, and the other parte requireth ſome learning. Wherefore as without Numbring a man can doe almoſte nothing, ſo wyth the helpe of it, you maye attaine to all things.

Sc. Yea ſir? Why? then it wer beſt to learn the Art of Numbring firſte of all other learning, and then a man néede learn no more, if all other come with it.

Maiſter. Nay not ſo: but if it be firſte learned, then ſhall a manne be able (I meane) to learne, perceyue, and attayne to other ſciences, whiche without it, he ſhoulde neuer get.

Scholer. I perceyue by your former wordes, that Aſtronomie and Geometrie depende muche of the helpe of Numbring, but that other Sciences, as Muſicke, Phiſicke, Lawe, and Grammer and ſuch like, haue any helpe of Arithmetike, I perceyue not.

Maiſter. I maye perceyue youre greate Clerkelineſſe by the ordering of your Sciences: but I wil let that paſſe now, bycauſe it toucheth not the matter that I entend, and I wyll ſhewe you howe Arithmetike doeth profit in all theſe, ſomewhat groſſely, according to your ſmall vnderſtanding, omitting other reaſons more ſubſtantial.

Fyrſte (as you recken them) Muſicke hath not only greate healpe of Arithmetike, but is made, and hath his perfectneſſe of it: for all Muſicke ſtandeth by Number & Proportion.

And in Phiſicke, beſide the calculation of Criticall dayes, with other thinges whiche I omitte, howe can anye manne iudge the Pulſe rightlye, that is ignoraunte of the proportion of Numbers?

And as for the Lawe, it is playne, that the man that is ignorant of Arithmetike, is neither meete to be a Iudge, neyther an Aduocate, nor yet a Proctour. For howe can hee wel vnderſtande another mannes cauſe appertayning to diſtribution of goods, or other dettes, or of ſummes of money, if he bee ignorante of Arithmetike? This oftentimes cauſeth righte to be hindered, when the Iudge either deliteth not to heare of a matter that he peceyueth not, or cannot Iudge it for lacke of vnderſtanding: This commeth by the ignoraunce of Arithmetike.

Nowe as for Grammer, mee thinketh you ſhoulde not doubte in what it needeth number, ſith you haue learned that Nounes of al ſortes, Pronouns, Verbs, & Participles, are diſtinct diuerſly by Numbers: beſides the varietie of Nounes of Numbre, and Aduerbs. And if you take away Number from Grammer, then is all the quantitie of Syllables loſte. And manye other wayes dothe Number helpe Grammer. Whereby were al kindes of meters found and made? Was it not by Number?

But howe néedefull Arithmetike is to all parts of Philoſophie, they may ſoone ſée, that doe reade eyther Ariſtotle, Plato, or anye other Philoſophers writing. For all theyr examples almoſte, and their probations, depende of Arithmetike. It is the ſaying of Ariſtotle, that he that is ignorante of Arithmetike is méete for no ſcience. And Plato his Mayſter wrote a like ſentence ouer hys Schoolehouſe dore. Let none enter in hyther (quoth he) that is ignorant of Geometrie. Séeyng he woulde haue al his Schollers experte in Geometrie, much rather he woulde the ſame in Arithmetike, wythoute whyche Geometrie cannot ſtande.

And howe néedefull Arithmetike is to Diuinitie, it appeareth, ſéeing ſo many Doctors gather ſo greate myſteries out of Number, and ſo muche doe write of it. And if I ſhould goe about to write all the commodities of Arithmetike in ciuil actes, as in gouernaunce of common weales in time of peace, and in due prouiſion and order of armyes in time of warre: For numbring of the hoſte, ſumming of their wages, prouiſions of victualles, vewing of Artillerie, wyth other armoure;

Beſide the cunningeſt pointe of all, for caſting of ground, for encamping of men, with ſuch other like. And howe manye wayes alſo Arithmetike is conducible for all priuate weales, of Lordes and all poſſeſſioners, of marchauntes, and all other occupyers, and generallye, for all eſtates of men, beſides Auditours, treaſorers, receyuers, ſtewards, bayliffes, and ſuche lyke, whoſe offices without Arithmetike is nothing. If I ſhould (I ſaye) particularlye repeate all ſuche commodities of this noble Science of Arithmetike, it were ynoughe to make a verye great booke.

Scholer. No, no ſir, you ſhall not neede: For I doubt not, but this that you haue ſaid, were ynoughe to perſwade anye manne to thinke this Arte to be righte excellent and good, and ſo neceſſarie for man, that (as I thinke nowe) ſo muche as a man lacketh of it, ſo muche he lacketh of his ſenſe and witte.

Maiſter. What? are you ſo farre chaunged ſince, by hearing the fewe commodities in generall? By likelihode you woulde be far chaunged, if you knew all the commodities particular.

Scholer. I beſéeche you ſir, reſerue thoſe commodities that reſt yet behynd, vnto their place more conuenient. And if ye will be ſo good as to vtter at thys tyme thys excellente treaſure, ſo that I maye bée ſomewhat enriched thereby, and if euer I ſhall be able, I wil requite your paine.

Mayſter, I am very glad of your requeſte, and I will doe it ſpéedely, ſith that to learne it you be ſo ready.

Scholer. And I to youre aucthoritie my witte doe ſubdue, whatſoeuer you ſay, I take it for true.

Mayſter. That is too muche, and méete for no man to be beléeued in all things, without ſhewing of reaſon. Thoughe I mighte of my Scholler ſome credence require, yet except I ſhew reaſon, I do it not deſire. But now ſith you are ſo earneſtly ſet this Arte to attaine, beſte it is to omitte no time, leaſt ſome other paſſion, coole this greate heate, and then you leaue off before you ſée the ende.

Scholer. Thoughe manye there be ſo vnconſtant of mind, that flitter and turne with euerye winde, whyche often beginne, and neuer come to the ende, I am none of their ſorte, as I truſte you partelye knowe.

For by my good will what I ons beginne till I haue it fully ended I woulde neuer blin

Mayſter. So haue I Founde you hytherto in deede and I truſte you will increaſe rather than go backe. For better it were neuer to aſſaye, than to ſhrinke and flée in the middle way. But I truſt you wil not ſo do, therefore tel me briefly. What call you the ſcience that you deſiere ſo greatly?

Scholer Why ſir? you knowe.

Mayſter. That maketh no matter. I woulde heare whether you knowe, and therfore I aſke you. For, greate rebuke it were to haue ſtudied a Science, and yet cannot tel howe it is named,

Scholer. Some call it Arſmetrik, and ſome Augrime.

Maiſter. And what dothe thoſe names betoken?

Scholer. That if it pleaſe you, of you wold I learne.

byne name
[illegible]

Maiſter. Bothe names are corruptly written, Arſmetrike for Arithmetik, as the Gréekes call it, & Augrim for Algoriſme, as the Arabians found it, whiche bothe betoken the ſcience of Numbring. For Arithmos in

Gréeke

Greeke, is called number: and of it commeth Arithmetike, the Art of Numbring. So that Arithmetike is a ſcience or arte teaching the maner and vſe of Numbring. This Art may be wrought diuerſly, with Penne or wyth Counters. But I wil firſt ſhew you the working wyth the Penne, and then the other in order.

Scholer. This I wil remember. But how many thinges are to be learned, to attayne this Arte fullye?

Maiſter. There are reckened commonlye ſeauen partes or workes of it.

Numeration, Addition, Subtraction, Multiplication Diuiſion, Progreſſion, and extraction of rootes: to theſe ſome men adde Duplation, Triplation, and Mediation. But as for theſe laſte three, they are contained vnder the other ſeauen. For Duplation and Triplation, are contained vnder Multiplication, as it ſhall appeare in their place. And Mediation is contained vnder Diuiſion, as I wyl declare in his place alſo.

Scholer. Yet then there remaine the firſte ſeauen kindes of Numbring.

Maiſter. So there dothe: Howbeit, if I ſhall ſpeake exactelye of partes of Numbring,

bring I muſte make but fiue of them: For Progreſſion is a compound operation of Addition, Multiplication and Diuiſion. And ſo is the extraction of roots. But it is no harme to name them as kindes ſeuerall, ſéeing they appéere to haue ſome ſeueral workig. For it forceth not ſo much to contēd for the number of them, as for the due knowledge and practiſing of them.

Scholer. Then you wil, that I ſhall name them as ſeauen kindes diſtinct. But nowe I deſyre you to inſtructe me in the vſe of ech of them.

Mayſter. So will I, but it muſte be done in order: for you maye not learne the laſt as ſoone as the firſte, but you muſt learne them in that order, as I did rehearſe them, if you wyll learne them ſpéedily and well.

Scholer, Euen as you pleaſe. Then to begin, Numeration is the firſte in order: what ſhall I doe wiih it?

Maiſter. Firſte you muſte know what the thing is, and then after learne the vſe of the ſame.

NVMERATION.

NVMERATION is that Arithmeticall ſkill, whereby we may duely value, expreſſe and reade anye number or ſumme propounded: or elſe in apte figures and places, ſet down any number knowen or named.

Scholer. Why? then me thinketh you put a difference betwéene the value and the Fygures?

Mayſter. Yea ſo doe I: For the valewe is one thing, and the figures are an other thing: and that commeth partely by the diuerſity of figures, but chieflye of the places wherein they be ſet.

Scholer. Then I muſte knowe here thrée thinges: the Value, the Figure, and the Place.

Mayſter. Euen ſo: but yet adde Order to them as the fourth. And firſte marke, that there are but tenne figures, that are vſed in Arithmetike: and of thoſe tenne, one doth ſignifie nothing, whiche is made like an o, and is called priuately a Cyphre, thoughe all the other ſometime he likewiſe named. The other nine are called Signifying figures, and be thus figured.

1 2 3 4 5 6 7 8 9

And this is their value.

j. ij. iij. iiij. v. vj. vij. viij. ix.

But here muſte you marke, that euery Figure hathe two values: One alwayes certain that it ſignifieth properly, which it hath of his forme: and the other vncertaine, which he taketh of his place.

A Place is called the ſeate or roome that a Figure ſtandeth in. And looke howe manye Figures are written in one ſumme, ſo manye places hathe that whole number. And the firſte place muſte bée called that that is nexte to the ryghte hande, and ſo reckoning by order towardes the lefte hande, ſo that that place is laſte, that is nexte to the lefte hande. As for example: If there ſtoode before you ſixe men in a rowe, ſide by ſide, and you ſhoulde tell them as they ſtand in order, beginning wyth the man that were nexte to youre righte hande: then hée that were nexte hym ſhoulde bée called the ſeconde, and ſo forthe to the fartheſt from your ryghte hande, whyche is the firſt and the laſte.

Scholer. Syr, I perceiue you well: ſo might I recken letters or anye other thing. As if I ſhoulde write eight letters after this order, a,b,c,d,e,f,g,h, now muſte I ſay h, is the firſt g the ij, f the iij, e the iiij, d the v, c the vj, b the vij, and a the viij.

Maiſter. That is wel done. And after the ſame ſort vſe hereafter, that what I declare by one example, doe you expreſſe by an other, and ſo I ſhall perceyue whether you vnderſtande it or no. And ſo paſſe ouer nothing, till you perceiue it well, and be experte therein.

Scholer. Sir, I pray you howe manye of theſe places be there in all?

Maiſter. There is no certaine number of them, but they are ſometimes more and ſomtimes fewer, according to the ſumme that is expreſſed. For ſo many as the figures are, ſo many are the places: and the laſte place is ſo called, not bycauſe it is laſte of all other, but it is the laſte of that preſent ſumme, and it maye bée the middle place in an other ſumme.

Scholer. Me ſéemeth I perceiue this verye well, as touchyng the order of reckoning of the places: But as for the number of them, you ſay there is no certaintie. Nowe there reſteth to declare the value of the figures by diuerſity of places, which you called, the Value vncertaine.

Maiſter. But firſte let me heare whether you knowe perfectly the certaine value.

Scholer. Yes ſir, as you wrote them, ſo I marked them.

Maiſter. Howe write you then fiue?

Scholer. By this figure 5.

Maiſter. And howe ſixe?

Scholer. Thus, 6.

Maiſter. Write theſe thrée numbers eache by it ſelfe as I ſpeake them. vij. iiij. iij.

Scholer. 7.4.3.

Maiſter. How write you theſe foure other, ij, j, ix, viij?

Scholer. Thus (I trow,) 2,1,6,8.

Maiſter. Nay, there you miſſe: Looke on mine example againe.

Scholer. Sir, truth it is, I was too blame, I tooke 6 for 9, but I will be warer héereafter.

Mayſter. Nowe then take héede, theſe certaine valewes euerye figure repreſenteth, when it is alone written withoute other Fygures ioyned to him. And alſo when it is in the firſte place, though manye other doe followe: as for example: This figure 9 is ix. ſtanding now alone.

Scholer. Howe? is he alone and ſtandeth in the middle of ſo many letters?

Maiſter. The letters are none of hys fellowes. For if you were in *Fraunce* in the middle of a M. Frenche men, if there were no Engliſh man with you, you woulde recken your ſelfe to be alone.

Scholer. So it is. Then 9 without more figures of Arithmetike, betokeneth ix, whatſoeuer other letters be aboute it.

Maiſter. Euen ſo, and ſo dothe it, if it be in the firſte place ioyned with other, how many ſoeuer do followe, as in this example, 3679. you ſee 9 in the firſte place, and doth betoken nine, as if he were alone.

Scholer. I perceiue that. And dothe not 7 that ſtandeth in the ſeconde place, betoke vij? and 6 in the third place, betoken vj? And ſo 3 in the fourth place, betoken three?

Maiſter. Their places be as you haue ſaid, but their valewes are not ſo. For as in the firſte place, euerye figure betokeneth hys owne value certaine onelye, ſo in the ſecond place euery figure betokeneth hys owne valew certaine ten times: as in the example, 7 in the ſeconde place is ſeuen times x, that is, lxx. And in the third place, euery figure betokeneth his owne valewe a hundreth times, ſo that 6 in that place betokneth vi. C. And in the fourth place, euery figure betokeneth his owne value a M. tymes, as in the foreſaid number 3. in the fourth place, ſtandeth for 3. M. And in the fifth place, euerye figure ſtandeth for his owne valewe x. M. times. And in the vi place a C. M. times. And in yͤ vii. place a M. M. times: And in yͤ viii. place x. M. M. ſo yͭ euerye place excéedeth the former x. times.

Scholer. As thus: if I make thys Number at al aduentures, 91359684, héere are eyght places. In the firſt place is 4, and betokeneth but foure: in the ſecond place is 8, and betokeneth x. times 8, that is, 80: In the third place is 6, and betokeneth ſixe hundreth: In the 4. fourth place 9 is nine thouſand. And 5 in the fifth place is x. M. times 5, that is, fiftie M. So 3 in the ſixte place, is a C. M. times 3, that is CCC. M. Then 1 in the ſeuēth place, a M. M. And 9 in the eighte place tenne thouſande thouſande times 9, that is xc. M. M. But nowe I can not eaſily nor quicklye reade it in order.

Mayſter, That ſhall you practiſe by thys meanes. Firſt putte a pricke ouer the fourth figure, and ſo ouer the ſeauenth. And (if you haue ſo many) ouer the tenth, thirteenth, ſixteenth, and ſo forth, ſtill leauing two figures betweene eche two prickes. And thoſe two roomes betweene yͤ pricks, are called ternaries.

Then begin at the laſt pricke, and ſee how many figures are betweene him and the end, which can not paſſe three, reconing hymſelfe for one: then pronounce them as if they were written alone from the reſte, and adde at the end of theyr valewe ſo many times thouſand as your number hath prickes.

After that come to the nexte three figures, and ſounde them as if they were aparte from the reſt, and adde to their valewe ſo manye times thouſandes, as there are prickes betwene them, and the firſt place of your whole number. And ſo do by euery other three Figures following, if you haue moe. As in example, 91359684. this was your number.

Put a pricke ouer 9 in the fourth place, and ouer 1 in the ſeauenth place, and then no more, (for your places come not to tenne) as thus: 91359684.

Nowe go to the laſt pricke ouer 1, and take it and the figure 9 that foloweth it, and value them alone.

Scholer. 91 that is xcj.

Maiſter. So is it : but then adde for the number of your prickes twice M.

Scholer. That is xcj, thouſande thouſande.

Maiſter. So is it. Then take the thrée other figures from one to the next pricke, and value them.

Scholer. 359. that is CCC.lix.

Maiſter. Now adde for the one pricke, that is betwéene them and the firſte place, M.

Scholer. CCC. lix. thouſande.

Maiſter. Then come to the other thrée figures that remaine.

Scholer. 684. that is, vj.C.lxxxiiij.

Maiſter. Nowe haue you valued all. And at the end of the laſte number you ſhall adde nothing, bicauſe there remayneth no pricke nor number after it : yet proue in an other number, as thus, 2 3 0 8 6 4 0 8 9 1 0 5 3 4 0.

Scholer. $2\,3\,\dot{0}\,8\,6\,\dot{4}\,0\,8\,\dot{9}\,1\,0\,\dot{5}\,3\,4\,0$. I haue pricked them as you taught me: but I am in doubt, whether I haue done well or no, bycauſe of the Cyphars: for I remember, you tolde me that they doe ſignifie nothing, and therefore I doubte whether I ſhoulde recken them for a figure in ſetting of the prickes: and againe, I know not wherefore they ſerue.

Maiſter. That wil I tel you now. Indéede they are of no value themſelues, but they ſerue to make vppe number of places, and ſo maketh the figure following them to bée in a further place, and therefore to ſignify the more value: as in this example 90, the Ciphre is of no value, but yet he occupieth the fyrſte place, and cauſeth 9 to be in the ſeconde place, and ſo to ſignifie tenne times 9, that is, xc. ſo that two Ciphres thruſteth the figure following them, into the thirde place, and ſo forth.

Scholer. Then I perceiue in the example aboue I haue pricked wel ynough: for thogh that Ciphre that is pricked ſignifie nothing, yet muſt he haue the pricke, bicauſe he came in the xiiij. place. Then wil I proue to number that ſūme. Firſt there is 230 M.M.M.M. and then foloweth 864 M.M.M. And what ſhall I nowe doe? There is a Cyphre in the third place, and no figure after him, but they that haue reckened.

Maiſter. He didde ſerue for them that you haue alreadye reckened, to make them in a place further than they ſhoulde be if he were away: and therefore nowe you ſhall let him goe. And ſo doe alwayes when he occupieth that place next before any pricke, whiche is the laſte of that Ternarie, and a Cyphre in the laſte place doth nothing.

Scholer. Then ſhal I ſaye but 89 M.M.

Maiſter. So, but goe forth.

Scholer. 105 thouſande. Nowe are all my prickes ſpent, and yet remaine 340, ſo that I muſte value them CCC.xl. only.

Mayſter. Nowe can you recken after thys ſorte: and remember, that euery ſuch roume ſo parted, is called a Ternarie or Trinitie.

Some doe parte ſuch great numbers with letters, after this manner.

c b a c b a c b a c b a c b a
2 3 0 8 6 4 0 8 9 1 0 5 3 4 0. In whyche example yée maye ſée, that *a* ſupplyeth the roume of your pricke. And ſome doe parte the numbers with lines after this forme.

230|864|089|105|340. where you ſée as many lines as you made prickes, and to one intente, ſaue that the lines doe more plainely parte euery thrée figures, according as they ſhoulde be valewed vnder one Denomination.

Scholer. Yea ſir, but if you ſhoulde ſhewe me a number ſo parted, I ſhoulde take it for many numbers, and not for one.

Maiſter. So might you doe, not knowing my meaning. But what if I didde ſet forth the number without lines, and your ſelfe (for the eaſe of reckning) did ſo part it with lines, woulde you forget wherefore ye did it, and then take them for many numbers?

Scholer. No I trowe not, but yet I doubte.

Mayſter. Then vſe that that you like beſt, for all the three wayes are to one intent, ſaue (as I ſaid) that the lines do more plainely diſtinct the denominations.

Scholer. What call you Denominations?

Maiſter. It is the laſte value or name added to any ſumme. As when I ſaye: CC. xxij. pounds: pounds is the Denomination. And likewiſe in ſaying: 25 men, men is the Denomination, and ſo of other. But in this place (that I ſpake of before) the laſte number of euery Ternarie, is the Denomination of it. As of the firſt Ternarie, the denomination is vnites, and of the ſeconde Ternarie, the Denomination is thouſandes: and of the thirde Ternarie, thouſand thouſandes, or millions: of the iiij. thouſande thouſand thouſandes, or thouſande Millions: and ſoforth.

Scholer. And what ſhall I call the value of the iij. figures that maye bée pronounced before the Denominators? as in ſaying: 203 000000, that is CCiij. millions. I perceyue by youre wordes, that millions is the Denomination: but what ſhal I call the CCiij. ioyned before the Millions?

Maiſter. That is called the Numerator or valewer, and the whole ſumme that reſulteth of them bothe, is called the Summe, value or number.

Scholer. Nowe is there anye thing elſe to be learned in Numeration? or elſe haue I learned it fully?

Maiſter. I might here ſhew you who were the firſte inuentors of this Arte, and the reaſons of all theſe things that I haue taughte you, but that wil I reſerue till ye haue learned ouer all the practiſe of this Arte, leaſte I ſhoulde trouble your witte, with ouer many things at the firſte.

☞ But yet this muſte you marke, that there are thrée kindes of number: one called Digits, an other articles, and the thirde mixte

umnumbers.

A Digit is any number vnder 10, as this:
1,2,3,4,5,6,7,8,9.

And 10 with al other that maye be diuided into tenne parts iuſt, and nothing remaine, are called Articles: ſuche are 10,20,30,40,50,&c.100,200,&c.1000.&c.

And that number is called mixt, that containeth Articles, or at the leaſte one article, and a digit: as 12,16,19,21,38,107,1005. and ſo forth. And for the more eaſe of vnderſtanding and remembraunce marke this: The diget number is neuer written wyth more than one figure, but the article and the mixt number are euer wrytten wyth more than one fignre. And thus they differ, that the article hathe euermore this Cyphre 0, in the firſte place: and the mixte number hathe euer there ſome Diget.

Scholer. By theſe laſte wordes, I perceiue it muche better than I did before, and now (I thinke) I wil neuer miſſe to knowe thoſe three aſunder.

Maiſter. If you remember nowe all that I haue ſayde, you haue learned ſufficientlye this firſte kinde of Arithmetike, called Numeration. Howbeit, I will yet exhorte you nowe, to remember bothe this that I haue ſaide, and all that I ſhall ſaye, and to exerciſe your ſelfe in the practiſe of it: For Rules withoutpractiſe, are but a light knowledge: and practiſe it is, that maketh menne perfect and prompt in all things.

And as you haue learned to gather and expreſſe the value of a ſumme propounded, and ſet downe before you : ſo muſte you practiſe to marke note, or write downe, with apte figures, and in due places, any number, onely named or recited to you, or of your ſelfe imagined: as for a proofe: How note you, or write downe this ſumme, fiue thouſande, two hundreth, fiftie and ſeuen.

Scholer. This troubleth me nowe, whether I ſhoulde beginne at the firſte figure or at the laſt. For reaſon (me thinketh) ſhould cauſe me to beginnne at the firſte: and yet if I write it as you ſpeake it, I muſte beginne at the laſte.

Maiſter. When you knowe youre places perfectly, you maye beginne where you liſt. But the more eaſe for your hande is to begin with the laſte, that is to ſay, as I did ſpeake them. Yet for the more ſuretie, a while you maye beginne with the firſte, repeating my wordes backewarde thus : Seauen, Fiftie, two hundreth, fiue thouſande : or elſe ſounding them all by their diget or valewer, as thus : ſeauen, fiue, two, fiue : for that way is eaſieſt. But then muſte you looke wel, whether there be any Ciphre in your ſumme that he may be ſette in his place. As if youre laſt valewer of youre ſumme (as you ſpeake it) be aboue 9, then is there a Cyphre in the firſt place. And if it be a hundred or aboue, then is there two Cyphres one in the firſt place, & an other in the ſeconde, and ſo forth.

But bicauſe this thing is ſuch that cannot be ſet forth without manye wordes, I think beſte here nowe at the ende of Numeration to adde a table eaſie and ready for the firſt exerciſe of it.

Lo, this is the Table.

The right ſide or hande.

The lefte ſide or hande.

The names of the digits, valewes certaine, or valewers.

x.M.of milliõs.	M.of millions.	C.of millions.	x.of millions.	Millions.	C.of thouſãdes.	x.Thouſandes.	Thouſandes.	Hundreds.	Tennes.	Unites.	The denominatours of the place or valewe vncertain
9	9	9	9	9	9	9	9	9	9	9	Nine.
8	8	8	8	8	8	8	8	8	8	8	Eyght.
7	7	7	7	7	7	7	7	7	7	7	Seuen.
6	6	6	6	6	6	6	6	6	6	6	Sixe.
5	5	5	5	5	5	5	5	5	5	5	Fiue.
4	4	4	4	4	4	4	4	4	4	4	Foure.
3	3	3	3	3	3	3	3	3	3	3	Thre.
2	2	2	2	2	2	2	2	2	2	2	Twoo.
1	1	1	1	1	1	1	1	1	1	1	One.
0	0	0	0	0	0	0	0	0	0	0	Ciphre.
Eleuenth.	Tenth	Ninth.	Eighth.	Seuenth.	Sixte.	Fifte.	Fourth.	Third.	Second.	Firſt.	The order of the places.

This Table (as you may ſee) hathe eleuen places, and in eche of them are ſette al the digites, whoſe certaine value is written in the right hande of the Table, & the value vncertaine on the left hande. So that by this table you may learne bothe howe to expreſſe anye number that you liſte, (if that it exceede not eleauen places) that is to ſay, lxxxx. thouſand Millions, and ſo maye you by the helpe of it, value all ſummes propoſed vnder the ſayde number.

For example: take the ſumme that I propoſed before, which was fiue thouſand, two hundred, fiftie and ſeauen. And if you wil expreſſe it, take the firſte number (as I ſpeake it) which is fiue M. whoſe valuer or certaine value is v. and his vncertaine value or denomination is M. Firſt you ſhall ſeeke at the right hand of the valuer v. Then ſeeke along vnder the tytle of Denomination towarde the left hande, til you find thouſands, and vnder it right at the foote of the Table, is the number of the place, that is the fourthe, wherein you muſte write youre diget or valuer fiue.

Afterwarde come to the ſeconde parte of the number, two hundred, whoſe valuer is 2, and his denomination C. Seeke two at the right hande of the Table, and goe along vnder the denominations towarde the lefte hande, til you come vnder C: then looke to the table, and there ſhall you ſee the number of the place, that is to ſay, three, wherein you muſte ſet your diget 2.

Then doe ſo by your other two numbers that remaine, and you ſhall finde fiue in the ſecond place for your fiftie, and 7 in the firſt place for your ſeauen. And thus maye you doe with other numbers.

Scholer. Maiſter I thanke you hartilye. I perceiue you ſeeke to inſtruct me moſt plainly and briefly, and not to hide youre knowledge with ſubtile wordes as many doe. For this rule is ſo plaine, that I can deſire it no plainer. And though it ſeeme ſomwhat long, yet I perceiue it to be a ſure way.

Maiſter. So is it, and thoughe it be long, yet it is neyther too long, neyther too plaine for yong learners that lacke practiſe: for this table is in ſteade of a teacher, to them that lacke one. But nowe I truſte I haue ſaid ynough of Numeration: which after you haue wel practiſed, then may you learne forth.

Scholer. Yet I praye you in one thing to tel me your iudgement. Why do men recken the order of the places backewarde, from the right hand to the lefte?

Maiſter. In that thing all men doe agree, that the Chaldeyes, which firſt inuented this Arte did ſet theſe figures as they ſet all their letters: for they wryte backewarde as you terme it, and ſo do they reade. And that may appeare in all Hebrue, Chaldeye, and Arabike bookes, for they bée not only written frō the right hande to the lefte, and ſo muſte bée read, but alſo the right end of the booke is the beginning of it: whereas the Gréekes Latines, and all nations of Europe, do write & reade from the left hande towarde the right: And all their bookes begin at the left ſide.

Scholer. That reaſon doth ſatiſfie me.

Maiſter. It neyther ſactſfieth mée, neyther lyketh me well, bycauſe I ſée that the Chaldeys and Hebrues doe not ſo vſe their owne numbers, as at another tyme I will declare. But this playne reaſon may beſt ſatiſfie you preſentely: That ſéeing in pronouncing of numbers we kéep the order of our owne reading, from the lefte hande to the right: And againe, we doe euer name the greater numbers before ẏ ſmaller: it was reaſon, that the leſſer places conteyning the leſſer numbers, ſhould be ſet on the right hand, and the greater places conteining the greater numbers, to procéede toward the left hande.

Scholer. This reaſon is to me ſo plaine, that it ſeemeth now againſte reaſon to make a doubte of that order. So that nowe for Numeration I am ſatiſfied: ſo that onelye practiſe ſhal make me fully ready and expert in it. And in the meane ſeaſon, I deſire to learne the other kindes of Arithmetike.

Maiſter. That is well ſaide: but what ſhould you next learne can you tel?

Scholer. I remember you ſaid that Addition was nexte.

Maiſter. Euen ſo, and what that is muſte you firſte knowe,

ADDITION.

Additiō is the gathering togither and bringing of two numbers or more into one totall ſumme: as if I haue 160 Bookes in the Latyne tong, and 136 in the Greeke tongue, and would know how many they be in all, I muſte write theſe two numbers one ouer an other, writing the greateſt number higheſt, ſo that the firſt figure of the one, bee vnder the firſte figure of the other. And the ſecond ſeconde vnder the ſeconde, and ſo forth in order.

When you haue ſo done, drawe vnder them a right line, then will they ſtand thus.

Nowe beginne at the firſte places, toward the right hande alwayes, and putte togither the two firſt figures of thoſe two numbers, and looke what commeth of them, write vnder them, right vnder the line. As in ſaying, 6 and 0, is 6. Write 6 vnder 6: as thus.

160
136

160
136
6

And then go to the ſecond figures, and doe likewiſe: as in ſaying, 3 and 6 is 9: write 9 vnder 6 and 3, as here you ſee.

160
136
96

And likewiſe do you with the figures that be in the third place, ſaying: 1 and 1 be 2: write 2 vnder thẽ, and then will your whole ſumme appeare thus.

160
136
296

So that now you ſee, that 160, and 136 doe make in all, 296.

Scholer. What? this is very eaſie to doe, me thinketh I can do it euen ſith.

There came through Cheapeſide two droues of cattell: in the firſt was 848 ſheepe, and in the ſecond was 186 other beaſtes.

Thoſe two ſummes I muſte write as you taughte me, thus.

```
 848
 186
-----
```

Then if I put the two firſt figures togither, ſaying: 6 and 8 they make 14. That muſte I write vnder 6 and 8, thus.

```
 848
 186
-----
  14
```

Mayſter. Not ſo, and here are you twice deceiued. Firſt, in going about to adde togyther two ſummes of ſundrie things, which you ought not to doe, excepte you ſeeke onelie the number of them, & care not for the things. For the ſumme that ſhould reſulte of that addition, ſhould be a ſumme neither of ſheepe, nor other beaſtes, but a confuſed ſumme of both. Howbeit ſometimes yee ſhal haue ſummes of diuers denominations to be added, of which I will tell you anone: but firſt I will ſhew you, where you were deceiued in an other pointe, and that was in writing 14, (which came of 6 and 8) vnder 6 and 8, which is vnpoſſible. For, howe can two figures of two places be written vnder one figure, and one place?

Scholer. Truth it is: but yet I did ſo vnderſtand you.

Mayſter. I ſaide indeede, that you ſhoulde write that vnder them, that did reſulte of thē both togither: which ſaying is alwayes true, if that ſumme do not exceede a Digit. But if it be a mixte number, then muſte you write the Digit of it vnder your figures, as I haue ſaid before: but and if it be an Article, then write o vnder them, and in both ſortes you ſhall kepe the article in your mind. And therfore when you haue added your ſeconde figures, which occupy the place of tennes, you ſhall put that 1 therto, which you kept in your minde: for though it were ten indeede, yet in that place it is but as one, becauſe, that euerie 1 of that place, is ten, for it is the place of tens. And in like manner: if you haue in the ſecond place ſo great a number, that it amounteth aboue 9, then write the digite, and reſerue the article in your mind, euer adding it to the next place following: and ſo of all other places, how many ſo euer you haue. And if you haue a mixt number, when you haue added your laſt figures, then write the digit vnder the laſt figures, and the article in the next place beyond them: ſo ſhall your number reſulting of Addition, haue one place more than the numbers which you ſhould adde togither.

Scholer. Now doe I perceiue you, and the reaſon of this is, (as I vnderſtande) becauſe that no one place can contain aboue 9, which is the greateſt figure that is, and then all tens or articles muſt be put to the next place following: for euerie place (as I may ſee) exceedeth the other place next before him, by 10.

Now (if it pleaſe you) I will returne to my example of Cattell: But I remember you ſaide, I might not adde ſummes of ſundry things togither, and that might I ſee by reaſon.

Maiſter. Trueth it is, if you ſeeke the due ſumme of anie things, but if you onelie ſeeke a bare ſumme, & haue no reſpect to the thing, then were it better to name the ſumme onely without anie thing, as in ſaying 848, without naming ſhepe, or any thing elſe. And likewiſe 186, naming nothing.

Now let me ſee: howe can you adde thoſe two ſummes?

Scholer. I muſt firſt ſet them ſo, that the two firſt figures ſtand one ouer an other, and the other each one ouer his fellow of the ſame place: then ſhall I drawe a line vnder them both. And ſo likewiſe of other figures, ſetting allwayes the greateſt number higheſt, thus, as followeth:

Then muſt I adde 6 to 8, which maketh 14, that is mixte number: therefore muſte I take the diget which is 4, and write it vnder 6 & 8 keeping ẏ article 1 in my mind thus.

848
186

848
286
4

Nexte that doe I come to the ſecond figures, adding them togither, ſaying, 8 and 4, make 12, to which I put the 1 reſerued in my minde, and that maketh 13, of which number I write the diget 3 vnder 8, and 4, & kepe the article in my mind thus:

848
186
34

Then come I to the thirde figures, ſaying: 1 and 8, make 9, and 1 in my minde maketh 10. Sir, ſhall I write the cipher vnder 1 and 8?

Mayſter. Yea.

Scholer. Then of 10 I write the cipher vnder 1 and 8, and kepe the article in my mind.

Mayſter. What nedeth that, ſeeing there followeth no more figures?

Scholer. Sir, I had forgotten, but I will remember better hereafter. Then ſeeing I am come to the laſt figures, I muſte write the cipher vnder them, and the article in a further place after the cipher, thus:

848
186
1034

Mayſter. So now ye ſée, that of 848, and 186 added together, there amounteth 1034.

Scholer. Nowe I thinke I am perfite in Addition.

Mayſt. That wil I proue by this example. There are two armies of ſouldiours: in the one are 106800, and in the other 9400: How many are there in both armies ſay you?

Scholer. Firſte I ſette them one ouer an other, beginning with the firſte numbers at the right hād, thus. But the neather nūber wil not match the ouer number.

106800
9400
————

Mayſter. That forceth not.

Scholer. Then doe I adde 0 to 0, and there amounteth 0, that muſt I write vnder yͤ firſt place, thus.

106800
9400
————
0

Mayſter. Well ſaide.

Scholer. Then likewiſe in the ſeconde place I adde 0 to 0, and there ariſeth 0, which I write vnder the ſecond place, thus.

106800
9400
————
00

Then I come to the thirde place ſaying: 4 and 8 make 12, of whiche I write the diget 2, and kéepe the article 1 in my minde, thus.

106800
9400
————
200

Then adde I 9 to 6, whiche maketh 15, to that I adde the article 1 that was in my mind, and it is 16. I write 6 vnder 6 and 9, and keepe one in my minde, thus.

106800
9400
―――――
6200

Mayſter. Why doe you not write bothe figures, ſeeing you are come to the laſt couple of numbers?

Scholer. May reaſon ſheweth me, that I muſt adde that article that is in minde, vnto the next figure of the ouer ſumme, thoughe there be no more in the neather ſumme.

Ma. That is well conſidered: then doe ſo.

Scholer. Then ſay I, 0 in the ouer ſumme, and 1 in my minde, maketh 1, that I write vnder 0: Then foloweth there yet one more in the ouer ſumme, which hath none to be added to it, for there is none in the neather ſumme, nor yet in my mind, therfore I thinke I muſt write that euen as it is.

Mayſter. Yea.

Scholer. Then doth my whole ſumme appeare, thus.

106800
9400
―――――
116200

Mayſter. If you marke this, you haue learned perfectly the cōmon addition of all ſummes which are of one denomination: ſo that ye obſerue this alſo, that in Additiō you muſt haue two numbers at the leaſt, or elſe how cā you ſay that you do adde? And euer let the greateſt number be writtē higheſt, for that is the beſt way, though it be not neceſſarie.

And forget not this, that if you haue many numbers to adde togither, you ſhall haue oftentimes an article of a greater value thā 10: ſometimes 20, ſometimes 30, ſometimes more, yea, peraduenture 100. Therefore, as you did with the article 10, ſo do with them, reſeruing them in your minde, and adding to the number next following, ſo many as their valuer or value certaine is: that is to ſay, 2 for 20, 3 for 30, and ſo forth of other. But if the ☞ article be 100, then muſt you not adde the article to the next figures following, but to the third figures from them, as I will ſhewe you anone by example. And if it chaunce the number to be ſuch, that it doe comprehende two ſundrie articles, (that is, one of tennes, and an other of hundreds) then muſte you reſerue them both in your minde, and adde the article of tennes, to the figures that folow next, and the article of hundreds, to the figure of the third place from thence.

Nowe take this example for all. I would ad theſe xij. ſummes in one, whiche I ſet after thys maner. Then do I beginne and gather the ſumme of the firſt figures, whiche commeth to 107. For firſt I take 9. ther x. times, and that is 90: then 9 and 8 is 17, that is in all 107. of whiche ſumme I write the 7 vnder the firſte figures, and then haue I an article of an hundred in my minde, whyche eyther I muſt keepe in my minde till I come to the thirde figures, which are in the roomes of hundreds, or elſe I maye for feare of forgetting, write this one (beeing of the third place in your accompte) vnder the third rowe of figures, making two lines, as you ſee here done. And then muſt I write the digites vnder the loweſt line: and this is the ſureſt way, when the ſumme is ſo great, that the addition of one row paſſeth 100.

4889
4599
2299
3699
2399
4090
1999
3198
299
699
899
499
389

When I haue ſo done, I muſt then come to the ſeconde rowe of figures, and adde them togither, whiche doeth make 115. of whyche ſumme I wꝛite the dygitte 5 vnder the ſame ſeconde row, and then I haue a mixt number remaining of two figures, of which the 1 (that ſtandeth foꝛ 10) muſt be added to the ſecond oꝛ next place after them that I did laſt adde. And the other (ŷ ſtandeth foꝛ 100) muſte be added to the thirde place from thence.

Scholer. That is to ſay, the fourth place from ŷ firſt line oꝛ row of figures.

Maiſter. Euen ſo. And thē wil the ſum appeare thus.

4889
4599
2299
3699
2399
4090
1099
3198
299
699
899
499
289

11
1

57

Then adde the third row of figures, with the two vnities betwéene the line, and the ſumme amoūteth to 50: of which I wꝛite the Cypher vnder the ſame thyꝛde rowe, and the 5 vnder the nexte figures toward the lefte hand. And with my pen I giue a daſhe to the two vnities betwéene ŷ lines, whoſe valew I haue already added vnder the loweſt line.

Then I adde the figures of the fourth row, with the 1 and 5 that are vnder them betwéen the two lines, & they make 29: then dashe I the 1, & the 5, with my penne, as I did befoꝛe the two unities: & ſo wꝛite under ẏ loweſt line the 9 (ẏ is the digit) under the fourth place: & the 2, that is the article, beyonde it, toward the left hand. So thoſe ſummes doe make 29057.

$$\begin{array}{r}
4889\\
4599\\
2299\\
3699\\
2399\\
4090\\
1099\\
3198\\
299\\
699\\
899\\
499\\
389\\
\hline
\cancel{5}\cancel{1}\\
\cancel{5}\cancel{1}\\
\hline
29057
\end{array}$$

Scholer. This ſeemeth ſomewhat harde, by the reaſon of ſo many nūbers togither. Howbeit I think if I do often pꝛoue euē with this ſame example I ſhal be able to do ſo ſhoꝛtly, wͭ any other ſumme.

Maiſter. So ſhall you. Foꝛ it is often pꝛactiſe that maketh a man quicke and ripe in all things: But becauſe of ſuche great ſummes there may chaunce to be ſome errour. I will teach you howe you ſhal pꝛoue whether you haue done wel oꝛ no.

Scholer. That were a great helpe and eaſe.

Maiſter. Begin firſt with the higheſt number, and then to all the other oꝛderlie, & adde them togither, not hauing regarde to theyꝛ places, but as though they were all unities:

and ſtill as your number encreaſeth aboue 9, caſt away 9. Then goe forth, euer caſting away 9 as often as it amounteth thereto: and ſo do till you haue gone ouer all the numbers that you intended firſt to adde, and whatſoeuer remaineth after ſuch addition and caſting awaie of 9, write it in ſome voide place by the ende of a line for the better remembraunce: & then putte togither the figures that reſult of the Addition, ſtill caſting away 9 alſo. And then that that remayneth, write at the other ende of that line: and if thoſe two figures be like, then haue you well done by likelihode: but if they be vnlike, then haue you miſſed. As for example in this preſent ſumme: The firſte figure of the ouer line is 9, lette him go: then 8 and 8 is 16, take awaye 9, and there remayneth 7, adde to it 4 that followeth, and that maketh 11. from whiche if you take 9, there reſteth 2: then come to the next row, whoſe firſt and ſeconde number are 9, therefore ouerpaſſe them both, and take the 5 to the 2 which did remaine in the firſt row, that maketh 7, putte thereto the 4 folowing, that maketh 11, thence take 9, and there remayneth 2: nexte that, goe to the thirde line, whoſe two firſte numbers you maye lette paſſe, aſte, bycauſe they are nines: then take the wo whiche with the other two that remayned in the ſeconde rowe, make 6: then goe to the fourth rowe, whoſe two firſte numbers et goe, and take the 6, to the [illegible] that remayned, and that maketh 12, take awaie 9, and there reſteth 3, which with the 3, that is next, maketh 6. And ſo goe thꝛoughe all the other numbers, and you ſhall finde that there remaineth 5, after you haue caſt away 9 as often as you find it: therfoꝛe wꝛite 5 at one end of a line in a voide place thus. 5 ———

Then gather all the figures of the totall ſumme which is vnder the loweſt line, and caſt away 9 as often as you finde it, as thus: ſeauen and 5 make 12, take awaie 9, and there reſteth 3, to that if you adde the 2 that is laſte (foꝛ you may let goe the 9) then doth it make 5, whiche you muſt wꝛite at the other ende of the line that you made in the voide place, and it wil be thus. 5 ——— 5

And then you ſee that thoſe two figures bee like, whereby you maye knowe that you haue done well, and ſo maye you pꝛoue in anie other.

Scholer. If it pleaſe you, I wil pꝛoue in an other ſumme.

Mayſter. With a good will.

Scholer. Then wil I take one of your former examples, which was this.

106800
9400
116200

Firſt in the higheſt line, 8 and 6 make 14, then 9 taken away, there remaine 5, to which I adde the 1 that foloweth, and that maketh 6. Then come I to the ſecond line, where I finde firſt 4 which with 6, maketh 10, from that I take 9, and there reſteth 1, the next figure is 9, and therefore I let him alone, ſo find I one remaining, which I ſet at the ende of a line thus. 1———
Then I come to the totall ſumme, and there I finde that al the figures put together make 10, from which I take 9 and there reſteth 1 alſo, which I put at the other ende of the line thus, 1———1
And becauſe they be like, I know that I haue wel added.

Addition of nūbers of diuerſe denominations.

Mayſter. So you knowe nowe both howe to adde two ſūmes or more together: and alſo how to proue whether you haue done well or no: which thing alſo you may doe beſt by Subtraction. But becauſe you canot yet ſkil of it, I will let that paſſe till anone, and will teache you now how to adde ſūmes of diuers denominations: whiche thing can neuer be but whē the one denomination is ſuch that it conteineth the other certain times. And yet you ſhal adde them to the other, not after this ſort as you did them that were of one denomination, but after ſuch a ſort as I will now ſhew you, that is to ſay.

If you haue a ſumme of diuers denominations, then loke that ye ſet euery denomination by himſelf, with ſome note or figure of his denomination, as they be wont to be written. Then write your other ſūmes ſo vnder that firſt, that euery one be ſet vnder the other of the ſame denominations, as for example: if your denominations be poundes, ſhillings, & pens, write pounds vnder poundes, ſhillings vnder ſhillings, and pens vnder pens, and not ſhillings vnder pens, nor pens vnder pounds.

Scholer. Nowe that you haue ſpoken it, me thinketh it needeth not to warne me of it, for it were againſt reaſō ſo to cōfound ſūmes: but yet if you had not ſpoken of it, peraduenture I ſhould haue bene deceiued in it.

Mayſter. If you doe ſay it is ſo plaine, I will ſpeake no more of it, but with an example make the matter to appeare euidently.

Firſt,

Firſte, one man oweth me 22 ℔, 6 s, 8 d. An other oweth mee 5 ℔, 16 s, 6 d. And an other oweth mee 4 ℔, 3 s, I would know what this is altogether. Therfore muſt I firſt ſet down my greateſt ſum & thẽ the other, euery one vnder his denomination greeing to the greateſt ſumme, as here you ſee.

℔	s	d
22	6	8
5	16	6
4	3	0

Then muſte I beginne at ſmalleſt numbers, (whiche muſt allwayes be ſet nexte the right hand) and adde them togither, and if the ſumme of them will make one of the next denomination, then muſte I keepe it in my minde till I come to that place, or elſe for more eaſineſſe write it vnder that place betweene the double line, and vnder that place muſt I note the reſidue, if there remaine anie of the ſame denomination, but if there remaine none, then neede I to write vnder it nothing. And this is all that you muſte marke in this Addition: for all other things ☞ are like to the other manner of Addition before mentioned. Therefore the chiefeſt point of this Addition is, to knowe the valewes of common coynes and rated ſummes, As how many ſhillinges be in a pounde: how many pence in a ſhilling, of which and of other like things, I will inſtruct you hereafter, in teaching of Reduction: But now I maye not diſturb your wit from the thing that we are about.

Therefore let vs returne to that former example, which I propoſed of thrée detters, whiche ſummes when I had ſette orderly they ſtode thus, with a double line vnder them:

℔	ß	d
22	6	8
5	16	6
4	3	

Then to adde them vnto one ſum, I muſt begin at the right hand, where the ſmalleſt denomination is, and adde thē togither firſt, ſaying: 6 and 8 make 14. Nowe ſéeing theſe 14 are pennies, and that 12 pence make one ſhilling, which is the next valewer, I take awaye 12 from 14, and there reſteth 2, which I write vnder the pennies, and for the other 12, which maketh 1 ſhilling I write 1 vnder the tytle of ſhillings, thus:

℔	ß	d
22	6	8
5	16	6
4	3	
	1	
		2

Then do I adde all the ſhillings togither, and finde them 25, to whiche I adde that 1, betwéne the two lynes, that maketh 26, but bicauſe that 20 ſhillinges do make 1 pound, I take away 20 from 26, and for that 20 I write 1 vnder the pounds betwéne the two lines, and the other 6 that remayneth, I write vnder the ſhillings, as appeareth in the exãple before.

℔	ß	d
22	6	8
5	16	6
4	3	
1	1	
	6	2

Then come I to the pounds, adding them all togither, and finde them to be 31: thereto I adde the 1 betwéne the two lines, & that maketh 32, whiche ſumme I write downe whole, becauſe there reſteth no greater denomination, and then my whole ſumme appeareth thus.

℔	ß	d
22	6	8
5	16	6
4	3	
1	1	
32	6	2

So is my totall ſumme, 32 ℔. 6 ß. 2 d. And this maye you proue in an other lyke ſumme.

Sc. Then will I caſte the whole charge of one moneths commons at Oxforde wyth batteling alſo.

Mayſter. Goe to, let me ſée how you can doe.

Scholerr. One wéekes commons was 11 d. *ob.* *q̃.* and my batling that wéeke was 2 d. *q̃.* *q.* The ſecōd wéekes commons was 12 d. and my batling 3 d. The third wéekes commons 10 d. *ob.* and my batling 2 d. *q.* *c.* The fourth wéekes commons 11 d. *q̃.* and my batling 1 d. *ob.* *c.* These eyghte ſummes woulde I adde into one whole ſumme, and therefore I will ſette them one ouer another, thus.

But I had forgotten, I ſhoulde haue ſet the greateſt ſumme higheſt.

d.	*ob.*	*q̃.*	*q.*	c.
11	1	1		
2		1	1	
12				
3				
10	1			
2			1	1
11		1		
1	1			1

Maiſter. So is it commonly beſt, howbeit, here it forceth not: and in ſuch ſummes as this is, that go by order of wéekes, dayes, or yeares, it is better to kéepe that order, than to alter them, and to ſette the greateſt number higheſt, for that ſerueth for ſuch ſummes as go not by order.

Scholer. Then if I haue ſet them well ynough, I will begin to ad them thus.

Firſte of the ſmalleſt valewers at the right hande, which are called cées, I find 2, and ſéeing that 2 cées, do make one q, I wil wꝛite nothing vnder the cées, but will wꝛite 1 q foꝛ 2 cées, vnder the rewes betwéen the lynes, as the example ſheweth.

d.	ob.	q̃.	q.	c
11	1	1		
2		1	1	
12				
3				
10	1			
2			1	1
11		1		
1	1			1
			1	

Then come I to yͤ next valewers, where I finde 2 q, and to them I adde the q that is betwéene the lines, and ſo are they 3 q: but becauſe 2 q, maketh one q̃, I wꝛite one q̃ vnder the farthynges betwéene the lines, and the q that remaineth muſt I wꝛite beneth the nethermoſte line vnder the rewes, thus.

d.	ob.	q̃.	q.	c.
11	1	1		
2		1	1	
12				
3				
10	1			
2			1	1
11		1		
1	1			1
		1	1	
			1	

Then come I to the farthinges, where I

finde 3,and the other q̃ that is betwéene the lines,maketh 4 farthings.And bicauſe 4 q̃ make iuſt 1 pennie , I ſhall write nothing vnder the farthings, but muſt write 1 vnder the pens,betwéene the lines.

Next that muſt I adde the halfe pence togither,of which there are 3.but ſéeing that 2. ob.make 1 d,I muſt write 1 vnder the pens betwene the lines : but how ſhal I do it,for there is 1 alreadie?

Maiſter. Haue you forgotten how I didde in addition of the great ſumme before ? you muſt ſet it vnder the other,ſo ſhal they bothe ſtande for 2. For if you ſhoulde ſet it before or behinde the other,they ſhould make 11.

Scholer. I remember it nowe, and I perceiue the reaſon. Then I wil write 1 ob,vnder the halfepence, and for the other two halfepens, which make 1 d,I write 1 vnder the pens : Then come I to the pens, & finde, that there are of them 52. then put I to them the 2 betwene the lines,and that maketh 54, which amoũteth to 4.ß.6 d:the 6 d I muſt write vnder the pens,and the 4 ß.I muſt ſet (I ſuppoſe) farther toward the left hande by themſelues.

Maiſter. Euen ſo.

Scholer. Then appeareth all my addition thus. And the ſumme is 4 li 6 d. ob.q.

li	d.	ob.	q̃.	q.	c
	11	1	1		
	2		1	1	
	12				
	3				
	10	1			
	2			1	1
	11		1		
	1	1			1
	1		1	1	
	1				
4	6	ob.		q.	

M. Now haue you done this well. But tell me, why did you write kewe, cœ, thus, q,c. & not rather thus q^c, as the faſhion is?

Schol. Becauſe I thoughte it was the beſt way for due gathering of euery denomination by himſelfe.

Mayſter. So was it in déede. Wel now, can you tell how to proue this addition, and ſuch other like of diuers denominatiōs, and to trie whether you haue done well or no?

Scholer. I would I could.

Mayſter. That ſhall you doe by this meanes. Firſt as you did begin to ad ſo recken agayne euery denomination by it ſelfe, and when you finde ſo many ſmall that doe make any other denomination, let them go, and kéepe in minde only the reſidue that wil make no greater denomination, and looke whether there be any ſuch like value vnder the nether lyne, and if there bée, you haue well done, and ſo goe from one denomination to an other, vnto the ende.

But here muſt you note, that in gathering of the ſummes, ye muſt recken thoſe figures that are written betwéene the lynes, with them that are written aboue them: as for an example, I will examine the ſumme that I did laſt adde, which ſtoode thus.

℔	ß	d
22	6	8
5	16	6
4	3	
1	1	
32	6	2

Firſte I finde 6 and 8, whiche maketh 14, from whiche I take 12, bicauſe it maketh one of the next denomination, and there remayneth 2, and vnder that place I ſée a like figure, therefore I knowe that well to be done. Then come I to the ß, where I find 1, 3, 16, and 6, that maketh 26, I caſt away 20, for they make another denomination, that is to ſaye poundes: and the 6 which remaineth, is lyke to the 6 that is written vnder them beneathe the loweſt lyne, therefore that is well done alſo. And thence I goe to poundes, where I finde 1, 4, 5, 22, that is 32, to whiche ſumme agréeth another lyke vnder it. Therefore I iudge all well done.

Scholer. I perceiue reaſon in this probation. Nowe will I attempte the ſame in the ſumme that I did adde, whiche when I had ended adding, ſtode as you maye ſée in the example following.

Firſte amongeſt the cées I finde but two, which make one q euen, therfore there muſt nothing bée vnder the line for them: And amongſte the kewes I fynde 3, of whiche two make 1 q̃, therefore I let them goe, and the one q, that is lefte, hath an other lyke vnder his place, therefore that is well done.

Then the Farthinges are iuſte 4, which make 1 d. and therefore I let them goe. Amongſt the halfe pence there is one od (for 2. muſt I caſt away, bycauſe they made one penny) and vnto it anſwereth a like ſum vnder it. The pens are 54. frō which I take away 48, that makes 4, ß, and the 6 remay-

ß	d	ob.	q̃.	q.	c
	11	1	1		
	2		1	1	
	12				
	3				
	10	1			
	2			1	
	11		1		1
	1	1			1
	1		1	1	
	1				
4	6	ob.		q.	

ning agrée to a like figure ſet vnder thẽ. And laſt of all remayneth the 4 ß, which the abiected pens did make: ſo I perceiue that I haue wel done. Now this will I not forget. But will this examination ſerue in all addition?

☞ Mayſter. It ſerueth for all addition of ſundrie denominations, if the addition be made with two lines, (as were theſe) elſe it wil not ſerue, bycauſe that thoſe ſummes whyche are here added betwéene the lines, in Addition by one line, are vnderſtanded and not written: but I let that way paſſe, bicauſe as it is commõ, ſo is it more deceiueable than this way, namelye if a mans memorie be either dull or troubled.

Scholer. Yet it were good to knowe that way alſo.

Another forme of Addition.

Maiſter. If you deſire to knowe it, this it is in fewe wordes. Doe euerie thing as you did in this ſorte of Addittion, ſaue that where you made here two lines, you ſhall make there but one: and thoſe ſummes that you did here write betwéene the lynes, you muſt kepe in your memorie, and vſe them (as you dydde here) each one when you come to his place.

Scholer. Then they differ not, but in this, that this addition with two lines leaueth nothing to memoꝛie, but wꝛiteth downe all: and the other way committeth certaine numbers to memoꝛie, as you taughte me in the firſt examples of addition of ſmall ſummes of one denomination. But what if a man vſe it (as you ſay men do commonly) how ſhal it be examined?

Maiſter. Séeing you are ſo deſirous of it, I will ſhewe both an example of the addition, and alſo the manner to examine it.

I pꝛopoſe theſe thꝛée ſummes to be added, and I gather firſte the pence, as I did in the other ſoꝛt, and I finde of them 8, 3, 9, that is 20, of which ſumme I bate away 12, whiche make 1 ß, and kepe that 1 in my minde, and the reſt, that is 8, I wꝛite vnder the pence.

℔	ß	d
12	8	9
6	7	3
3	6	8

Then do I adde the ſhillings togither, and finde of them 6, 7, 8, that is 21, whereof I bate 20, that make 1 ℔, whiche I kéepe in minde, and to the other 1 that remaineth, I adde that one that came of ẏ pens ⁊ was in my mind, whiche make 2, and them I wꝛite vnder the ſhillings.

Then doe I recken the pounds togither, 3,

6, 12,

6, 12, that is 21, and to them I adde the 1 is my mind that remayneth of the ſhillings, which make 22, them doe I write vnder the poundes, and then my ſumme totall appeareth to be 22 ℔, 2 ſ, 8 d.

℔	ſ	d
12	8	9
6	7	3
3	6	8
22	2	8

Another forme of proofe.

Now to examine this ſum and all ſuche like, you ſhall doe thus. Firſte beginne at the lefte hande with the poundes, and take from them that are aboue the lyne, 9, as often as you can: then that that remayneth ſhall you double, and ioyne it with the ſhillings, and take awaye 9 from that as often as you can, and whatſoeuer remayneth, yée ſhall take for it thrée tymes ſo muche, and putte to the pence: then take from all that ſumme 9, as often as you can, and what ſo remayneth after you haue wythdrawen 9 as often as you can, write that at the ende of a line, as I taughte you in the other Addition.

And then come to the ſum vnder the line, beginning with the pounds, and doe euen as you did with the ſummes aboue the line, till you come to your pennies: and if the figure of the ſumme that remayneth after caſting awaye 9, (as often as you can) doe agrée wyth the other that remayned before of the other ſumme, whyche you did write at the ende of the line, then haue you done well, elſe not: and for an example, I will examine that laſte ſumme which was thus:

li	s	d
12	8	9
6	7	3
3	6	8
22	2	8

Firſt I ſhal begin at the left hād with the pounds, putting them togither, whiche make 21, in which ſumme I finds 9 twice, (for twice 9 is 18) that I deducte, and there remayneth 3: that 3 muſt I double (as I ſaide) bicauſe it is the remayner of the poundes, and it will be 6. Then gather I the ſumme of the ſhillings, whiche is 21, to the whiche I adde the foreſaide 6, and then it is 27, wherein I finde 9 thrée times, and there remayneth nothing. This remayner ſhould I take thrée times, but thrée times nothing, is nothing: therefore in this place is there nothing left to be added to the pennies. Wherefore I muſte take the ſumme of penies alone, whiche is 20, from thence if I take 9 twice, there remayneth but 2, whych I putte vnto the ende of a lyne thus,

thus. 2 ———

When I come to the poundes of the vnder number or totall ſumme, and there I finde 22, from whiche I take awaye 9 twice, and there remayneth 4 : that 4 I double, and it is 8, then doe I adde that 8 to the ſhillings, and it maketh 10, from which I withdrawe 9, and there reſteth one : then doe I take that 1 thrée times, and it maketh 3, whyche I adde to the 8 d. and it maketh 11, frō whiche if I bate 9, there reſteth 2, which is equal to the number noted at the ende of the line : and therby I perceiue that I haue done well.

Scholer. But I doe not ſée the reaſon of this.

Maiſter. No : no more doe you of manye things elſe, but hereafter will I ſhewe you the reaſons of all Arithmeticall operations : for this I iudge to be the beſte trade of teaching, firſt by ſome briefe precepts to inſtruct a learner ſomewhat in the vſe of the Arte, before he learne the reaſons of the Art, and then may you afterward more ſoner make him to perceiue the reaſons : for harde it is to occupie a young learned witte with both the arte and the reaſons of it all at once : howbeit hée

The reaſon of this proofe.

The beſt trade of teaching.

ſhall neuer be cunning in deede in an arte, that knoweth not the reaſon of euerie thing touching it. But for this worke, bycauſe the reaſon is eaſie, I wil ſhew it you nowe. You knowe that if one pounde do remaine, it being tourned into ſhillings, woulde make 20 ß, in whiche number there is 9 contained twice, and 2 ß beſide. And therefore for one pounde you ſhal take 2 ß, and ſo for euerie one pound 2 ß.

Scholer. I ſee it well, for if there remained 7 li, after the nines were caſt away, I muſt take 14 ß for that 7 li. And ſo haue I caſte away 14 times 9 ß, and yet remayneth of euerie pound 2 ß which maketh 14 ß.

Maiſter. Like wayes in ſhillings, whiche containe 12 d: for euerie ſhilling, if you abate 9 pence there reſteth 3 pence.

Scholer. It is plaine ynoughe. And ſo if 5 ſhillings doe remaine, I muſte take for it 15 d, that is three pence for euerie ſhilling, and yet in that ſo doing, I haue caſte awaye fiue times nine pence.

Maiſter. Other workes haue as good reaſon, but I wil not ſtande aboute yeelding reaſons now.

Scholer. Yet one thing more I praye you ſhew ſhewe me, why did you write your number that remayned (after you hadde withdrawen all the nines) at the ende of a line: for I ſawe no reaſon why that line did ſerue.

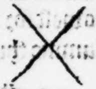
Maiſter. Did you euer marke a Carpenter when he wrought?

Scholer. Yea many times.

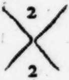
Maiſter. And haue you not ſeene him when he hath taken meaſure of a boarde, that hee hath pricked it, and hath with a twitch of his hande drawen a line from the pricke that hee made?

Scholer. Yes I haue marked that and haue ſeene ſome marke 3 or 4 lines by the pricke, ſome alſo haue I ſeene make a croſſe by it, but that I perceiued was for the eaſie finding of their pricke.

Maiſter. And euen ſo is this line for the eaſie finding of your remainer, and therefore ſome doe make a croſſe, thus.



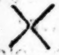
And ſet the one remainer aboue the croſſe, and the other vnder the nether part of the croſſe, as if I ſhould ſet my two remainers thus.

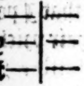


Another kind of proofe moſt vſuall and apteſt of all.

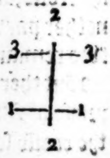
But there is another ſort of proofe of Addition, to which the croſſe ſerueth more meeter: & that is whe the addition is of diuerſe denominations: and I would examine euerie denomination by it ſelfe, which waye though it be not much vnlike to the firſt proofe that I brought of ſuche diuerſe ſummes, yet will I declare it, leaſte you ſhoulde thinke that I would hide it from you.

You muſte make ſo manie lines in your croſſe, as you haue ſundrie denominations: as if you haue but two denominations, then you may make it thus, that the ouer part and the nether part may ſerue for one denomination, and the two ſides for the other. And if you haue thrée denominations, as poundes, ſhillings, and pennies, then muſte you make thrée lines thus. The vpright line may ſerue for poundes, and the higheſt thwart line for ſhillings, and the loweſt for pens: as for example I will take a ſumme thus added.





℔	ſ	d
16	12	5
12	8	1
9	2	7
38	3	1



For the proofe of the which, bycauſe it contayneth thrée denominations, I muſt make a croſſe of thrée lines, as in the page afore. Thē I recken firſt at the righte hand the pennies: 7, 1, 5, make 13, from whiche I take 12 for the next denomination, that is to ſaie, a ſhilling, and there reſteth 1, which I muſte write at one end of the neather thwart line.

After that I gather the ſumme of the ſhyllings, 2, 8, 12, which maketh 22. to them I put one that I toke of the pennies, and that maketh 23: from thoſe I take 20, the quantitie of the next greater denomination, that is to ſay, a pound, and there reſteth 3, which I write at the ende of the higheſt thwart line.

Thirdlie, I adde togither the pounds, 9, 12, 16, whiche make 37, to them I adde the 1 that came of the ſhillings, and then there is 38, wherein I finde 4 times 9, and 2 ouer, that 2 I write on the vpright line.

That doone, I come to the totall ſumme, and examine it, beginning at the pennies, where I finde but one, and cannot take 9 frō him, therfore I ſet him at the other ende of yͤ neather thwart line: Then I come to the ſhillings, where I finde only 3, which bicauſe it is leſſe than 9, I ſette it at the other ende of the line of ſhillings, that is, the ouermoſte thwart line.

Laſt of all, of the 38 ℔, I take foure times 9, which is 36, and there remayneth 2, which I write vnder the vpright line.

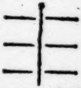
Thē I conſider euerie number, comparing it to the number that is againſte it, and bycauſe I finde them to be euerie one like hys match, I know that I haue wel done.

Scholer. This croſſe I perceiue doeth ſerue for thoſe thrée denominations, pounds, ſhillinges, pennies. But what if I had *ob*, *q̃*, *q*, and *c*?

Maiſter. You thinke you be at Oxforde ſtil, you bring forth ſo faſte your *q* and *c*. Theſe lines, as I haue ſaide, doe ſerue for thrée denominations, ſuch as they be: as here they do ſerue for poundes, ſhillings, and pennies: but if yée haue no poundes in your ſumme, then may they ſerue for ſhillings, pennies, and halfe pennies: yea for *q̃*, *q* and *c*, if you haue no greter denomination, ſo that you remember that the vpright line ſerueth for the greateſt denomination, and the higheſt thwarte line, for the nexte, and the loweſt for the leaſt,

And ſo if you haue foure denominations

you muſt make your croſſe with ſo many lines. And if that your ſumme be of more denominations, make ſo many lines in your croſſe. And thus will I make an ende of Addition.

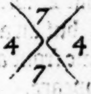


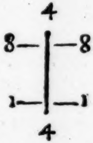
Examples of Addition.

℔	ſ
262587	6
41635	12
28124	2
471	4
332818	4

℔	ſ	d
340	17	10
28	6	8
13	13	4
382	17	10

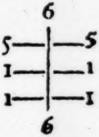
The Proofes.





An other Example.

℔.	ſ.	d.	q̃.
22		6	2
2	3	4	
	10	2	3
24	14	1	1



Scholer.

THen haue I learned the two firſt kindes of Arithmetike: nowe as I remember, doeth folowe Subtraction, whoſe name me thinketh doth ſounde contrarie to Addition.

Maiſter. So is it in déede: for as Addition encreaſeth one groſſe ſumme by bringing manye into one, ſo contrarie waies, Subtraction diminiſheth a groſſe ſumme by withdrawing of other from it, ſo that Subtraction or Rebating is nothing elſe, but an art to withdrawe and abate one ſumme from an other, that the Remainer may appeare.

Scholer. What do you call the Remainer?

Maiſter. That you maye perceyue by the name.

Scholer. So me thinketh: but yet it is good to aſke the trouth of all ſuch things, leaſte in truſting to myne owne coniecture, I bée deceyued.

Maiſter. So is it the ſureſt waye. And as I ſee cauſe, I wyll ſtill declare thyngs vnto you ſo plainelie, that you ſhall not neede to doubte. Howbeit, if I doe ouerpaſſe it ſometimes (as the manner of men is to forget the ſmall knowledge of them to whome they ſpeake) then do you putte me in remembraunce your ſelfe, and that way is ſureſt.

And as for this worde that you laſte aſked me, take you this deſcription: The Remainer is a ſumme lefte after due Subtraction made, which declareth the exceſſe or differẽce of the two other numbers: as if I woulde abate or ſubtract 14 out of 18, there ſhould remaine 4, which is called the remayner, and is the difference betwéen thoſe two numbers 14 & 18. Remainer.

Scholer. I perceiue then what Subtraction is: Nowe reſteth to knowe the order to worke it.

Maiſter. That ſhall you doe by this meanes. Firſte you muſte conſider, that if you ſhould go about to rebate, you muſt haue two ſundrie ſummes propoſed, the firſte which is your groſſe ſumme or ſumme totall: (and it muſt be ſet higheſt) and then the rebatement or ſumme to be withdrawen, which muſt be ſet vnder the firſte (whether it bee in one parcel or in many) and that in ſuche ſort, that the firſte figures be one iuſte ouer an other and ſo the ſeconde and thirde, and all other folowing, as you did in Addition: then ſhal you drawe vnder them a line, and ſo are your ſummes duelie ſet to beginne youre working.

Then beginne you at the righte hande (as you did in Addition) and withdrawe the nether number out of the higher, and if there remaine anye thing, write that righte vnder them beneth the line: and if ther remaine nothing (by reaſon that yͤ 2 figures were equal) then write vnder them a ciphar of noughte. And ſo doe you with all the other figures, euermore abating the lower out of the higher, and write vnder them the Remainer ſtill, til you come to the ende. And ſo will there appeare vnder the line what remayneth of youre groſſe ſumme, after you haue deducted the other ſumme from it, as in this example.

I receiued of your father 48 ℔, of whiche I haue layde out for you 36 ℔: nowe woulde I knowe what doeth remaine? and therefore I ſet my numbers thus in order: Firſt I write the greateſt ſumme, and vnder him the leſſer, ſo that the figures at the right ſide be euen one vnder another, and ſo the other, thus.

$$\begin{array}{r} ß \\ 48 \\ \hline 36 \end{array}$$

Then do I rebate 6 out of 8, and there reſteth two, which I write vnder them right beneath the line, thus.

$$\begin{array}{r} ß \\ 48 \\ 36 \\ \hline 2 \end{array}$$

Then I go to the ſecond figures, and do rebate 3, out of 4, where there remaineth 1, which I write vnder them righte, and then the whole ſumme and operation appeareth thus.

$$\begin{array}{r} ß \\ 48 \\ 36 \\ \hline 12 \end{array}$$

Whereby it appeareth, that if I withdraw 36, out of 48, there remaineth 12.

Scholer. Nowe will I proue in a greater ſumme: And I wil Subtract 2367924 out of 3468946. Thoſe ſummes I ſet in order thus

$$\begin{array}{r} 3468946 \\ 2367924 \\ \hline 2 \end{array}$$

Then doe I beginne at the righte ſide, and deducte 4 out of 6, and there reſteth 2, whiche I write vnder them. Then goe I to the ſeconde figures, and withdrawe 2 out of 4, and there remaine two, whiche I ſet vnder thẽ alſo: then I take 9 out of 9, and there reſteth 0, which I write vnder them: for you ſay, that if the figures be equall, ſo that nothing remain, I muſt write this ciphar 0 vnder them.

Maſter. It was well remembred, nowe go forth.

Scholer. Then I come to the fourth place and draw 7 out of 8, and there remayneth 1, which I write vnder them alſo. Then in the fifte place I take 6 from 6, and there reſteth nought, for it I write vnder them a ciphar, 0: Then in the ſixt place 3 rebated from 4, there remayneth 1, which I write vnder them: and likewiſe in the vij. & laſt place, 2 taken from 3, there is lefte 1, whiche I write vnder them: ſo haue I done my whole working, and my ſummes appeare thus. Whereby I ſee, that if I rebate 2367924, out of 3468946, there remayneth 1101022.

3468946
2367924
1101022

Maſter. Thys is well done. And that you maye be ſure to perceiue fullye the Art of Subtractiõ. let me ſee how can you ſubtract 52984732 out of 8250003456.

Scholer. Firſte I ſette downe the greateſt ſumme, and after that I wꝛite vnder it the leſſer number, beginning at the righte ſyde : and then my figures will ſtand thus.

8250003456
52984732

24

Then take I 2 from 6, and the reſte is 4 whiche I wꝛite vnder them : then doe I withdꝛawe 3 from 5, and there remayne 2, which I wꝛite vnder them. Then take I 7 out of 4, but that I cannot, what ſhal I now doe?

Note.

Mayſter. Marke well what I ſhall tell you now, how you ſhall doe in this caſe and in all other like. If any figure of the nether ſumme be greater than the figure of the ſumme that is ouer him, ſo that it cannot be taken out of the figure ouer him, then muſte you put 10 to the ouer figure, and then conſider how muche it is, and out of that whole ſumme withdꝛawe the nether figure, and wꝛite the reſt vnder them. Can you remember this?

Scholer. Yes, that I truſt I ſhall. Now then in mine example where I ſhoulde haue taken 7 out of 4 and coulde not, I put 10 to that 4, which maketh 14, from it I take awaye 7, and there reſteth 7 alſo, whiche I

wꝛite

write vnder them.

Mayſter. So haue you done well, but nowe muſte you marke another thing alſo: that whenſoeuer you doe ſo put 10 to any figure of the ouer number, you muſt adde one ſtil to the figure or place that followeth next in the nether line, as in this example there followeth 4, to which you muſt put 1, and make him 5, & then go on as I haue taught you.

Schol. Then ſhall I ſay: 4 and 1 (which I muſt put to him for the 10 that I added to 4 before) make 5, which I ſhould take out of 3, but that cannot be, therefore muſt I put to it alſo 10, & then it will be 13, from whiche I take 5, and there reſteth 8 to be written vnder them: and becauſe of that 10 added to the 3, I muſt ad 1 to 8 that followeth in the nether line, & that maketh 9, which I ſhould take out of 0, and cannot, therefore I put thereto 10, and that maketh 10: from 10 I take 9, and there remayneth 1, whiche I write vnder them.

Then doe I adde 1 likewiſe to the nexte figure beneath, which is 9, and that maketh 10, that 10 ſhould I take out of the figure aboue, but I cannot, for it is 0, therefore I put 10 to it, and ſo take I 10 out of 10, & there reſteth 0 to be written vnder them. Then come I to the next figure which is 2, and to him doe I ad 1, which maketh 3, that 3 I can not take out of nought, therefore of ẏ nought I make 10, and thence do I take 3, ſo remaineth there 7 to be written vnder them. Likewiſe do I put 1 to 5 that followeth, and then is it 6, that would I take out of 5, and cannot, therefore I adde 10 to that 5, and make it 15, from which I rebate 6, and there remaineth 9, which I write vnder them. Now haue I ſpent all the nether figures, & what ſhall I doe more?

Maſter. You ſhould haue added 1 to the next figure following (if there had bene any) becauſe you added 10 to the laſt figure before of the ouer line: but ſeeing there is no figure following, you muſt adde that 1 to the place following, and then deducte that 1 from the number aboue.

Scholer. Then ſhall I ſaye becauſe I borrowed 10 to the ouer 5, I muſte put 1 in the next place beneath, that is vnder 2: then muſt I ſubtracte that 1 from 2, and there reſteth 1, to be written vnder that, in the ninth place. Nowe I haue no more to ſubtracte, for there is neuer any figure remayning beneath, nether yet any vnite to be added, becauſe I borrowed not 10 to the figure laſte before, and yet is there 8 remayning in the ouer line, which (I thinke by reaſon) ſhoulde be ſet at the ende of the figures in the loweſt row whiche is vnder the line, for becauſe there was nothing taken from it.

Mayſt. That is wel conſidered, and reaſon teacheth ſo in dæde.

Scholer. But ſyr I beſéeche you, ſhall I alwayes when any number ſo remayneth alone (as this 8 did) write him vnder the lyne ſtraight againſt his owne place?

M. Yea, what elſe? Whether they bée one or many: and this wel remembred, you haue ſufficiently learned Subtractiõ. How be it, becauſe of certaine thinges that might deceiue you, if you did not take good héede to your working, I wil propoſe to you another example of many numbers to be ſubtracted, as thus.

I receyued of a friende of mine to kéepe 3869 Crounes, of which at one time I deliuered him againe 500, at one time 368, and at another time 440, and an other tyme 80, and an other time 64: nowe would I know how many doeth reſt behinde. Therefore firſte I ſet downe my groſſe ſum, and a lyne vnder it: & vnderneath it I ſet all the parcels, thus: and vnder them a double line.

2869 *Crowns receiued.*
500
368
440 *Deliuered.*
80
64

Then firſte I begin at the firſt place, & gather togither the ſum of al thoſe lynes (ſaue the ouermoſt) in their firſt figures, and ſo do I with all the figures of the ſecond place, & ſo forth as I did in Addition, ſaue that I leaue out the higheſt row of numbers (as the line warneth me) and that ſumme ſo gathered betwéen the double line, doe I ſubtract out of the higheſt row of numbers, and the remayner doe I ſette vnder ỹ nethermoſt line: as for example.

2869 *Receyued.*
500
368
440 *Deliuered.*
80
64
1452
1417 *Reſt behinde.*

I ſet the ſummes as before: than doe I gather the firſt figures together, where I fynde but 4 & 8, that make 12, (for thrée Cyphers increaſeth no ſumme in addition, as you learned before) of the 12 therefore doe I write the digite 2 betwéene the double line, and kéepe the article in my mind, til I come to the seconde place, where I finde 6, 8, 4, 6, that make 24, to them I put the article in my mind, and it is 25, of which I write 5 vnder the ſeconde place, and kéepe the digite 2 in my minde for the third place, where I finde 4, 3, 5, that make 12, to the which I adde the 2 in my minde, and it maketh 14, thereof I write the 4 vnder the thirde place: and becauſe there remayneth no more figures to be added, I write the digit 1 in the fourth place, as you ſée in the example.

Then come I to ſubtracting of this ſūme betwéene the lines, for by Addition it is equall to the fiue parcels ouer it. Therefore I procéede to ſubtracte it from the ouermoſte ſumme, ſaying: 2 from 9, remayne 7, to bée written vnder thē beneath the loweſt line. Then in the ſeconde place I take 5 from 6, and there reſteth 1, to be written vnder thē. Then in the third place, 4 from 8, reſteth 4. Laſt of all in the fourth place, 1 from 2, remayneth 1. And thus I ſée that after thoſe 5 ſummes are ſubtracted from 2869, the Remainer is 1417.

Scholer. This I perceyue: but is there no ſhorter way and more ſpeedier?

Mayſt. Yes, when you are a while exerciſed in it: for you may as faſt as you can gather the numbers togither, withdrawe them out of the higheſt ſume, if ſo be it, that all the parcels which you doe gather, do not exceede nine, but and if they exceedes nyne, then muſt you ſubtract onelye the digit that is in it, and reſerue the article till the nexte place, where you ſhall adde it with the other figures and ſo ſubtract the whole out of the figure aboue them: but and if in this place the ſumme of the parcels do exceede 9, then (as I ſayd before) ſubtract the digit only, and reſerue the article to the nexte place, and ſo ſtill goe forth, till you haue ended your working.

An abridgement of the former maner of Subtraction.

As for example: In the laſt ſumme propoſed, I gather firſte in the firſte place 4 and 8, that maketh 12, of whiche I deducte the digitte 2 out of 9, and write vnder the remayner, which is 7, and the article 1 I keepe in my minde. Then in the ſecond place I gather the parcels 6, 8, 4, 6, which amount to 24, to that I adde the article 1, which I haue in my minde, and then is it 25, then do I take 5 (that is the digitte in this number) frõ 6, that is in the ſeconde place of the higheſt ſumme, and there remayneth but 1 to bée written vnder them, and nowe doe I kéepe the article 2 in my minde ſtill. Then in the thirde place, 4, 3, 5, maketh 12, and the article 2 in my minde maketh 14: then take I 4 (which is the digitte) from 8, that is ouer them, and there reſteth 4, whiche I write vnder them. Then haue I the article 1 yet in my minde, which I ſhoulde adde to the parcels next following, but ſéeing there is no number following, I take that digit alone, and deduct him out of the next ſũme aboue, which is 2, and then is the remayner 1, which I write in the fourth place vnder 2. Loe, now haue you a ſhorter way.

Scholer. I like both wayes well, and I perceiue both well, yet as in the one the working ſéemeth ſomewhat long, ſo in the other it leaneth very much (me ſéemeth) to remembrance, and therefore maye cauſe errour quickly, except a man haue a quick and an exerciſed remembraunce.

Mayſter. What? would you then haue ſuche a way that ſhould not be ſo long as the one,nor ſo ſhort as the other?

Scholer. Yea,if there were any ſuche.

Mai. Than doe thus: ſtill as you gather your parcels,when they excéed a digit,& maketh him 10 or more, take the article, and write him betwéene two lines (as in the firſt example) vnder the next place towarde the left hande: and then deducte the digitte from the figure that is ouer him, and wryte the remayner. And then when you gather the next parcels,you ſhall adde to them the figure that is vnder them,betwéen the two lines.And if it excéede 9,do as I ſaid before, write the article vnder yͤ next place betwéen yͤ lynes,and ſubtracte the digitte from the Figure that is ouer thoſe parcelles: and if that all the parcelles togither and the number betwéene the lines do make but a digit, then deduct it wholy from the figure aboue: as in this example. I would ſubtracte oute of 4 0 3 0 8 9 6 4,

theſe thrée parcelles
20003428.
10002342.
10101461.

40308964
20003428
10002342
10101461

Therfore I ſet the firſte in order due: &

then I gather the parcels of the firſte place, which are 8,2,1, that is 11: of whiche I take away the article, and ſette him vnder the ſecond place betwene the lines: and the digit 1 that remaineth, I deduct out of 4, and there reſteth 3 to be written vnder the firſt place beneath the loweſt line. Then come I to the ſecond place, and gather the parcels of it, 6,4, 2, and the 1 betweene the lines, which make 13, of whiche I take the article, and ſet him vnder the third place betwéen the lines, and the digit 3 I take from 6, and there remaineth 3, which I write vnder the ſecond place beneth the loweſt line. Than in the thirde place I find 4,3,4, which with the 1 betwéen the lines, doe make 12, therefore I write the article agayne vnder the fourth place, and the digit 2 I take from 9, and there remayneth 7 which I write vnder them beneath the loweſt line.

And thē come I to the fourth place, where I gather 1,2,3 & the 1 betwéene the lines, that maketh 7, which becauſe it is but a digitte I plucke from 8, and the Remayner is 1. and muſt be written vnder them in the fourthe place. After that come I to the fifte place, where are onely thrée cyphars, which make nothing, thẽ ſhould I take that, that is to ſay nothing, from the figure ouer them, whiche is alſo a cyphre, therefore I muſt ſaye thus: if I take nought from nought, there remayneth nought: ſo muſt I write a cyphre vnder them. Thẽ in yͤ firſt place I find but 1, which I take out of 3 ouer him, and the Remainer is 2, that muſt be written beneth the loweſt line in the firſt place. So I go to the ſeuenth, where I find only cyphres, and in the groſſe ſum ouer them a cyphre alſo, therefore muſt I write their remainer (whiche is nothing) with a cyphre alſo. Then in the eighte and laſt place, I gather 1, 1, 2, that make 4, which if I take out of that 4 that is ouer thẽ, there will nothing remaine. And that muſt be noted with a cyphre beneath the loweſt line, as I haue often ſayde, and ſo haue I ended my worke, and the figures ſtand thus.

$$\begin{array}{r} \underline{40308964} \\ \left.\begin{array}{r} 20003428 \\ 10002342 \\ 10101461 \end{array}\right\} \\ \hline 1 \\ \hline 00201733 \end{array}$$

Scholer. Syr, I remember you taughte me that cyphers ſhoulde not come in the laſt place, for becauſe they ſerue onelye to encreaſe the valewe of other Figures whiche followe them, and ſerue not for thoſe figures that goe before them: and nowe in your example you haue ſet two ciphers in the two laſte places.

Maſter. I commende you for your rememoraunce. And truth it is, I ſhould not haue ſet them here, but only becauſe that I would make you plainly to perceiue the art of Subtraction. Therfore ſeeing that you do now perceiue it, whenſoeuer you ſhoulde write downe a cyphar, looke whether anye other figures be yet behinde. And if not than let go the cyphar alſo, for it needeth not to write him in any latter places, where no other figure doth followe, excepte it be (as I did) to teache the vſe of Subtraction the playner.

Therefore my figures muſt ſtande thus when I haue ended my worke.

40308964	*Charge.*
20003428 10002342 10101461	*Diſcharge.*
111	
201733	*Reſt.*

Scholer. So I woulde thinke by yͭ you taught me before. And now I beleeue I could ſubtract any ſummes.

Maiſter. So may you if you haue marked what I haue taught you. But becauſe this thing (as all other) muſt be learned ſurely by often practiſe, I will propounde here two examples to you, wherein if you often exerciſe your ſelfe, you ſhalbe ripe and perfecte to ſubtract any other ſumme lightly, for in them is contained all the obſeruaunces of whole number. And becauſe you ſhal perceaue ſomewhat both how to doe it, and alſo whether it be well done when you haue proued to doe it, therefore haue I written vnder them, both the Remaynders: And to one of them alſo adioyned his proofe.

308964	*Lent.*
103145	*Repayde.*
102597	
101024	
11	
2198	*Reſt.*
308964	*Proofe.*

125614	*Debt.*
842	*Payde.*
681	
201	
11	
124390	*Reſt to pay.*

Scholer. Sir, I thank you. But I thinke I might the better doe it, if you did ſhewe me the working of it.

Mayſter. Yea, but you muſt proue your ſelfe to doe ſome thinges that you were neuer taught, or elſe you ſhall not be able to doe any more than you were taughte: And that were rather to learne by rote (as they call it) then by reaſon. And agayne there is nothing in this example or anye other of whole number, but I haue taughte you the rules of them alreadie.

Scholer. Then I truſt by practiſe to attaine the vſe of it. And is this all that I ſhall learne of Subtraction?

Mayſter. Yea, ſauing that (as you haue ſéene in Addition) there are numbers of diuers denominations, in whiche the working is not muche vnlike, yet without ſome inſtructions be giuen of it, it might ſéeme to a learner more difficulte, then in déede it is. Therefore I will briefly ſhewe you the vſe of it onely, by one example or two.

A certayne man owed to me 14 ℔, 12 ß, 8 d, of which he paide me at one tyme 4 ℔, 6 ß, 8 d: at an other time 3 ℔, and at an other 2 ℔, 3 ß, 4 d, and laſt of all, 6 ß, 8 d.
Now would I know what remaineth vn-

payde yet, therefore I ſet my ſummes thus.

℔	ß	d
14	12	8
4	6	8
3		
2	3	4
	6	8

Scholer. Syr, I pray you why doe you write 2 ℔, for the common ſpeeche vſeth rather to ſay 40 ß.

Maiſter. We muſt here vſe the denomination that is greateſt in any ſumme, ſo that we maye not write according as we vſe to ſpeake, ſaying: 16 d. 18 d: or likewayes, 7 grotes, 8 grotes: 24 ß. 40 ß 48 ß. and ſuche other, but we muſte write euerye denomination that is in any ſumme by it ſelf, namely ſhillinges and poundes. So muſt we write for theſe ſummes now named, 1 ß, 4 d: 1 ß, 6 d: 2 ß, 4 d: 2 ß, 8 d: 1 ℔, 4 ß: 2 ℔, 8 ß: and ſo forth of other like.

Scholer. So that we maye not write in Arithmetike pennies, when the ſumme amounteth to ſhillings, nor ſhillinges when the ſum maketh poundes. Nowe if it pleaſe you, ende your example.

Mai. When my ſummes are ſo ſet as I ſhewed, then muſt I begin with the ſmalleſt denomination, ſaying: 8, 4, 8, are 20. which ſumme becauſe it is pence, & 12 pence

℔	ß	d
14	12	8
4	6	8
3		
2	3	4
	6	8
	1	
4	16	

doe make 1 ß, I muſt take from that 20 (which commeth of the parcels) 12. & for them write betwéene yͤ lines vnder yͤ ſhillings, than the 8 d, that remayneth I take out of yͤ higheſt ſum, which is 8 alſo, & then remayneth nought: wherfore vnder the pence I write nothing. Then come I to the ſhillinges, and gather the parcels 6, 3, 6. whiche with the 1 betwéene the lines, make 16, that muſt I take out of the ſumme that is ouer it. But ſéeing that ſumme is but 12, I cannot take 16 out of 12, I muſt borrow one of the 14 ℔, and put to the 12, and that maketh 32, for 1 ℔ is worth 20 ß, then take I 16 out of 32 and there reſteth 16 to be written vnder the ſhillinges. Then come I to the poundes, whoſe parcels are 2, 3, 4 that is in all 9, and one more muſt I adde therto, becauſe of the 1 that I borrowed before vnto the 12 ß, and than is there 10, whiche I muſt take oute of 14, ſo doth there remain 4 to be written vnder the poundes: ſo doeth my remainer appeare to be 4 ℔, 16 ß.

Scholer. This doe I perceiue verie well, and if there be none other thing to be learned in Subtraction, then maye I come to Multiplication, foꝛ that you reckened to bée in oꝛder next.

Maiſter. We haue done in déede wyth the arte of Subtraction, as touching the woꝛking. But yet befoꝛe we go to multiplication, I wil inſtruct you how to examine your woꝛk whether it be well done oꝛ no, and that is by caſting away 9 as oftē as you can finde it, as you did in Addition, ſauing that you muſte here examine the higheſt number alone, and note the reſidue of it at a lines end, as you did in Addition.

A proofe of Subtraction in numbers of one denomination.

And when you haue done with the higheſt number, then examine all the other togither, caſting thence 9 as often as you can: and if the laſt remayner be like the other, then haue you done wel.

But if you haue diuerſe denominations in your ſumme, yet foꝛ them all ſhall you make but one ſeuerall line, as you did in Addition, remembꝛing to begin the examination at the greateſt denomination, and to double the remayner of pounds, and triple the remainer of ſhillings, as you did alſo in Addition.

A proofe in Subtraction amōg diuerſe denominations.

℔	ſ	d
14	12	8
4	6	8
3		
2	3	4
	6	8
4	16	

As for a proofe, I will examin this work where in the higheſt line I finde of poũds 14. from thence I bate 9, & there reſteth 5, which I doe double, becauſe they are pounds, and thẽ are they 10 thereto I ad the 12 & it maketh 22 frõ whiche I take 9 twiſe, & there reſteth 4, whiche becauſe they are ſhillinges, I triple, and then are they 12, thereto I adde the 8, & then are they 20 thẽce take I twiſe 9, and yet reſteth 2, whiche I write at the one ende of a line thus, 2 ———

Then I examine all the other parcelles & the remayner together, euery denominatiõ by it ſelfe. And firſt of poundes I finde 4, 3, 2, 4 that is 13. from which I take 9, & there reſteth 4. that doe I double, and it maketh 8, to it doe I put the ſhillinges, 6, 3, 6, 16. that is 31 (for the one betwéen the lines muſt not be reckoned nor none in that ſpace) and that maketh in all 39. Where hence I take 9, foure times, and there remayneth 3. yͭ doe I take thrée times, and it is 9, wherfore I caſt it away: then doe I take the pennies 8, 4, 8. that maketh 20, from which I take 9 twiſe, and there reſteth 2. whiche I wꝛite at the other end of the pꝛoofe lyne. And bicauſe I ſée that thoſe two numbers are equall, I ſaye that I haue well wꝛought.

And if you will you may make foꝛ euerye denomination a line, as you learned in Addition: but then muſt you beginne your examination at the ſmalleſt denomination, as you did in Addition, foꝛ their pꝛoofe is altogither like, ſauing that in Addition you examine the nethermoſt ſumme alone, and all the other togither: and in Subtraction yée muſt examine the higheſt number alone, & al the other togither. And if you marke it wel, it is euen all one, foꝛ that ſumme that in Addition is loweſt, in Subtraction is higheſt: and that ſumme is called the Groſſe oꝛ Totall ſumme.

Groſſe oꝛ total ſumme.

Therefoꝛe if you marke what I ſayde in Addition, you may eaſilye perceiue what is to be done foꝛ the pꝛoofe of Subtractiõ. And to the intent that you may perceaue it yͤ better, I will ſhew you an other pꝛoofe of Subtraction, and that ſhall be by Addition, thus. Dꝛaw vnder yͤ loweſt nũber (which is your remayner) a line: then ad that number, and all the other that you did ſubtract befoꝛe, to-

An other proofe of Subtractiõ.

gether, and wꝛite that that amounteth, vnder the loweſt line: and if the ſumme that commeth thereof, be equall to the higheſt of the ſubtraction, than was the ſubtraction well wꝛought, oꝛ elſe not. As foꝛ example: in the laſte ſummes, which ſtoode thus.

℔.	ß.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	

Firſt I adde 3, 4, 8, that maketh 20: whereof I take 12 awaye, bycauſe they make one ſhilling, and wꝛite foꝛ them 1 vnder the ſhillings: and the 8 that is lefte, I wꝛite beneath the loweſt line, then adde I the ſhillings 6. 3. 6. 1. 16. that make 32: from whiche I take 20, and foꝛ it I wꝛite 1 vnder the poundes, and the 12 that remayneth, I wꝛite vnder the ſhillings. Then come I to the poundes, adding them togither, which are 4. 3. 2. 1. 4. that maketh 14: then doe I wꝛite 14 vnder the ℔, and ſo haue I ended the Addition. And I ſee that the loweſt line of number and the higheſt be like, wherfoꝛe I know that I haue wel done. Foꝛ my figures appeare as you may plainlie ſee in the page folowing.

And thus nowe haue I taughte you the arte of Subtraction, and the means to pꝛoue whether it be well wꝛoughte oꝛ not.

And this laſt pꝛoofe of Subtraction is moſt apteſt and beſte allowed of any other pꝛoofe: whether it be of ℔ ſ. d. oꝛ any other groſſe ſome whatſoeuer.

℔.	ſ.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	
14	12	8

Nowe and you remember, I omitted in teaching the pꝛoofe of Addition one way which I ſaid was by ſubtraction.

Scholer. Trueth it is, and then was it deferred, becauſe that I had not thē learned the feate of Subtraction, whereby I ſhould haue pꝛoued it, but now I thanke you, I haue wel learned the arte of Subtraction, & the pꝛoues of it, both by 9. and by addition. And nowe I would be glad to know, howe I may pꝛoue Addition by Subtraction.

The profe of Addition by Subtraction.

Maiſter. Then marke you this. Whē you haue ended your addition: take the numbers all that you did adde, to the higheſt ſumme, and deduct oꝛ ſubtract them from the groſſe ſumme that doth reſulte, and if the remayner be like to the higheſt number, then haue you done well, elſe not.

As for example. I take one of the ſummes that I did adde before, which was this that followeth here.

```
106800
  9400
------
116200
```

Then doe I come to the middle number (becauſe here in this example are onelye thrée numbers) and ſubtracte that from the nether nūber, beginning at the right hande, and firſt I ſaye, 0 out of 0, there remaineth 0: that write I vnder an other line. Then againe, 0 in the ſecond place from 0, remaineth 0, vnder it I write 0 alſo. Next that in the third place, 4 out of 2 wil not be, therfore I ad to that 2, 10, and make it 12 from that I take 4 and there reſteth 8. Then ſaye I farther: 9 in the fourth place, and 1 (whiche I muſt adde for the 10, borrowed before) make 10, that muſt I take from 6: and becauſe I canne not, I adde to the 6, 10, and then is it 16: from then I take 10, and there reſteth 6, to be written vnder them. Againe in the fiſt place where I finde nothing written, I muſt ſette 1 for the 10 laſt borrowed, and that 1 doe I take from the 1 vnder him, and ſo remaineth noughte, wherefore I write downe a cypher 0. Now haue I done with the ſubtraction: and yet in the groſſe ſumme remayneth 1, whiche I muſt ſette right in the ſame place, in the remayner, and ſo the remayner appeareth to bee lyke vnto the higheſt ſumme of the Addition, as here appeareth. Wherefore I ſay that the Additiō was wel wroughte. And note, that if you had ſubtracted the vppermoſt from the product or totall ſūme, then the reſidue thereof woulde be equall to that middlemoſt number. But if the parcelles which you added, be more then two: (as thrée foure, fiue, ſixe, or more) than from your groſſe or totall ſumme ſubtracte firſt one of the percels: and note that newe reſidue. Out of that new reſidue, ſubtract another of your percels, (whiche you will) and Note that ſecond new reſidue. And if you haue no mo percels added, but thrée, than is that ſecond new reſidue equall & alike to the thirde parcel, whiche you haue not (as yet) ſubtracted, if you haue wrought wel: both in your firſt Addition, and now in youre ſubtracting

```
106800
  9400
------
116200
------
106800
```

tracting. And ſo in this wiſe, (if you haue foure, fiue, or more parcels) maye you procéede to make your ſelfe ſure of your totall ſumme, firſt, by Addition of the ſaide parcels, produced and gathered. And thus may you doe in any other ſumme of one denomination or many: Sauing that I wil tel you by the way, the laſte manner of proofe that was ſhewed you in Addition, is the beſte, and the apteſt proofe for the Rule of Addition.

Scholer. Sir, I thanke you moſte heartily.

Maiſter. Therefore nowe will I make an ende of Subtraction, and will inſtruct you in Multiplication.

MVLTIPLICATION.

Multiplication is ſuch an operation, that by two ſummes produceth the third: whiche third ſumme ſo many times ſhall containe the firſte, as there are vnities in the ſecond.

cond. And it ſerueth in the ſteade of manye Additions. As for example. When I would know how many are 30 times 48: if I ſhould ad 48, thirtie times, it would be a lōg work. Therfore was this worke of Multiplication deuiſed, which ſhall doe that at once, that Addition ſhould do at many times.

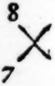
Scholer. I perceiue the commoditie of it partlie, but I ſhall not ſee the full profit of it, till I know the whole vſe of it. Therefore ſir I beſeech you, teache me the working of it.

Maiſter. So I iudge it beſte, but becauſe that greate ſummes can not be multiplyed, but by the multiplication of digits, therefore I thinke it beſt to ſhewe you firſt the way of multiplying them: As when I ſaye, 8 times 8, or 8 times 9 &c. And as for the ſmall digits vnder 5, it were but follie to teach any rule, ſeing they are ſo eaſie, that euerie childe can doe it. But for the multiplication of the greater digits, thus ſhal you do.

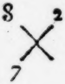
Firſte ſet your digittes one ouer the other righte, then from the vppermoſt downward, and from the neathermoſt vpwarde, drawe ſtraighte lines, ſo that they make a croſſe, commonlie called Saincte Andrewes croſſe, The diffe-rence.

as you ſee here. Then looke how many eche of them lacketh of 10, and write that agaynſt eche of them, at the end of the lynes, and that is called the Differences, as if I would know how many are 7 times 8, I muſte write thoſe digittes thus.

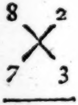
Digis difference.



Then doe I looke how much 8 doth differ from 10, and I finde it to be 2, that 2 doe I write at the right hande of 8, at the end of the line, thus.



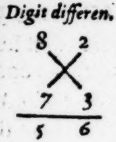
After that, I take the difference of 7 likewiſe from 10, that is 3, and I write that at the righte ſide of 7, as you ſee in this example.



Then doe I drawe a lyne vnder them, as in Addition, thus.

Laſte of all I multiplie the two differences, ſaying: 2 times 3 make 6, that muſte I euer ſet vnder the differences, beneth ẏ line: then muſt I take the one of the differences (whiche I will, for all is like) from the other digit (not from his own) as the lines of

the

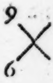


the croſſe warne me, and that that is left, muſt I write vnder the digits. As in this example. If I take 2 from 7, or 3 from 8, there remaineth 5: that 5 muſt I write vnder the digits: and then there appeareth the multiplication of 7 times 8, to be 56. And ſo likewyſe of anye other digits, if they be aboue 5, for if they be vnder 5, then will their differences be greater than themſelfe, ſo that they cannot be taken out of them. And agayne, ſuch little ſummes euerye childe can multiply, as to ſay: 2 times 3, or 4 times 5, and ſuch like.

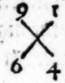
Scholer. Truth it is. And ſeeing me ſeemeth that I vnderſtand the multiplying of the greater digits, I will proue by an example how I can doe it. I would knowe howe many are 9 times 6.

Mayſter. It is all one in value to ſaye 9 times 6, or 6 times 9: but yet the order is beſt to put the leſſe ſumme firſte, ſaying: 6 times 9, and ſo of al other ſummes.

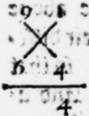
Sch. Then would I know, howe many are 6 times 9: therefore I ſet yͤ digits thus, & make the croſſe thus.



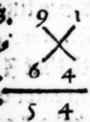
Then doe I ſet their differences at the right ſide: the difference of 9 which is 1 againſte it, and the difference of 6, whiche is 4 againſte it alſo, as in this example.



And vnder them I drawe a line: Then doe I multiplie the digites togither, ſaying: one tyme 4 maketh 4, that 4 doe I write vnder the differences thus.



Then take I one of the differences from the other digite, as one from 6, or elſe 4 from 9, and eache wayes there reſteth 5, which I do write vnder the digits thus. And ſo appeareth the multiplication of 6 times 9, to bee 54. Thus I ſee the feate of this manner of multiplication of digittes.



Maiſter. Now mighte you goe ſtraight to the multiplication of greater numbers, ſaue that both for your eaſe and ſuertie in working, I wil drawe you here a table, whereby ſhall appeare the multiplication of all digits, and this is it that followeth in the next page.

1	1	2	3	4	5	6	7	8	9
	2	4	6	8	10	12	14	16	18
		3	9	12	15	18	21	24	27
			4	16	20	24	28	32	36
				5	25	30	35	40	45
					6	36	42	48	54
						7	49	56	63
							8	64	72
								9	81

In which figure, when you would knowe
he producte in anye multiplication of digits,
eke your firſt or laſt digit in the greater fi-
gures, and from it go righte forthe towardes
he right hande, til you come vnder the num-
er of your ſecond digite, which is in the high-
ſt rowe: and then the number that is in the
eeting of the rowes of little ſquares (whiche
ome directlie from both your propounded di-
gits) is the multiplication that amounteth of
hem. As if I woulde knowe by this table the
ultiplication of 7 times 9, ſeeke firſt 7 in the
greater figures, and then go right forthe to-
ward the right hand, till you come vnder 9 of

the higheſt rowe, in which place where you ſo come vnder the other digit (as here for ex ample you come vnder 9) is alwaies contai ned the ofcome or product, which you ſeeke: and that place we tearme to be in the com mon angle, in reſpect of the two numbers ſo taken on the outſides, as here in that com mon angle, where yͤ rewes of little ſquare (directly proceeding from 7 and 9) do meete, you haue 63, which 63 is the ſumme of th multiplication of 9 by 7.

Scholer. This is very good and ready. And ſo may I find the multiplication of any digits. But now how ſhall I doe in greate ſummes?

Maiſter. When you would multiplie a ny ſumme by an other, you ſhal marke tha it is the meeteſt order to ſet the greateſt nū ber higheſt, which is the place of the nūber must be multiplied: and likewyſe the leſſe number vnder it, for that is the place of th Multiplier or multiplicatour, that is to ſay, the nūber by which multiplicatiō is made: and is in Engliſhe alwayes put before this worde, Tymes: in ſuche ſpeaking when I ſay, 20 times 70. And the number that fol loweth this word, Times; is that which

muſ

muſt be multiplied.

Therefore when I would multiplie one number by another, I muſt write the greateſt higheſt, and the leſſer vnder it, as in Addition. And vnder them muſt I drawe a line. As for example: If I would multiplie 264 by 29, I muſt ſet them thus.

```
264
 29
———
```

Then muſt I multiplie euery figure of the higher rowe by euery figure of the nether rowe: and that that amounteth, I muſt ſet vnder the line, as thus. Firſt I doe multiplie 4 by 9, ſaying: 9 times 4 (or 4 times 9, whiche is all one) and that maketh 36, as the table before of digittes doeth declare, of that 36 I muſte write the 6 that is the digitte, vnder the 9, and the 3 in the next place towarde the lefte hande.

```
264
 29
———
 36
```

Then come I to the ſeconde figure of the higher rowe, and ſaye: 9 times 6 make 54, of which I write the 4 vnder the 3, and the 5 vnder the next place (as the reaſon willeth me) thus.

```
364
 29
———
536
 4
```

After that come I to the next figure, which is 2, and do multiply it by 9, & that maketh 18:

whereof I write 8 vnder the thirde place, and the article 1 in the fourth place, thus.

$$\begin{array}{r} 264 \\ \underline{29} \\ 1536 \\ 84 \end{array}$$

And ſo haue I ended the firſt figure of yͤ multiplier. Wherfore I giue it now a fine daſh with my pen.

Then beginne I with the nexte figure, and multiply it into all the higher figures, as thus.

$$\begin{array}{r} 264 \\ \underline{2\cancel{9}} \\ 1536 \\ 84 \\ 8 \end{array}$$

Firſt, 2 times 4, make 8, that doe I write vnder the ſecond place: for euermore the digite or firſt figure of multiplication that amounteth of the firſt figure of the higher number, muſt be ſet vnder the multiplier of it, and the other in their order, toward the left hand.

Scholer. I vnderſtande you thus: that the digit of the ſumme amounting of the multiplication of the firſt figure of the higher row, by the firſte figure of the lower row or multiplyer, muſt be ſet vnder the firſt place: & that that amounteth of the ſame firſt figure by the ſeconde multiplier, muſte be ſet vnder the ſeconde place, & ſo of the other, if there be more multiplyers.

Maiſter. So meane I indéede: and if

there amount but a digit, then muſt it be ſet vnder the multiplyer.

And now to go forth: I multiplie by the ſame 2, the ſecond figure of the higher rowe, which is 6, ſaying: 2 times 6, make 12: whereof I write the digit 2 vnder the thirde place, & the article 1, I write vnder the fourth place.

$$\begin{array}{r} 264 \\ 2\cancel{9} \\ \hline 1536 \\ 184 \\ 28 \end{array}$$

Then do I multiplye the laſt figure of the higher ſum, by that ſame 2, ſaying: 2 times 2 is 4: whiche I write vnder the fourth place. And ſo haue I ended the whole multiplication: wherefore I alſo giue the 2 a daſhe with my pen, thus: and ſo I do euer aſſone as I haue diſpatched any digit by whiche I multiplie. And the ſumms ſtand thus.

$$\begin{array}{r} 264 \\ \cancel{2}\cancel{9} \\ \hline 1536 \\ 184 \\ 428 \end{array}$$

Than muſt I draw a line vnder all thoſe ſummes that amount of the multiplicatication, and muſt ad all them into one ſumme, as in this example you may ſee.

$$\begin{array}{r} 264 \\ \cancel{2}\cancel{9} \\ \hline 1536 \\ 184 \\ 428 \\ \hline 7656 \end{array}$$

Where in the firſt place I find but 6, and therefore write I it vnder the line. Then in the ſeconde place 843, make 15, whereof I write 5, and keepe one in my minde, and ſo forth, as you learned in Addition. And ſo appeareth the whole ſumme to be 7656, which amounteth of the multiplication of 264 by 29.

Scholer. If there be no more to be obſerued in it, then can I do it, I ſuppoſe, as by this example I ſhall proue. I would multiplie 1365, by 236, wherefore I ſet them thus.

```
1 3 6 5
  2 3 6
-------
```

Then doe I multiplie 5 by 6, ſaying: 6 times 5 make 30: of whiche I write the ciphre in the firſt place, and the article 3 in the ſeconde place.

```
1 3 6 5
  2 3 6
-------
    3 0
```

Then do I by the ſame 6, multiplie the ſeconde fygure of the higher ſumme, whiche is 6, ſayinge: 6 tymes 6 make 36: of which I write the 6 vnder the ſeconde place, and the 3 vnder the thirde place.

```
1 3 6 5
  2 3 6
-------
  3 3 0
    6
```

Then doe I multiplie the thirde figure whiche is 3, by the ſame 6, and that maketh 18: of that I ſette the 8 vnder the thirde place, and the 1 in the fourth place.

$$\begin{array}{cccc}1&3&6&5\\ &2&3&6\\ \hline 1&3&3&0\\ &8&6&\end{array}$$

Then come I to the laſt figure of the higher ſum, and multiplie it by 6, ſaying: 6 tymes 1 make 6: that doe I write vnder the fourth place. And ſo haue I ended the firſte multiplier, and daſh him ſlightly with my pen.

$$\begin{array}{cccc}1&3&6&5\\ &2&3&\cancel{6}\\ \hline 1&3&3&0\\ 6&8&6&\end{array}$$

Then begin I with the ſecond multiplier, and ſay firſt 3 times 5, that maketh 15, of which I ſette the 5 vnder the ſecond place, becauſe that the multiplier is there, and the 1 I ſet vnder the thirde place.

$$\begin{array}{cccc}1&3&6&5\\ &2&3&\cancel{6}\\ \hline 1&3&3&0\\ 6&8&6&\\ &1&5&\end{array}$$

Then come I to the ſeconde figure, that is 6, and multiplie it by 3, whiche maketh 18, of which I ſette the 8 vnder the thirde place, and the article 1 in the fourth place.

$$\begin{array}{cccc}1&3&6&5\\ &2&3&\cancel{6}\\ \hline 1&3&3&0\\ 6&8&6&\\ 1&1&5&\\ &8&&\end{array}$$

Than come I to the third figure, which is 3, and multiply it by 3, ſaying: 3 times 3, make 9, whiche becauſe it is but one digit, I ſet vnder the fourth place.

1365
236
1330
686
115
98

And then comming to the laſte figure 1, I multiplye it by 3, and it maketh 3. which I ſet in the fift place, and then haue I ended two of the multiplyers, and the ſummes ſtande thus. And then I giue 3 his daſhe.

1365
236
1330
686
115
398

1365
236
1330
686
115
398
10

Then come I to the thirde multiplier, and multiplye it into euery figure of the higher ſumme, and firſte I ſaye: 2 times 5 makes 10, of whiche I ſet the cipher vnder the multiplyer in the third place, and the article 1 in the fourthe place.

And ſo multiplying the ſeconde figure 6, by that ſame 2, there amounteth 12: whereof of I write the digitte 2, vnder the fourth place, and the article 1, vnder in the fift place.

```
 1365
  236
-----
 1330
 686
 115
398
110
 2
```

Nowe doe I multiplye by the thirde figure of the higher ſumme, which is 3, and that maketh 6: which I ſet vnder the fifte place, as appeareth in the example following.

```
 1365
  236
-----
 1330
 686
 115
398
110
62
```

Than come I to the laſt place, and multiplye that 1 by 2, and there amounteth 2, which I ſette in the ſixte place, and then doth the ſum ſtande thus.

```
  1365
   236
------
  1330
  686
  115
 398
 110
262
------
322140
```

And ſo haue I ended the whole multiplication.

But now (as you taughte mee) to knowe what this whole ſumme is, I muſt ad all thoſe parcelles together, and then vnder the line will appeare as you maye ſee the groſſe or totall ſumme, that is, 322140.

That

Mai. That is well done.

Sch. Then me thinketh I would call it well done, when I knew whether I had well done or no.

Maiſter It may be tryed by 9, as addition was, but the ſureſt proofe is by Diuiſion, and therefore I will reſerue that till you haue learned the art of Deuiſion.

ye abrid. uſe multipl. by

And before we paſſe from Multiplication I wil yet ſhew you another way of multiplication, which is counted of ſome men, and is in déede, both more readyer, and more certayne, whiche differeth nothing from this that I haue taughte you, ſaue that it doeth vnderſtande alwayes the articles, and ioyne them to the nexte ſumme, and therefore I will declare it onelye by an example.

If I woulde multiplie 1542, by 365, I muſt ſet them as I ſayde before, and then do I multiply 2 by 5, and it maketh 10, of which I write the article vnder the firſte place, and kéepe the digit 1 in my mind.

```
1542
 365
----
   0
```

Then ſaye I forth: 5 times 4 doe make 20, and the 1 in my minde, are 21, thereof I write the 1 vnder the ſecond place,and keepe the 2 in my minde.

$$\begin{array}{r} 1542 \\ \underline{365} \\ 10 \end{array}$$

Then come I to the third figure 5. ſaying: 5 times 5 make 25.& the 2 in my minde make 27,whereof I write the 7 vnder the thirde place, and keepe the article 2 in my mynd.

$$\begin{array}{r} 1542 \\ \underline{365} \\ 710 \end{array}$$

Then comming to the laſt figure, I ſaye: 5 times 1 make 5, and 2 in my mind make 7: that doe I write vnder the fourth place.

$$\begin{array}{r} 1542 \\ \underline{365} \\ 7710 \end{array}$$

And then haue I ended my firſte multiplier,and therefore I daſhe it. Then doe I likewiſe with the ſecond multiplier,ſaying: 6 times 2 make 12, therof I write the digit 2 vnder the ſeconde place,and keepe the article 1 in my minde.

$$\begin{array}{r} 1545 \\ \underline{36\cancel{5}} \\ 7710 \\ 2 \end{array}$$

Then ſay I forth: 6 times 4 maketh 24 and 1 in my minde make 25. ſo I ſet that 5 vnder the thirde place,and keepe the 2 in my minde.

$$\begin{array}{r} 1542 \\ \underline{36\cancel{5}} \\ 7710 \\ 52 \end{array}$$

Then multiplie I forth,ſaying: 6 times 5, maketh 30, and 2 in minde make 32, whereof I write the 2 vnder the fourth place and, keepe the 5 in my minde.

$$\begin{array}{r}1542\\ 36\cancel{5}\\ \hline 7710\\ 252\end{array}$$

Then do I multiplie the laſt figure 1 by 6, and it maketh 6, to yͭ I adde the 3 in my minde, and it maketh 9, whiche I write in the fiſt place.

$$\begin{array}{r}1542\\ 3\cancel{6}\cancel{5}\\ \hline 7710\\ 9252\end{array}$$

And ſo haue I ended tħ. figures of the multiplier.

Than with the thirde & laſt multiplier, do I likewayes, and ſay firſt: 3 times 2 make 6: which I write in the thirde place vnder the multiplyer.

$$\begin{array}{r}1542\\ 3\cancel{6}\cancel{5}\\ \hline 7710\\ 9252\\ 6\end{array}$$

Than by that 3 doe I multiplye likewaies the ſeconde figure 4, and it maketh 12, whereof I write the digitte 2 vnder the fourth place, and the article 1 I keepe in minde.

$$\begin{array}{r}1542\\ 3\cancel{6}\cancel{5}\\ \hline 7710\\ 9252\\ 26\end{array}$$

Than come I to the thirde figure 5 ſaying: 3 times 5 maketh 15, and the 1 in my minde make 16, thereof I write the 6 vnder the fift place, and keepe the article 1 in my minde.

$$\begin{array}{r}1542\\ 3\cancel{6}\cancel{5}\\ \hline 7710\\ 9252\\ 626\end{array}$$

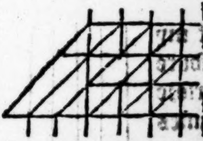
Then come I to the laſt figure, whiche is 1 and multiply it by 3, & it maketh 3, thereto I adde the 1 in my mynde, and it maketh 4, which I write in the 6 place. And then haue I ended yͤ multiplicatiõ, and the figures ſtand in order thus.

$$\begin{array}{rrrrrr} & & 1 & 5 & 4 & 2 \\ & & & 3 & 6 & 5 \\ \hline & & 7 & 7 & 1 & 0 \\ & 9 & 2 & 5 & 2 & \\ 4 & 6 & 2 & 6 & & \end{array}$$

Which parcels if I adde into one ſumme, it will be 562830, which is the groſſe or total ſum of al that multiplication

Scholer. Well, this maner of multiplicatiõ I perceaue: but what other ſorts haue you?

Mai. There is one way that is wrought by a checker Table, made thus.



Loke howe manye places your ſumme hath that you woulde multiplie ſo many ſquares muſt you make in your table, from the righte ſide to the lefte: and ſo manye from the higher parte to the lower, as there be places in your multiplier. Then ſet downe your greateſt ſumme firſt on the toppe of the table, euery figure in due order, in a ſquare alone, I mean in thoſe

Another vvay of multiplication.

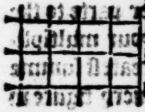
ſquares that be opē and uncroſſed. And likewiſe in thoſe like ſquares at the right hand, ſet downe your multiplicatour or multiplier, the laſt figure in the higheſt place, and ſo downward, that the firſt figure maye bée in the loweſt place.

Scholer. Sir if it pleaſe you, me thinketh then I underſtand you beſt, when you doe not ſtand long in telling the rule before examples: But propoſe ſome example, and then in declaring it, bring in the rules withall.

Mai. In déede, that way is eaſieſt for a yong learner, therefore will I euen ſo doe. Take this example: now I would multiplie 2336 by 25.

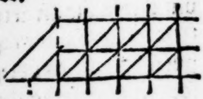
Firſt I conſider that my greateſt number hath foure figures or places, & therfore I make ſo manye roomes betwéene lines thus.

Than I ſée that of my multipliers there are two, wherefore I drawe ſo manye lines a croſſe the other, that there maye be two roomes betwéene them thus.



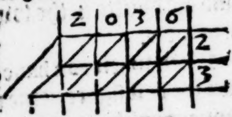
But you muſt not forget to ſet

let the endes of the lines run out, as it appeareth in this Patron, for in thoſe open ſquares muſt your two firſt numbers, and al the total ſumme be ſet.

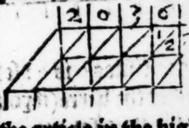


Thē draw a croſſe bar through euerye cloſe ſquare, ſo that it maye reache down to the loweſt ouerthwart lyne, as in this form. And than is your checker formes prepared.

Then ſette downe your firſte or greateſt ſumme on the top, & your multiplyer on yͤ right ſide in yͤ open ſquares, thus.

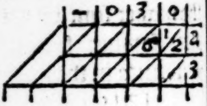


Then begin to multiplie the firſt figure of the higher ſum, by the higheſt of the multiplier, ſaying: 2 times 6 make 12, that 12 muſt you write in the ſquare that is againſt the 2 and the 6, but in ſuche manner that the digit be ſette in the nether corner of the ſquare, & the article in the higher corner: as you may ſee in this example.

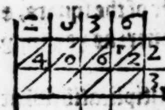


And ſo of euery other multiplicatiō, what euer amounteth you muſte write in the common ſquare, which is againſt both thoſe figures, by which you doe multiplie. And if that ſumme doe make but one digitte, than muſt it be ſet in yͤ lower corner of yͤ ſquare, but if it make an article, than write the article in the higher corner, and let the cipher go (if you will) euermore, for here it ſerueth for nothing, ſeing the lines doe diſtincte the places: but if the ſumme amounting of ſuch multiplication doe make a mixte number, then write the article in the higher corner, and the digitte in the lower corner, as I did by that 2.

Then when you haue multiplied and ended the firſt figure, come to the nexte, & multiplie it in like maner, as in ſaying: 2 times 3 is 6: that 6, becauſe it is but a digitt, you ſhal ſet in the nether corner of the ſquare, next vnder 3, thus.



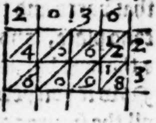
Then go forth, ſaying: 2 times 0 is 0: ſet that vnder the barre (if you liſt) in the third ſquare.



Then forth and ſay: 2 times 2 make 4, that ſet in the laſt ſquare vnder the barre.

barre, ſo haue you ended the firſt multiplyer: Daſhe him.

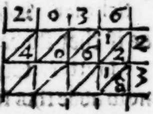
Come nowe to the ſeconde multiplier, and ſaye: 3 times 6, make 18, of whiche ſumme, the article 1 muſt be ſet aboue the barre, in the ſquare that is nexte to the 3 (as you ſee here) and the 8 vnder the barre.



Then ſay 3 times thre make 9, ſet it in ẏ next ſquare beneth the bar. Then 3 times 0 is 0, write it in the next ſquare, or let it go, for all is one.

Scholer. I perceiue it well: for here the lines diſtinct the places, wherefore cyphers do only ſerue, and therefore here they nede not to be.

Mayſter. Then ſaye farther: 3 tymes 2, make 6: write that in the laſt ſquare, then will the whole figure ſtand thus.



Sch. Now could I (me ſemeth) do like againe. But how ſhal I do now to gather the ſumme?

Mai. Marke firſte the order of the places in this figure, and ſo ſhall you perceaue the reaſon of gathering them into a ſumme.

The ſloape barres do part the places; for yͤ firſt place is the loweſt corner (in al ſuch figures) of the nethermoſt ſquare next the right hand: & all the halfe ſquares betwéene that barre and the next, ſtandeth for the ſecond place, and ſo the roome betwéene that & the next bar, is the third place: & ſo forth. Now if you perceaue this, then muſt you ad all the figures of one place together, as if you had an Addition of diuers ſummes.

Scholer. If I vnderſtand you right, then muſt I take here in this example 8 to be in the firſt place: 9, 1 and 2, in the ſecond: 0, 6, 1, in the third: 6, 0, in the fourth: 4 in the fift: and the ſixt place hath no figure.

Mai. You ſay well, and the reaſon is bicauſe the multiplication ſeruing to yͤ ſquare, made but a digit.

Scholer. Then it is all one, as if they ſtode thus.

1 2
6 1
4 6 0 9 8

Mayſter. Euen ſo it is: and now adde this ſum, and there wil appeare the totall of the multiplication to be 46828.

A proofe vvithout ſquares.

And if you will ſée the agréement of this maner of multiplication, and the other that you learned before, then multiply thoſe two ſummes (that is 2036, by 23) after the firſt maner without ſquares.

```
2036
  23
----
```

Scholer. You meane to ſet them thus in order. And then multiplie 3 into 6 make 18: 3 times 3 make 9, 3 times 0 is 0 then 3 times 2 make 6: whiche muſt be ſet thus.

```
2036
  23
----
  18
609
```

Then do I likewiſe with the ſecond multiplier, ſaying: 2 times 6 make 12, 2 times 3 make 6 2 times 0 is 0, & 2 times 2 make 4, which when I adde to the other then will the whole multiplication ſtãd thus.

```
 2036
   23
-----
   18
 609
   12
406
-----
46828
```

Mai. So that you may ſee in euery place the ſame figures, as they were in the multiplication by ſquares, though they differ in heighte and lowneſſe of places, but being added togither, they make one ſumme.

And thus now ye haue learned thꝛee ſoꝛts of multiplicatiõ, which liketh you beſt, that may you vſe.

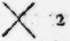
Yet are there other foꝛms, but ſith they nothing differ from theſe thrée in effect, but only in ſetting of the nūbers, I will ouerpaſſe them till a more méeter place and time. And now wil I inſtruct you in Diuiſion, ſo that you thinke your ſelf ſufficiently to perceaue what I haue taught you.

Sch Yes ſir I thanke you, but I doe not perceaue how to examine my workes, to trie whether I haue well done or no.

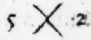
Mai. That is commonly vſed by ẏ proofe of 9, as you learned before in Addition and Subtraction, ſaue that it hath this wayes diuers from them.

Firſte you muſt make a croſſe after this maner.

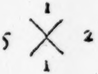
Then muſt you examine your ſum that ſhoulde be multiplyed, and looke what remaineth after caſting awaye of 9, that ſette you at the one ſide of the croſſe: then examine the multiplier, and whatſoeuer remayneth in it, after caſting away 9 as often as you can, write that at the other ſide of the croſſe: then muſt you multiplie thoſe two numbers togither, and looke what amounteth thereof, if it be vnder 9, write it at ẏ higher part of ẏ croſſe: but if it be aboue 9, then take thence 9 as often as you canne, and write the reſte at the heade of the croſſe. As in the laſte example of multiplication, the number to be multiplied is 2036, wherein is once 9, and 2 remaineth, which I write at one ſide of a croſſe thus.



Then doe I examine the multiplier, which is 23, wherin there is no 9 but 5 in al, that 5 therfore I ſet at the other ſide of the croſſe, thus.



Then doe I multiplie by 2 and it maketh 10, from whiche I withdraw 9 and there reſteth 1 that 1 do I ſet at the head of the croſſe, then doe I examine the groſſe ſumme, amounting of the multiplication, whiche is 46828. wherein I finde 9 three times and 1 remayning, that 1 I ſet at the foote of the croſſe and then I ſee it to agree with the other 1 at the toppe of the croſſe, and ſo know I that I haue done well: for if they two did differ, then were my worke vaine, and the multiplication falſe.



This is the common proofe, but the moſte certaine proofe is by Diuiſion, of which I wil anone inſtruct you.

Scholer Sir, what is the chiefe vſe of Multiplitiplication?

Maiſter. The vſe of it is greater then you can yet vnderſtand:howbeit, theſe plaine cõmodities it hath, that if you would reſolue anye great and whole valure into many ſmall and leſſe portions: as if you woulde chaunge poundes into ſhillings, pence or anye other greater or ſmaller parcels, by multiplication, yée ſhal do it ſpéedely and eaſily. Alſo if you ſhoulde néede to adde one ſumme to it ſelfe, or to anye other oftentimes, you ſhall doe it by Multiplication muche more ſpéedelie, readily, eaſily and ſurelye, then by often and ſundrye Additions. Take you theſe commodities groſſely ſhewed for an anſwere at this time, and hereafter I wil more abundantlye make you to perceiue the vſe of it.

DIVI-

DIVISION.

Scholer.

WEll ſir, then in Diuiſion I praye you to inſtructe mee. But me thinketh by the name of it, that it ſhoulde be al one with Multiplication: for I call that Diuiſion, when any thing is parted into diuerſe & many partes.

Maiſter. You take it as it is taken commonlye, howbeit, if you marke wel, you ſhall perceiue that it is quite contrary to Multiplication, and doth not parte one thing or fewe things into many, but contrarywayes it bringeth many parcels in to fewe, but yet ſo, that theſe fewe taken togither, are equal in value to the other many: for by Diuiſion pence are tourned into ſhillings, and ſhillings into poundes: as for example of 120 ſhillings, it maketh 6 poundes, ſo are 120 tourned into 6, which is a ſmaller number : but then if you conſider the denominatours, you ſhal ſee that they are ſuch, that one of the latter is equal to 20 of the firſt, and ſo in value the ſummes are one, though in number they doe farre differ, and the latter ſumme is the leſſer, and ſo is it alwaies in diuiſion how be it, yet in the working, the ſumme is parted by an other, & therof doth it take the name.

Scholer I thinke I ſhal better vnderſtand the reaſon of the name, when I know the vſe of the worke, therefore now would I gladlye learne that.

Diuiſion what it is.

Maiſter. Diuiſion is a diſtributing of a greater ſumme by the vnities of a leſſer. Or Diuiſion is an Arithmetical producing of a thyrde number, in reſpect of two propounded numbers : which thirde number ſhal ſo oftē containe an vnit, as the greater of the two propounded numbers doth contain the leſſer. So that, euen as Multiplication did ſeeme to ſerue in ſteede of manye Additions, ſo Diuiſion maye ſeeme to be in place of many Subtractions : Bicauſe that third number briefly expreſſeth, how many times the leſſer of your two propounded nūbers may be Subtracted, from yͤ greater: As in practiſe wil more plainly appeare. Therefore (as you may perceyue) vnto Diuiſion are required thrée nūbers: the firſt, which ſhould be diuided, and that muſte (generally) be the greater: and the ſeconde, by which the other muſt be diuided, and that is (generallie) the leſſer, & is called yͤ Diuiſor. And the thirde which aunſwereth to the queſtion, How many times: and therefore is called the Quotient.

The firſt muſt be firſte written and the ſeconde ſo ſet vnder it that the laſt figure of the lower number be right vnder the laſte of the higher, contrarie ways to the worke of the other kindes of Arithmetike: for in them the two firſt figures were ſet euer méete one vnder the other, but in Diuiſion the laſt figures muſt be ſet méet, except it chance ſo, yͭ the laſt figure of the Diuiſor be greater then yͤ laſt of the higher number, for then you ſhal ſet the laſt of the Diuiſor, vnder the laſt (ſaue one) of the higher number, as for example.

A generall rule for placing the figures.

An exception.

If you ſhoulde diuide 365 (which are the ſumme of the dayes of a yeare) by 28 which are the dayes of a common moneth, then ſhoulde you ſet them thus.

365
28

But if you would diuide thoſe 365 dayes, by 52. whiche is the number of weekes in one yeare, then ſhould you ſet them thus.

365
52

Likewayes if I would diuide the ſame 365 by 4, whiche is the ſumme of the quarters of a yeare, then muſte I ſet them thus.

365
4

Scholer. Syr, this doe I vnderſtande, but howe nowe ſhoulde I do to diuide the one by the other?

Maiſter. You muſt beginne with the laſte figure next the left hande, and ſee howe many times the laſt figure of the diuiſor may be taken out of the laſt figure of the ouer number, and that ſhall you note within a crooked line toward your right hand. As for example.

I woulde diuide 365 by 28, then ſet I thoſe two ſummes thus.

365(
28

And I looke howe manye times I may finde 2 (which is the laſt figure of the diuiſor) in 3, (which is the laſt of the number to be diuided) and conſidering that I can take 2 out of 3 but once, I make a crooked line at the right hande of the numbers, & with in it I ſet 1, and that is called the Quotiente number, as I tolde you. Then bycauſe that when 2 is taken out of 3, there remaineth 1, I muſt write that 1 ouer 3, and deface or cancell the 3 and the 2, then will the figures ſtand thus.

Quotient number.

1
$\cancel{3}65$ (1
$\cancel{2}8$

Then muſt I goe to the nexte figure of the diuiſor, and take it likewayes ſo many times out of the figures that be ouer it, and looke what doth remaine, that I muſt write ouer them, and cancel them, as in this example.

Therefore now I do take once 8 out of 16, and there remayneth 8; whiche I muſt ſet ouer the 6, and cancel or croſſe out the 15. and the 8 of the diuiſor: And then will the figures ſtande thus.
And ſo haue I once wrought.

$\cancel{1}8$
$\cancel{3}\cancel{6}5$ (1
$\cancel{2}\cancel{8}$

Scholer. So I perceiue that you take the nether figure not only out of the other that is right ouer him, but out of that with the other alſo that remayneth before, and are written toward the left hand.

Maiſter. So muſt you doe: for you muſt ſo take the diuiſor out of the ouer number, that there remain not ouer it ſo great a ſumme as it ſelfe is, for then were your worke in vaine.

But yet againe here muſt you marke, that when you ſeeke how many times the laſte figure of the diuiſor may be found in the number ouer him, that you looke alſo whether you may as often finde al the figures folowing in thoſe that are aboue them, (conſidering al the remainders if there be any) if not, take youre Quotient leſſe by one and then proue again, and ſo ſtil, til you find a meete Quotient: And by that meete quotiẽt muſt you alwaies multiplye your diuiſor, and the product ſet vnder your diuiſor, ſo ẏ his firſt figure ſtand vnder the firſte figure of your diuiſor, and the ſecond vnder the ſecond, and ſo forth: and then ſubtract that product from the number to be diuided, that ſtandeth directlie ouer it, as you haue ſeene me do.

Note.

☞ When you haue thus wrought once, than muſt you beginne againe and write your diuiſour a newe, nearer towarde the righte hande by one place, as in this example, you ſhall ſette 2 vnder 8, and 8 vnder 5, thus

$$\begin{array}{llll} & \cancel{2} & 8 & \\ \cancel{3} & \cancel{8} & 5 & (1 \\ & \cancel{2} & \cancel{8} & 8 \\ & & 2 & \end{array}$$

Then (as before) ſeeke how many times you may take your diuiſor out of the number ouer him now.

Scholer. That may I do here 4 times.

Maiſter. Truth it is that you may finde 2, foure times in 8: but then marke whether you can finde the figure following ſo manye times in the other that is ouer him. Can you finde 8 foure times in 5?

Scholer. No, neither yet once.

Maiſter. Therefore take 2 out of 8, once leſſe.

Scho. That is 3 times.

Ma. Well, then 3 times 2 make 6: if I take 6 out of 8, there remayneth 2: which 2 wyth the 5 following, make 25, in whiche ſumme I finde 8 iij times alſo, and therefore I take 3 as a true quotient, and write it within the crooked line of the quotient, before the one thus.

Marke hovv to conſider this kind of remainer.

$\cancel{2}$ 8

3 $\cancel{1}$ $\cancel{6}$ 5 (13

$\cancel{2}$ $\cancel{1}$ 8

Then ſaye I: 3 times 2 make 6, then 6 out of 8 reſteth 2, therefore I cancel the 2 and the 8 and write ouer it the 2 ẏ doth remaine, thus.

2

$\cancel{2}$ $\cancel{8}$

$\cancel{3}$ $\cancel{6}$ 5 (13

$\cancel{2}$ $\cancel{1}$ 8

$\cancel{2}$

Then do I take 8 as many times out of 25, ſaying: 3 times 8 make 24, and if I take 24 out of 25, there remaineth 1: ſo then I cancell 25 and 8, and ouer the 5 I ſet 1, thus.

$$\begin{array}{cccc} & \cancel{2} & & \\ \cancel{1} & \cancel{8} & 1 & \\ \cancel{3} & \cancel{6} & \cancel{5} & (13 \\ \cancel{2} & \cancel{8} & \cancel{8} & \\ & \cancel{2} & & \end{array}$$

Or you mought (after you founde 3 to be a fit quotiẽt) ſtraight way haue multiplied the whole diuiſor 28, by that 3, at once; which giueth 84, whiche being ſet vnder 28, and duelie ſubtracted from 85, of the number diuided, giueth 1, ẏ remainer of ẏ whole diuiſion: as before you hadde. Worke which waye you liſt: here you ſee alſo the forme.

$$\begin{array}{cccc} & \cancel{8} & 1 & \\ \cancel{3} & \cancel{6} & \cancel{5} & (13 \\ \cancel{2} & \cancel{8} & \cancel{8} & \\ & \cancel{2} & 4 & \\ & 8 & & \end{array}$$

And now haue I done with diuiding, for I can finde my diuiſor 28 no more in the ouer ſumme.

Scho. No, except you would part the 1 that remayneth, into 28 partes.

Mai. That is wel ſaide, and ſo muſt we doe in ſuch caſes, whẽ there remaineth any thing: but I wil let that paſſe nowe, and wil make you perfect in diuiſion of whole numbers, and wil hereafter teach you particularly of brokẽ numbers, called Fractions.

Now if you do perceiue the order of diuiſion, then do you diuide this ſumme, 136280 by 452.

Scho. Firſte I ſet downe the number that ſhould be diuided, then doe I ſet the diuiſour vnder it, ſo that the laſte figure of it be righte vnder the laſt figure of the ouer number. Then wil it be thus.

$$\begin{array}{r} 136280 \\ 452 \end{array}$$

Maiſter. Can you take the laſt of your diuiſour (which is 4) out of 1, which is the laſt of the ouer number?

Sch. I had forgotten, becauſe the laſt of the diuiſor cannot be taken out of the laſte of the ouer number, in ſo much as it is the greater, therefore muſt I ſet the diuiſor one place more foreward, toward the right hande thus.

$$\begin{array}{l} 136280 \\ \;\;452 \end{array}$$

And then muſte I looke how often I maye finde the laſt figure of the diuiſour (that is 4) in 13, which thing I maye do 3 times, therfore doe I ſay: 3 times 4 is 12, whiche I take out of 13, and there remaineth 1. Then do I make at the right hande of my ſummes a croked line, and write before it my quotient 3: and I cancell 13 and 4, and ouer the 3 I ſet ẏ 1 that remaineth, and thē the figures ſtand thus.

$$\begin{array}{l} \;\;1 \\ \cancel{1}\cancel{3}6280\,(3 \\ \;\;\cancel{4}52 \end{array}$$

Then do I multiplie the ſame quotient into euerie figure of the diuiſor, and withdraw the ſumme that amounteth out of ẏ numbers ouer them, as firſt I ſaye: 3 times 5 make 15, which I take from 16, and there reſteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remaineth, thus.

$$\begin{array}{l} \cancel{1}\,1 \\ \cancel{1}\cancel{3}\cancel{6}\,2\,6\,0\ (3 \\ \cancel{4}\cancel{5}\,2 \end{array}$$

Then doe I ſay lykewaies, 3 times 2 makes 6, whiche I take out of 12 and there reſteth 6, therefore I cancell the 12 and the 2, and ouer the 2 I write 6 that remayneth, thus.

$$\begin{array}{l} \cancel{1}\cancel{1}\,6 \\ \cancel{1}\,3\,\cancel{6}\cancel{2}\,8\,0\ (3 \\ \cancel{4}\cancel{5}\cancel{2} \end{array}$$

Then ſhould I ſet forward the diuiſor, into the next place toward ẏ right hand, thus.

$$\begin{array}{l} \cancel{1}\cancel{1}\,6 \\ \cancel{1}\cancel{3}\cancel{6}\cancel{2}\cancel{8}\,0\ (3 \\ \cancel{4}\cancel{5}\cancel{2}\,2 \\ \quad 4\,5 \end{array}$$

Maiſter. But you maye ſe, that ouer the 4 is no figure, therfore muſt I ſet the diuiſor yet forewarder by an other place.

And marke, when ſoeuer it chanceth ſo, ẏ you ſhould ſet forwarde the diuiſor, and that it can not ſtande there, bycauſe there is no nūber ouer the laſte place, or if there be any, it is leſſer then the laſte figure of the diuiſor, then muſt you remoue the diuiſor yet once again: and becauſe that his firſte place of remouing ſerued not to ſubtracte him ſo much as once, therefore ſhal you write in the quotient a Cipher 0. And if you ſhould by chaunce neede to do ſo oftētimes, for euerie time write a cipher in the quotiente. The reaſon of this, will I ſhew you hereafter.

Scholer. Then muſt I ſet my ſummes thus.

$\cancel{1}\cancel{1}6$

$\cancel{1}\cancel{3}\cancel{6}\cancel{2}80$

$\cancel{4}\cancel{5}\cancel{2}52\,(30$

4

And bicauſe I remoued the diuiſor, ſo ẏ I ouerſkipped one place, I muſt write a cipher in the quotient: & then muſt I ſeeke a new quotient, as in this exāple I muſt ſay, How many times 4 is there in 6? and ſith it can be but once, therfore do I write 1 in the quotient, and then ſaye I: 1 time 4, takē out of 6, remaineth 2, I cancel the 6 and the 4, & write 2 ouer thē thus.

2

$\cancel{1}\cancel{1}\cancel{6}$

$\cancel{1}\cancel{3}\cancel{6}\cancel{2}80\,(301$

$4\cancel{5}\cancel{2}52$

4

2

$\cancel{1}\cancel{1}\cancel{6}$

$\cancel{1}\cancel{3}\cancel{6}\cancel{2}80\,(301$

$\cancel{4}\cancel{5}\cancel{2}52$

$\cancel{4}$

Then ſaye I againe, once 5 out of 28, remaineth 23, I let the 2 ſtande as it did, [illegible]d ouer ẏ 8 I ſet 3, cancelling the 8 and the 5 vnder it, thus.

$$\begin{array}{cccccccl}
 & & & 2 & & & & \\
 & \cancel{1} & \cancel{1} & \cancel{6} & 3 & & & \\
\cancel{1} & \cancel{3} & \cancel{6} & \cancel{2} & \cancel{8} & 0 & (301 & \\
 & \cancel{4} & \cancel{5} & \cancel{2} & \cancel{5} & 2 & & \\
 & & & \cancel{4} & & & &
\end{array}$$

Maiſt. You might as well haue ſaide, once 5 out of 8, & ſo remayneth 3, but now go forth.

Scho. Then once 2 out of 0, can not be, what ſhal I now doe?

Ma. Borow of the next number that is behinde (for there is 230) and doe as you learned in Subtraction in a like caſe.

Scholer. Then muſt I borrowe 1 of the 3 comming behinde nexte, and make that 0 to be 10: and then take I 2 out of 10, and there reſteth 8. And becauſe I borrowed one of the 3, I muſt cancel the 3, and write 2 ouer it: then doth the figure ſtand thus.

$$\begin{array}{cccccccl}
 & & & \cancel{2} & 2 & & & \\
 & \cancel{1} & \cancel{1} & \cancel{6} & \cancel{3} & 8 & & \\
\cancel{1} & \cancel{3} & \cancel{6} & \cancel{2} & \cancel{8} & \cancel{0} & (301 & \\
 & \cancel{4} & \cancel{5} & \cancel{2} & \cancel{5} & \cancel{2} & & \\
 & & & \cancel{4} & & & &
\end{array}$$

Maiſter. Nowe haue you done, and yet remaineth 228, and your quotient ſhew you, that if you diuide 136280 by 452 you ſhal finde your diuiſour in your greter number 301. that is CCC times & once and 228 remaining.

And in the other example, where I diuided

365 by [illegible], the quotient was 13, & 1 remayned: whereby I know that in a yeare (which containeth 365 daies) there are 13 monthes reckening 28 dayes (or 4 weekes) iuſte to a moneth, and 1 day more.

Sch. Why then do we cal a yeare but 12 monethes.

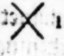
Ma. Of that at a more conuenient time wil I fullie inſtruct you: but nowe it is not conuenient to entangle your minde with other things, than do directlie pertaine to your matter. Therefore if you remember what you haue hearde, you haue learned a ſhort maner of diuiſion, which I would haue you often to practiſe, ſo that you maie be perfect in it, and hereafter I wil ſhew you certaine other proper points touching it.

Scholer. Then I pray you, yet tel me, how I ſhal examine and trie my worke, whether I haue done wel or no, that thoughe no man be by me to tell me, yet I may perceiue it my ſelfe.

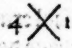
Maiſter. Some men (yea and commonlie) do trie it by the rule of 9, as in all the other kinds, ſaue that their order is this. Firſt they caſt away 9, as often as they can, out of the diuiſor, and that remaineth, they ſet at

Trial of the work

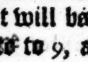
one ſide of a croſſe: As in our firſt example, the diuiſor was 28, from which you may take 9 thrée times and 1 remaineth which they ſet by a croſſe, thus.



Then do they likewiſe examine the quotient (which in our example is 13) and from thence they caſt away 9 as oft as they can, and the remainer they ſet at the other ſide of the croſſe, and then multiplie they together thoſe ij. remainers: and to it that amounteth they adde the remainer of the diuiſion, if there were any, from that whole ſumme they withdraw 9 as often as they can, and the reſt they ſet at the head of the croſſe: as in our example the quotient is 13, from which take 9, and there remaineth only 4 and therefore muſt you ſet 4 at the other ſide of the croſſe, thus.



Then multiplie 4 by 1, and it yeeldeth but 4, thereto adde the remainer of the diuiſion (which was 1) and it will bée 5, which ſumme doth not amount to 9, and therefore muſt be ſet wholy at the head of the croſſe, as you ſée here.

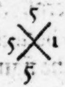


And this number on the

heade of the croſſe, is the firſt proofe, to whych if you finde an other like in the number that was diuided, then haue you done wel.

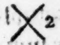
Therefore now ſhal you likewiſe examine the whole ſumme that was diuided, and take away 9 as often as you can, and that that remaineth, ſet at the foote of the croſſe: and if it be equal to that in the head of the croſſe, then haue you wel done, elſe not.

As in our example the whole ſumme was 365, which maketh 14, from that take 9, & there reſteth 5, which ſet at the foote of the croſſe, thus.

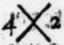


And you ſhal ſee that they agree: therefore haue you wel done.

Scholer. Nowe will I likewiſe examine our ſecond example, where yͤ diuiſor was 452 whiche maketh 11: from thence I take 9, and the 2 that remaineth I ſet at the right ſide of a croſſe, thus.

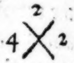


Then examine I the quotient, which was 301, where I finde but only 4 that do I ſet at the other ſide of the croſſe thus.

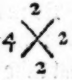


Then do I multiplie 4 by 2,

and it maketh 8, to that do I ad the remayner of the diuiſion (which was 228, and maketh 12) and they two make 20, wherein I finde twiſe 9, and 2 remaining, that 2 muſt I ſette at the heade of the croſſe, thus.



Then doe I examine the whole number to be diuided, which was 136280, where I finde twiſe 9, and 2 remaining, which I ſet at the foote of the croſſe, thus.



And becauſe that it doeth agrée with the figure at the head of the croſſe, I know that the diuiſion was well wrought.

Mai. This is the common proofe, howbeit, the more certaine working is by the contrary kinde, as to proue Diuiſion by multiplication, thus.

Multiplie the quotient by the diuiſor, and if the ſumme that amounteth be equall to the ſum that ſhould be diuided, then haue you well diuided, elſe not.

Howbeit, this muſt you marke, yͭ if there remained any thing after the diuiſion, that muſte you adde to the ſum that amounteth of the multiplication: as in our firſt exãple the quotient was 13, and the diuiſor was 28: Now multiplie the one by the other, & the ſumme will bee 364: to that if you adde the one that remained after the diuiſion, than will it be 363, which was the ſumme that ſhould be diuided, & therefore I knowe that I haue well done.

Scholer. Now will I proue the ſame in the ſecond example, whoſe diuiſor was 452, and the quotient 301: theſe doe I multiplye together, and there amounteth 136052, to which if I adde the 228 that remayned, then will it be 136280, which was the whole ſumme to be diuided, and therefore I perceiue that I haue well done.

Mai. This is the ſureſt way to examine Diuiſion by Multiplication: and contrarie waies the ſureſt proofe of Multiplication, is Diuiſion.

And therefore now wil I ſhew how you may proue Multiplication by Diuiſion.

A proofe of Multiplication by Diuiſion.

When you haue ended multiplicatiõ, and would know whether you haue well done or not, ſet the groſſe ſum that amounteth of the Multiplication ouermoſt, and diuide it by the multiplier: and if the quotient be the ſame number that ſhoulde be multiplied, then haue you well wrought, elſe not: as in that example where we multiplied 264, by 29, the groſſe ſumme was 7656.

Nowe if you will knowe whether that multiplication be true, you ſhall diuide that 7656 by the multiplier, 29: and you ſhall perceiue that the quotient will bée 264, and that is a token that you haue well wrought.

Scholer. By your patience I wil proue that: and firſt I ſet down the groſſe ſumme & the multiplier, not after the rule of multiplication, but after the rule of diuiſion; for now that nũber is become the diuiſor, that was before the multiplier, I ſhall
ſet them therefore thus. 7656
Then ſhall I ſéeke howe many 29
times 2 in 7, that may be 3 times,
and 1 remayneth: but then maye not 9 bée found ſo often in 16, therefore muſt I take a leſſer quotient, that is to ſay, 2: then ſay I twiſe 2 maketh 4, which I take out of 7, & there remaineth 3; then do I cancell 7 and 2,
and ouer 7 I write 3 and in
the quotient I ſet 2, ſo the fi- 3
gures ſtand thus. $\cancel{7}656\,(2$
Then ſay I forth, 2 times $\cancel{2}9$
9 make

9 make 18 which I bate out of 36, and there reſteth 18 then cancell I 3 and ouer him ſet 1, and likewiſe I cancell 6 and 9, and ouer them I ſet 8, ſo that thus ſtand the figures.

$$\begin{array}{lllll} & 1 & & & \\ & \cancel{3} & 8 & & \\ & \cancel{1} & \cancel{6} & 5 & 6 \;(2 \\ & \cancel{2} & \cancel{9} & & \end{array}$$

Then doe I ſette forwarde the diuiſor by one place, and ſeeke a newe quotient, that is to ſaye, how many times 2 are in 18, which I finde to be 9 times, but then can I not finde 9 ſo many times in 5, therefore I take a leſſer quotient, as to ſay, 8: but yet is that to greate, for if I take 8 times 2 out of 18, there remaineth but 2 and I can not finde 8 tymes 9 in 25, therefore yet I take a leſſer quotient, that is 7, which is alſo to great, for if I take 7 times 2 out of 18, there reſteth 4, but nowe I cannot take 7 tymes 9 out of 45, therefore yet I ſeeke a leſſer quotient, as to ſaye, 6: then ſaye I, 6 times 2, make 12, that I take out of 18, and there remaineth 6, ſo I cancell the 18 & the 2, and write 6 ouer 8, thus

$$\begin{array}{lllll} & 6 & & & \\ \cancel{3} & \cancel{8} & & & \\ \cancel{1} & \cancel{6} & 5 & 6 & (26 \\ \cancel{2} & \cancel{9} & 9 & & \\ & \cancel{2} & & & \end{array}$$

Then ſay I forth: 6 times 9 maketh 54, that take I out of 65 and there remaineth 11

$\cancel{1}$ 6
$\cancel{3}$ $\cancel{8}$ 1
$\cancel{7}$ $\cancel{6}$ $\cancel{5}$ 6 (26
$\cancel{2}$ $\cancel{9}$ $\cancel{9}$
$\cancel{2}$

and the figures ſtand thus. Then muſt I ſet forth the diuiſor againe, and ſeeke a newe quotient, whiche will be 4: for though I maye finde 2 in 11 fiue tymes, and 1 remayne, yet I cannot finde 9 ſo often in 16, therefore I ſet the figures thus.

1
$\cancel{1}$ $\cancel{8}$
$\cancel{3}$ $\cancel{8}$ 1
$\cancel{7}$ $\cancel{6}$ $\cancel{5}$ 6 (264
$\cancel{2}$ $\cancel{9}$ $\cancel{9}$ 9
$\cancel{2}$ 2

And yͤ 4 in the quotiẽt I multiplie into the figures of the diuiſor, ſaying: 4 times 2 maketh 8, which I take out of 11, & there reſteth 3, therefore I cancell the 11 & the 2 and ſet 3 ouer the firſte place of 11, thus.

$\cancel{1}$
$\cancel{1}$ $\cancel{8}$ 3
$\cancel{3}$ $\cancel{8}$ $\cancel{2}$
$\cancel{7}$ $\cancel{6}$ $\cancel{5}$ 6 (264
$\cancel{2}$ $\cancel{9}$ $\cancel{9}$ 9
$\cancel{2}$ $\cancel{2}$

And then doe I ſay forth, 4 times 9 maketh 36, which I take frõ 36, and there remaineth nothing, ſo that yͤ quotient of this diuiſiõ, where 7656 is diuided by 29, is 264, which doth declare that if 264 be multiplied by 29, the ſum will be 7656. And thus I perceiue now how both Multiplicatton is proued by Diuiſion, & diuiſion alſo by Multiplication.

Mayſter. Nowe haue I ended the fiue cōmon kinds of Arithmetike: for as touching Mediation, Duplation, triplation, and ſuch other, they are no ſeueral kinds of Arithmetike, but are contained vnder the other: for Mediation is contained vnder Diuiſion, & is nothing elſe but diuiding by 2: and ſo are duplation and Triplation contained vnder Multiplication: for Duplation is nothing elſe but multiplying by 2, and triplation is multiplying by 3, of which I wil only propoſe an exãple, for the rules you haue heard alreadie.

An example of Mediation.

If you woulde mediate or diuide into 2, this ſum, 4531010, you ſhal ſet 2 for the diuiſor, & work as you learned before, as thus.

$$\begin{array}{cccccccl} 4 & 5 & 3 & 1 & 0 & 1 & 0 & (\\ 2 & & & & & & & \end{array}$$

Then I finde 2 in 4 two times, therefore my quotient muſt be 2: ſo I cancell 4 & 2, and remoue the diuiſor forward, thus.

$$\begin{array}{cccccccl} \cancel{4} & 5 & 3 & 1 & 0 & 1 & 0 & (2 \\ \cancel{2} & 2 & & & & & & \end{array}$$

Then againe I finde 2 in 5 twiſe, and 1 remaining, ſo I write 2 againe for my ſecõd number of the quotiente, and cancell 5 and 2, and ouer 5 I ſet 1, thus.

$$\begin{array}{cccccccl} & & 1 & & & & & \\ \cancel{4} & \cancel{5} & 3 & 1 & 0 & 1 & 0 & (22 \\ \cancel{2} & \cancel{2} & & & & & & \end{array}$$

Then remoue I the diuiſor forward and ſeeke a new quotient, which is 6: then ſaye I, 6 times 2 make 12, take that out of 13, and there reſteth 1, ſo I cancell 2 and 13, and ouer 3 I ſette 1 thus.

$$\begin{array}{cccccccl} & \cancel{2} & 1 & & & & & \\ \cancel{4} & \cancel{5} & \cancel{3} & 1 & 0 & 1 & 0 & (226 \\ \cancel{2} & \cancel{2} & \cancel{2} & & & & & \end{array}$$

Then remoue I the diuiſor forward, and ſeeke a new quotient which is 5, then take I twiſe 5 out of 11, and there reſteth 1, ſo I cancell the 2 and the laſt figure of 11, and let the firſte ſtande thus.

$$\begin{array}{cccccccl} & \cancel{2} & \cancel{1} & & & & & \\ \cancel{4} & \cancel{5} & \cancel{3} & 1 & 0 & 1 & 0 & (2265 \\ \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} & & & & \end{array}$$

Then remoue I the diuiſor forward, and ſeeke a newe quotient, which is 5: then take I 2 fiue times out of 10, and there reſteth nothing.

Then remoue I againe the diuiſor forwarde, thus.

$$\begin{array}{cccccccl} & \cancel{2} & \cancel{1} & & & & & \\ \cancel{4} & \cancel{5} & \cancel{3} & \cancel{1} & \cancel{0} & 1 & 0 & (22655 \\ \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} & \cancel{2} & & & \end{array}$$

But bicauſe I can not finde the diuiſor in the number ouer it, I muſte ſette a cipher in the quotiente, and remoue the diuiſor to the next place, as appeareth in the figure before.

$$\begin{array}{l} \quad\ \ \cancel{1}\ \cancel{1} \\ \cancel{4}\ \cancel{5}\ \cancel{3}\ \cancel{1}\ \cancel{0}\ 1\ \cancel{0}\ \ (226550 \\ \cancel{2}\ \cancel{2}\ \cancel{2}\ \cancel{2}\ \cancel{2}\ \cancel{2}\ \cancel{2} \end{array}$$

Then ſeeke I a new quotient which I finde to be 5, for ſo many times maye I haue 2 in 10. Then haue I fully ended this Mediation or Diuiſion by 2, and the quotiēt is this, 2265505, which is the halfe of 4531010, as you maye trie by Duplation: for double that quotiēt, or multiplie it by 2, and the ſame number will amount.

Duplation.

I will no longer tarry about theſe, ſeeing they are but members of the other kindes. But here now wil I teache you certayn eaſie formes both of Multiplication & of diuiſion, and firſt of multiplication.

Eaſie formes of multiplication.

If you would therfore multiplie any ſum by 10, you ſhall neede to doe no more but ad a cypher before his firſt place, as for exāple: 36 multiplied by 10, make 360.

Likewiſe if you wil multiplie any ſumme by 100 put two ciphers at his beginning.

So if you would multiplie any ſum by a thouſand, ad three ciphers to the beginning of it.

Scholer. This doe I well perceiue, and alſo the reaſon of it.

Mai. I will omit all reaſons till our next méeting, when I ſhall tell you the reaſon of all other partes of Arithmetike alſo: and as to our matter now, looke (as I haue tolde you) that you both remember it, and alſo often practiſe it.

But if you would multiplie any number by 5, marke firſt whether the number be euen or odde; and if it be euen take the halfe of it, and write a cipher at the beginning of it, as for example: I would multiplie 2564 by 5, I take the half of it, which is 1282 (as you may know by Mediation) and before it I ſet a cipher, thus, 12820, and this is fiue times 2564.

And thus may you do with any other euē ſumme, that you would multiplie by 5.

But if the ſumme be od, as for example 2563, thē muſt you take the leſſer half of it, or (if you wil) take away 1 from the firſt figure (as here take 1 from 3) & then take the halfe of the reſt, and ſette before it 5: as of 2563, the leſſer half is 1281, for here I take but 1 for the half of 3: and if I put 5 before that leſſer halfe, then haue I multiplied it 5 times,

times,as thus,12815.

Scholer. What meane you by the leſſer halfe.

Mai. There is no iuſt half of any od number,therefore if we diuide an od number into 2 parts,as nigh equall as can be,yet will the one halfe exceede the other halfe by one, as for example.The two neareſt halfs of 9, are 5 and 4: and likewiſe of 15, are 7,& 8, where you ſee that ẏ one part ſtill is greater then ẏ other by 1. Now it is eaſie to knowe which is the greater halfe, and which is the leſſer halfe.

Scho. Then I perceaue you,and can doe likewiſe (I doubt not) with anye ſumme. For if it be not very eaſie to part into halfs, then will I doe it by Mediation eaſilye ynough.

Mai. That is a ſure way. And now haue you learned howe to multiplie eaſilie by 5, 10,100,1000:and of like maner maye you doe with any other of that ſort.

But now if you will multiplie by 20,30, 40,and ſo forth:or by 200,300, and ſuche like,where there is one cipher in the firſte place,or many orderly in ẏ firſt places, you ſhal take away thoſe ciphers,and multiplie the ſum only by the other figure or figures, (if they be many) & then at the beginning of the ſum that amounteth, ſhal you ſet ſo many cyphers as you toke away.

Example of 2873, which I would multiplie by 300. Firſt I caſt away the 2 ciphers from the multiplier, & I multiplie the ſum by only 3 that is left, and it amounteth to 8619: before which I put the two ciphers that I toke awaye before, and then is it 861900. And that is the ſumme that amoũteth, when 2873 is multiplied by 300.

Scholer. And if there were two or more figures beſide the ciphers, I muſt only take away the cyphers, and multiplie by yͤ other figures, as I learned before: as if I woulde multiplie 93648 by 25000, I ſhould take away the thrée cyphers, and multiplie the ſame by 25, and then at the beginning of that totall ſumme, ſhould I adde the thrée cyphers againe.

Mai. Euen ſo: but and if it chaunce the number that ſhould be multiplied, or both the ſummes as wel the number that ſhould be multiplyed as the multiplier, to haue ciphers in their firſt places, euermore caſte away the Ciphers, and worke by the reſt.

But remember to reſtore as many ciphers to ẏ amounting ſũme, as you bated before, as in this example : 30200 ſhall be multiplied by 206 : I ſhall only take awaye the two ciphers from the greater number, and then multiplie 302 by 206, and afterwarde ad the two cyphers againe. But if I would multiplie the ſame 30200 by 2060, I ſhall not onely take awaye the two ciphers from the number that ſhould be multiplied, but alſo I may take away the one cipher from the multiplier, and then muſt I adde 3 Ciphers to the ſumme that amounteth : but take héed that you take away no cipher that commeth after any ſignifying figure, as in this laſt example, you may not take awaye that in the fourth place of the higher number, neither that in the third place of ẏ multiplier : how be it, yet this you may doe : If one cipher or more come in the middeſt of your ſummes, you may multiplie by ẏ other figures, and ouerſkippe them, but ſo, that you giue euerye figure his due place, as thus : I will multiplie 3026 by 2004, therfore I ſet them thus.

```
  3026
  2004
  ----
    24
1208
```

And thus doe I multiplie then: Firſte 4 times 6 make 24: I ſette the 4 vnder the firſt place, and keepe the 2 ſtill in my mind: Then ſaye I againe: 4 times 2 maketh 8, and the 2 that is in my minde, maketh 10, I ſette downe the Cypher 0, and keepe the article one in my minde. Then 3 tymes 4, is 12, and the 1 in my minde maketh 13, I ſette downe the Figure 3, and keeping the 1 ſtill in my minde hauing no more places of yͤ vpper number to multiplie it withall: I put it downe nexte 2 in the fift place.

But now when I come to the next place being a Cipher 0: I let it go, becauſe it multiplieth nothing: And likewiſe the ſecond cipher.

But then when I doe come to the 2, and multiplie it into the 6 of the ouer number, you muſt take heede (according as I taught you in multiplication) that the firſt number amounting of the multiplication, be ſette righte vnder the multiplier, and the other orderly towarde the left hand, according as you may ſee in this example.

$$\begin{array}{r}
3026 \\
2004 \\ \hline
12104 \\
6052 \\ \hline
6064104
\end{array}$$

whiche being finiſhed with the Addition thereof gathered togither, will ſtand as this example ſheweth.

Which is indeede wrought ſo muche ſooner and ſhorter by ouerſkipping of the two ciphers whiche elſe woulde haue hadde two workinges more as by the ſame example here alſo ſet down doth appeare.

```
   3026
   2004
-------
  12104
  0000
 0000
6052
-------
6064104
```

Scholer. Sir, I thanke you: for I ſee greate eaſe in this waye of Multiplication, and if you canne ſhewe mee ſuche like in Diuiſion, you ſhall greatlye further me.

Mayſter. Yes, I will teache you ſome eaſie ways in diuiſion alſo, and firſt this: If you would diuide any ſumme by 10, you ſhall onely with your penne make a ſquare line, betweene the firſt figure of your ſumme and the ſeconde, and than haue you done: for the whole number that followeth the line, ſtandeth for the quotient and the figure that is before the line, is the remainer: as for example, 3648 diuided by 10, will ſtand thus.

Eaſie formes of Diuiſion.

364|8

Where 364 is the quotient, and betokeneth that ſo many times are 10 in 3648: and the 8 after the line, is the remainer, which cannot be diuided into 10, but by breaking it into fractions, wherwith I wil not meddle yet.

And ſo likewiſe if you would diuide any ſomme by 100, with your pen, you ſhal cut away the two firſt figures, & if ye would diuide by 1000, you muſt cut away the 3 firſt figures, & ſo of any other diuiſor, whoſe laſt figure is 1, & the other be ciphers, looke how many ciphers the diuiſor hath, and ſo many figures at the beginning ſhal you cut away with the ſquire line, and they ſtand alwaies for the remainer becauſe they are leſſe than the diuiſor, and cannot be diuided by it, and the other figures that be behinde ẏ line, ſtād for the Quotient.

But now if your diuiſor haue any other figure in his laſt place than 1, and in al his other places haue ciphers, looke howe manye ciphers there be, cut away ſo many of ẏ firſt figures of the nūber that ſhould be diuided, and diuide the reſt that followeth the line, by that figure that is in the laſt place, as if it were the whole diuiſor.

Example of 64284. whiche I would diuide by 300, here muſt I cut away the two firſt figures,(for ſo many ciphers my diuiſor hath) and muſt diuide the reſt by 3 whiche is the figure in the laſt place of the diuiſor. Firſt therefore I part away the two firſte figures,& the ſumme ſtandeth thus. 642|84

Then do I diuide 642, by 3, and the quotient is 214. for in 6 I finde twice 3, & in 4 once, and 1 remayning, which 1 with the 2 next before, doth make 12. wherein I finde 3 foure times: and this is a readie way to turne ſhillings into poundes: for ſith one pound doth contain 20 ſhillings, I muſt diuide the whole nũber of ſhillings by 20 therfore eaſilie to do it, I ſee that my diuiſor hath one ciphre, and therefore I cut away one figure from the beginning of the whole ſumme of ſhillings, & then I do mediate or diuide by 2 the other figures or ſumme that foloweth.

Scho. I wil put an example.

If you would diuide 64287 ſhillings by 20, that is to ſay: If I would turne ſo many ſhillings into poundes, I muſt caſt away yͤ firſt figure, that is 7. & diuide the reſt, that is 6428 by 2 ſo ſhal the Quotient be 3214, whereby I know that 64287 ſhillings, make 3214 poũds, and 7 ſhillings remayning.

Mai. Nowe proue by Multiplication whether you haue wel done or no.

Scho. The quotient is 3214. whiche I doe multiplie by the diuiſor 2, and it doth amoũt to 6428.

Maiſt. Hereby may you perceiue not onlie that you haue wel done, but alſo how by diuiſiõ you may turn ſhillings eaſilie into poũds: And contrarie waies, by Multiplication you may turne pounds into ſhillings.

An other manner of the abridgement.

But here ſhal you ſee amongſt diuers men, diuerſe formes of ſuche diuiſion, but if you marke what I haue tolde you, you ſhall perceiue eaſilie al their waies: for ſome men doe not cut away ſo manie of the firſt figures of the ſum ẏ they would diuide, as there are cyphers in the firſte places of their diuiſor, but they ſet al their ciphers orderly vnder the firſt places of the number that they would diuide, and then with the other figure (or figures if they be manye) they diuide the reſte of their ſumme. Example. If they would diuide 725931, by 3400, they ſet their ſums thus.

```
7 2 5 9 3 1
3 4     0 0
```

And then doe they diuide orderlie till they come to the ciphers: foꝛ there they ſtay & ende their woꝛke, as in this example: They ſeeke how often 3 may be founde in 7, which is 2 times, and one remayning, therefoꝛe they ſet 2 in the quotient, and cancel 3 and 7, & ouer 7 they ſet yͤ 1 yͭ remained, thus.

$$\begin{array}{ccccccl} 1 & & & & & & \\ \cancel{7} & 2 & 5 & 9 & 3 & 1 & (2 \\ \cancel{3} & 4 & & & 0 & 0 & \end{array}$$

Then do they go foꝛthe ſaying: two times 4 maketh 8, whiche they take out of 12, and there remaineth 4, thus.

$$\begin{array}{ccccccl} \cancel{1} & 4 & & & & & \\ \cancel{7} & \cancel{2} & 5 & 9 & 3 & 1 & (2 \\ \cancel{3} & \cancel{4} & & & 0 & 0 & \end{array}$$

Then renew they the diuiſoꝛ foꝛward, and ſeeke how often 3 maye be founde in 4, which is but once, and 1 remaineth, then ſet they 1 in the quotient, and cancell 3 and 4, & ouer them they ſet that 1, thus.

$$\begin{array}{ccccccl} & 1 & & & & & \\ \cancel{1} & \cancel{4} & & & & & \\ \cancel{7} & \cancel{2} & 5 & 9 & 3 & 1 & (21 \\ \cancel{3} & \cancel{4} & 4 & & 0 & 0 & \\ & 3 & & & & & \end{array}$$

Then take they once 4 out of 15, & there reſteth 11. Oꝛ elſe moꝛe eaſilie: Take once 4 out of 5, & there reſteth 1, ſo they cancel the 4 & 5, and ſet 1 ouer them, thus.

$$\begin{array}{ccccccl} & 1 & & & & & \\ \cancel{1} & \cancel{4} & 1 & & & & \\ \cancel{7} & \cancel{2} & \cancel{5} & 9 & 3 & 1 & (21 \\ \cancel{3} & \cancel{4} & \cancel{4} & & 0 & 0 & \\ & 3 & & & & & \end{array}$$

Then ſet they foꝛth yͤ diuiſoꝛ againe, & ſeeke how manye times 3 are in 11, whiche they finde 3 times, and 2 remayning : ſo they ſet 3 in the quotient,and cancell 11 and 3, and ouer them ſetteth 2, thus.

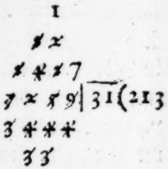
$$\begin{array}{ccccccl} & \cancel{1} & 2 & & & & \\ \cancel{1} & \cancel{4} & \cancel{1} & & & & \\ \cancel{7} & \cancel{2} & \cancel{5} & 9 & 3 & 1 & (2\,1\,3 \\ \cancel{3} & \cancel{4} & \cancel{4} & 4 & 0 & 0 & \\ & \cancel{3} & 3 & & & & \end{array}$$

Then do they multiplie 4 by 3, whiche maketh 12,that withdrawe they out of 29, and there reſteth 17. of whichs the 7 muſt be ſet ouer the 9, and the 1 ouer the 2,thus.

$$\begin{array}{ccccccl} & 1 & & & & & \\ \cancel{1} & \cancel{2} & & & & & \\ \cancel{1} & \cancel{4} & \cancel{1} & 7 & & & \\ \cancel{7} & \cancel{2} & \cancel{5} & \cancel{9} & 3 & 1 & (2\,1\,3 \\ \cancel{3} & \cancel{4} & \cancel{4} & \cancel{4} & 0 & 0 & \\ & \cancel{3} & \cancel{3} & & & & \end{array}$$

And now are the two ciphers next enſuing, ſo that the diuiſor can no more be ſet forward, and therefore is the diuiſion ended , and the remainer is 1731.

Nowe the quotient,which is 213, doth declare,that if you diuide 725931,by 3400 you ſhall finde it therein 213 times, and there remayneth 1731,ſo ſhall you finde it, if you worke as I taught you, by cutting away the two firſte figures,bycauſe of the two ciphers. But this muſt you marke (as you maye perceiue by this laſt example) that if there be left ☞ any other remainer in the ſumme that was behinde the ſquire line, that the remayner muſt be ſet to the latter ende of the firſte remayner, whiche was cut awaye wyth the ſquire line: as if you woulde diuide 725931 by 3400, after the forme that I taught you, then woulde your ſummes appeare thus.



So that 17, whiche remayneth after the line, muſt be ſette to the 31 (that was cut away wyth the line) in higher places, as you ſee here: where that 17 with the 31, do make 1731.

An other inuention of eaſie Diuiſion.

And here woulde I make an ende of Diuiſion, ſauing that there commeth to my minde one late inuention of eaſie Diuiſion whiche I will brieflie ſet forthe to you, ſo that if you finde eaſe in it, you maye vſe it. Bicauſe that the hardeſt point in Diuiſion, is the readie and eaſie finding of the quotient number: and againe, if that be truely knowen, all the reſt is but light to be done: therfore this wayes ſhall you quickelye and truelye finde the quotient.

Firſt

Firſte write the nine figures of number: I meane 1 2 3 4 5 6 7 8 9, not a long as I haue ſet them now, but vp and downe as in this forme. And at the left ſide of them draw a long line, as you ſee here: Then conſider the diuiſor, by whiche you intende to worke, and ſette it on the lefte ſide of the long line, righte againſte 1, and for a diſtinction drawe a line vnder it: then multiplye your diuiſor orderlie by eache of thoſe figures, beginning with 2, and ſo goe downewarde till you haue ended all. And looke what doth amount of the multiplicatiõ of each figure into the diuiſor, then write it againſt the figure wherby you did multiply.

1
2
3
4
5
6
7
8
9

Sc. By example I may perceiue it better.

Maiſter. Take this example 2 6 3 8 4 5 to be diuided by 64 thẽ muſt I ſet the 9 figures as I ſaide before, and the diuiſour muſt I ſet againſt the 1, thus. Thẽ muſt I multiply yͤ diuiſor by ech figure orderly: firſt by 2 and it maketh 128, which I muſt ſette againſt 2 at the left hand.

64	1
	2
	3
	4
	5
	6
	7
	1
	9

hande,

Then multiplie 64 by 3, and it maketh 192, whiche is ſette againſt 3. Then 4 times 64, make 256, ẏ ſet I by 4. Then ſaie I, 5 times 64 make 320, that ſet I againſte 5. Then 6 times 64, make 384, that ſette I againſte 6. Then 7 times 64 make 448, whiche I ſet againſt 7.

Further I ſay: 8 times 64, make 512, which I ſet by 8. And laſt of all I ſaye: 9 times 64, make 576, whiche I ſette againſte 9. And then the will ſtand thus.

64	1
128	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

And ſo is the table ended, by which you may eaſilie find the Quotient, as you ſhal ſée by example now.

Do you ſet down ẏ numbers (as you learned before) according to the order of diuiſion.

Sch That is thus.

```
2 6 3 8 4 5
  6 4
```

Mai Now looke what number ſtandeth ouer the diuiſor, reckning therto al them that be behinde it toward the left hand.

Scholer Then are there ouer the diuiſour, 263.

Maiſter. That is iuſte : nowe ſeke in the table on the lefte ſide, whether you can finde 263.

Sch. It is not there.

Maiſter. Then take the number that is next to it, beneath it: I meane a leſſer number than 263, but of all the leſſer numbers that the table hath, take you that that goeth nigheſt to 263.

Sch. That is 256.

Maiſter. So is it : and marke this euermore, when you can not finde iuſtlie in the table that ſumme that is ouer your diuiſour, then note that that is nexte beneath it of any ſumme that is in the table, and looke at the righte hande of the line what figure or digit that is againſt that ſumme, and take that digit for your quotient, and then worke on, as you learned before: for now haue I tolde you the whole vſe of this table.

Howbeit, yet that you maye be ſure to vnderſtande it, I will ſee you end thys example of Diuiſion by it.

Nowe therefore begin againe.

Scholer. Firſt I ſette downe the ſummes after the common maner, thus.

263845
64

Then do I looke ouer the diuidend, and finde there 263. Now to know how many times 64 may be taken out of 263, I reſort to the table aforeſaid, and ſeeke for the number 263, but it is not there, therefore as you had me, I take a leſſer number, the next to it that I can finde in the table, and that is 256, which number hath againſte it on the right hande this digite 4, which I muſt take for the firſt figure of my quotient.

Then do I (as I learned before) multiply that quotient into euerie figure of the diuiſor orderly, withdrawing the ſumme thereof, as mounting out of the ouer ſum: as here I ſay firſt: 4 times 6 make 24, ſo I take ẏ out of 26, ſaying: 4 out of 6 remaineth 2, which I write ouer ẏ 6: then 2 out of 2 remaineth nothing, then cancel I 2 and 6, and alſo 6 in the diuiſor, and the ſumme ſtandeth thus:

$$\begin{array}{l} \quad 2 \\ \cancel{2}\cancel{6}3845\ (4 \\ \cancel{6}4 \end{array}$$

Then doe I likewiſe ſay forth: 4 times 4 make 16, which I take out of 23, & there reſteth 7 to be ſet ouer 3, & that 3 with the 2 behinde it and the 4 vnder it, muſt be cancelled, as you ſee here.

$$\begin{array}{l} \quad \cancel{2}7 \\ \cancel{2}\cancel{6}\cancel{3}845\ (4 \\ \cancel{6}\cancel{4} \end{array}$$

Then haue I done wyth the

the figure of the quotient.

Maiſter. Now ſette forwarde your diuiſour, and ſéeke a newe quotient as you ſought this.

Scho. Then thus ſtandeth the figures. ſo that ouer the diuiſour I ſée 78, which I ſéeke in the table, and cānot finde it: therefore I take the next leſſer, and that is 64 the diuiſor it ſelfe.

$$\begin{array}{l} \cancel{2}7 \\ \cancel{2}\cancel{6}\cancel{3}845 \quad (4 \\ \cancel{6}\cancel{4}4 \\ \quad 6 \end{array}$$

Mai. So muſt you do when there is none other.

Sch. Then againſte it I finde this digit 1, which I muſte ſet in the quotient before 4, thus.

$$\begin{array}{l} \cancel{2}7 \\ \cancel{2}\cancel{6}\cancel{3}845 \quad (41 \\ \cancel{6}\cancel{4}4 \\ \quad 6 \end{array}$$

Then multiplie I 6 by 1, and it is but 6 ſtil.

Note.

Maiſter. You néeds not goe about to multiplie when the Quotient is 1, for 1 doth neither multiplie nor diuide, but in ſuch caſe only ſubtract the diuiſor out of the number that is ouer it,

Scholer. Then I take 4 out of 8, and there reſteth 4, & 6 out of 7 there remayneth 1, ſo I

cancell

cancel thoſe numbꝛes, & wꝛite the remainers ouer their places, thus.

1

$\cancel{2}\cancel{7}4$

$\cancel{2}\cancel{6}\cancel{3}\cancel{8}45 \quad (41$

$\cancel{6}\cancel{4}\cancel{4}$

$\cancel{6}$

Then ſet I foꝛward the diuiſoꝛ again, thus.

1

$\cancel{2}\cancel{7}4$

$\cancel{2}\cancel{6}\cancel{3}\cancel{8}45 \quad (41$

$\cancel{6}\cancel{4}\cancel{4}4$

$\cancel{6}6$

Where I ſée ouer the diuiſour 144, whiche I ſéeke in the table, and finde it not: therefoꝛe I take the nūber in yͤ table yͭ is nexte thereto, beneathe it, whiche I finde to be 128, againſt which in the right ſide I finde 2, which I take foꝛ my quotient, and that doe I multiplie firſte into 6, and thereof commeth 12, which I take out of 14, and then remaineth 2, that 2 I ſet ouer 4, and cancell the other figures, 1, 4 and 6, thus.

$\cancel{1}2$

$\cancel{2}\cancel{7}\cancel{4}$

$\cancel{2}\cancel{6}\cancel{3}\cancel{8}45 \quad (412$

$\cancel{6}\cancel{4}\cancel{4}4$

$\cancel{6}\cancel{6}$

Then ſay I foꝛ the: 2 times 4 are 8, whiche I take out of 24, and there remaineth 16, of whiche I wꝛite the 6 ouer 4, & the 1 ouer 2 and cancell 2,4 and 4, thus.

1

$\cancel{1}\cancel{2}$

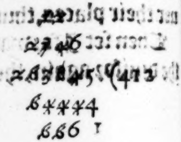
$\cancel{2}\cancel{7}\cancel{4}6$

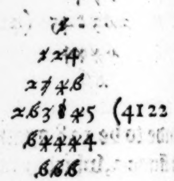
$\cancel{2}\cancel{6}\cancel{3}\cancel{8}\cancel{4}5 \quad (412$

$\cancel{6}\cancel{4}\cancel{4}\cancel{4}$

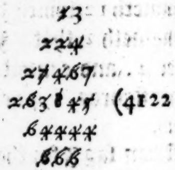
$\cancel{6}\cancel{6}$

Now againe I ſet forwarde the diuiſor thus. And ſeeing ouer it 16, I ſeeke that in ẏ table, but finde it not, therefore I take ẏ next leſſer, which is 128, againſt which I find 2: that do I ſet into the quotiēt, and by it I multiplie firſt 6, & therof cōmeth 12, which I take out of 16 and ther reſteth 4 thē cācel I 1, 6, and 6, & ouer 6 I ſet 4 thus.





Then do I multiply 4 by 2, and it maketh 8, which I take out of 45, & there remaineth 37, as in this example.



And nowe haue I done.

Mai. Well, now I ſee that you can worke by this kinde of diuiſion, as farre forth as I taught you.

Scho. Yea ſir, I thanke you, and I find in it much eaſe and certainenes.

M. Yet one thing I doubt whether you perceiue: what if you did finde in the table the nūber that ſtandeth ouer the diuiſor, what wold you next doe?

Scho. I thinke I ſhoulde take the digit againſt it on the left hand for the quotient.

Ma. So is it: and as often as you ſeke in the table and finde your number iuſt, the digit againſt it is your true and iuſt quotient. I cal that a true quotient alſo, if it be ẏ right quotient that you ſhoulde take, though your diuiſor multiplied by the ſame, do not cleare or ſubtract the number ouer it, but there doth ſomewhat remaine, as it chaunced in al the examples that you did worke by. But if it ſhoulde chance (as it doth often) that your diuiſor multiplied by your quotient, do ſubtract clean the number ouer it, thē cal I that quotient not only a true quotient, but alſo a iuſte quotient, becauſe it doth iuſtlie conſume the number ouer the diuiſor: and that chaunceth euermore when the nūber ouer the diuiſor is iuſtly found in the table.

Marke the diuerſitie betvveene a true quotient and a iuſt quotient.

Scho. This I ſhal remember.

Mr. But yet one eaſie pointe more I wyll tel you in this ſort of diuiſion, therfore marke it wel, When you haue founde in the table, either the ſame ſum that is ouer the diuiſor, other ỹ next beneath, (for lack of the other) thẽ looke what digit ſtandeth againſte it, take that for your quotient. And bicauſe it is ſome paine to multiply the diuiſor by the quotient, you ſhal not næde to doe it, but only take the number that you founde in the table, and ſubtract that from the ouer number: for if you do multiply the diuiſor by the quotient, that will be the number that ſhal amounte, therefore is this way more eaſier.

S. So is it, and alſo more cetainer for ſuch as I am, ỹ might quicklie erre in multiplying, eſpeciallie being ſmallye practiſed therein.

M. Thẽ proue in ſome brief exãple whether you can do it, and ſo wil we make an ende.

Scholer. I would diuide 38468 by 24, therefore firſt I ſet ỹ table as here foloweth: Then ſet I the two ſums of Diuiſion thus.

38428
24

And ouer the diuiſor I finde 38, which I ſæke in the table & finde it not, therefore I take the nexte beneath it which the table hath, & ỹ is 24 the diuiſor it ſelf: againſt which is ſet

24	1
48	2
72	3
96	4
120	5
144	6
168	7
192	8
216	9

1, which I take for the quotient, which I ſet in his place. And now I neede not to multiplie the diuiſor by it, but only to withdrawe the diuiſor out of the 38, that is ouer it, and ſo remaineth 14, as thus.

14
~~38~~468 (1
~~24~~

Then ſet I forwarde the diuiſor, and finde ouer it 144, as appeareth: then ſeeke I yͤ number in the table and finde it, and againſt it is 6, therefore I ſet 6 before 1 for my quotient, and I take that 144 for the iuſte multiplication of the diuiſor by that quotient, and therefore without any newe multiplication I do ſubtract the 144, from the other 144, and there reſteth nothing, as you may ſee.

14
~~38~~468 (1
~~24~~4
2

~~14~~
~~384~~68 (16
~~244~~
~~2~~

Therfore I ſet forward the diuiſor: but ſeing it will not bee in the next place, (for then ouer 2 woulde be nothing) I ſet it forwarde twice, as you ſee here.

~~14~~
~~384~~68 (16
~~244~~44
~~2~~22

And for becauſe that I could not ſet it in yͤ nexte place following, therefore

I ſet a cipher in the quotient, as you ſee.

Then loke I ouer the diuiſor, and finde 68, which I can not finde in the table, therefore take I the next beneath it, which I finde in the table, and that is 48, and againſt it ſtandeth 2, whiche I take for the quotient. And then without any multiplying of the quotient in to the diuiſor, I doe ſubtract that 48 from 68, & there reſteth 20, as here appeareth.

~~1~~~~4~~20
~~8~~~~6~~~~4~~~~6~~8
[illegible]

And ſo haue I ended the whole diuiſion.

Maiſter. In verie great ſummes to be diuided by great diuiſors, I thinke there is no better way thã this for any mã to vſe, though he be neuer ſo expert. And that eſpeciallie, if one great diuiſor be oftẽ to be occupied about diuiding many and diuers great ſummes. As commonly happeneth in Aſtronomical workings, and Geometricall, about the ſignes, both ſtraight and reuerſed: as if it be your fortune and deſire to wade to the profoundnes of Geometricall and Aſtronomicall calculations demonſtratiue, you will ſone confeſſe. Whereof, an other time ſhall better ſerue to ſpeake. Now can you ſufficiently ſkil in theſe

kindes

kinds of Arithmetike.) And now for the farther vſe of theſe two laſt, that is multiplication and diuiſion: I will briefly ſhew you the feate of Reduction, by the way.

REDVCTION.

REduction is, by which al ſummes of groſſe denomination maye be turned into ſums of more ſubtile denomination. And contrarye waies, all ſummes of ſubtile denomination, maye be brought to ſums of groſſer denomination.

Scho. What call you groſſe denomination, and ſubtile denomination?

Groſſe denominatiō.

Mai. That I call a groſſe denomination, whiche doeth containe vnder it many other ſubtiller or ſmaller: As a pound in reſpecte to ſhillinges, is a groſſe denomination: for it is greater than ſhillings, and contayneth many of them. And ſhillings in compariſon

to

Subtile denominatiō.

to poundes, or a ſubtile denomination, for becauſe they are leſſer then poundes, and many of them are contained in one of the other: as ſo, likewyſe of other thinges, whatſoeuer thing is compared to other, if it be greater and containeth many of them, it is a groſſer denomination: but if it bee leſſer, ſo that manye of them are in the other, then are they called ſubtile denominations: whereby you may perceaue, that one denomination may be called a groſſe denomination, and alſo a ſubtile (that is to ſay, a great and ſmall) in diuers compariſons. For ſhillinges compared to poundes are a Subtile or ſmall denomination: but compared to pennies, they are a groſſe or greate denomination.

Scho. Nowe I vnderſtande the name, I pray you teache me the vſe.

To reduce groſſe denominatiōs to ſubtile.

Ma. The vſe is eaſily learned, if you remēber what you haue learned before. For if you will reduce any ſumme of a groſſe denomination, into a ſumme of a ſmaller or ſubtiller denomination, you muſt conſider howe many of that ſubtiler denomination doe make one of the groſſer denomination, and by that number or numerator doe you multiplie the other ſumme : as if you woulde reduce 20 poundes into ſhillinges, you muſt conſider that in a pound are included 20 ſhillings, therfore multiplie the one 20 by the other 20, and there will amounte 400, whereby you may knowe, that in 20 pounde are contained 400 ſhillinges. Likewiſe if you would reduce 30 ſhillinges into pennies, conſidering that in 1 ſhilling, are 12 pennies, you muſt multiplie 30 by 12, and it will be 360 : whereby you find, that in 30 ſhillings, are contained 360 pennies. And thus may you reduce any groſſe denomination into a more ſubtiller, by multiplication, if you know how many of the leſſer doe make the greater : of which thing I will anone giue you a bréefe table for the moſt accuſtomed kinds of money, weights, meaſures, and tyme, and ſuch like, whereby you maye knowe howe often eche ſubtile denomination is contained in the Groſſer, when you ſhall néede it for the foreſayde kinde of Reduction. And alſo the ſame ſhall ſerue you, if you would reduce any ſumme of a ſubtiler denomination, into a ſumme of a groſſer denomination : For in ſuche Reduction you muſt conſider (as in the o-

To reduce ſubtile denominatiõ to groſſe.

ther forme) howe manye of the ſmaller doe make the greater, and by that number muſt you diuide the other ſumme, and the quotient will declare howe many of the greater denomination, are comprehended in that ſumme, as for example: If you woulde know how many ſhillinges are contayned in 3240 pence, conſider that 12 pennies doe make 1 ſhilling: you muſt diuide that 3240 by 12, and your quotient wil be 270, whereby you know that ſo many ſhillinges are in 3240 pennies. But and you would knowe farther, how many pounds are in thoſe 270 ſhillinges, ſeeing that euery pound contayneth 20 ſhillinges, diuide that 270 by 20, and it will be 13, and 10 remaining, whereby you may know that in 3240 pennies, or 270 ſhillings, are 13 poundes and 10 ſhillings. For euermore the remainer muſte be named by the name or denomination of the ſumme that was diuided, which in this place were ſhillinges. And thus maye you doe with any other kindes of denominations.

Wherefore to the intent that you maye haue a lighte knowledge in the common Coines, weights, meaſures, and ſuch other, I haue prepared here a briefe table, whiche ſhall ſuffiſe to you at this time, til hereafter at more conueniente opportunitie I maye inſtruct you more exactlie in the ſame.

Note (gentle Reader) theſe valewes of Engliſhe coines, as they were when this Authour firſt publiſhed his Booke. But in our time, (namelie An. 1582.) they are much diuerſe. Therefore ſomething to pleaſure thee in this purpoſe. I haue for thy benefit at the end of Reduction, ſet downe and annexed a Table, not onlie of our coines, but alſo of the moſt parte of Chriſtendome, with their iuſt waighte and values, currant in this Realme of *England*; as by the ſame ſhall plainlie appeare.

A Table for Engliſh coines. *An,* 1540.

English Coines.

A Soueraine.
Half a Soueraine.
A Roiall.
Halfe a Roiall.
A quarter Roiall.
An old Noble.
Halfe an old Noble.
An Angell.
Halfe an Angell.
A George Noble.
Half a George Noble.

A quarter Noble.
A Croune.
Halfe a Crowne.
A Croune.
A Grote.
A harpe Grote.
A penie of 2 pence.
A dandie pratte.
A pennie.
An half pennie.
A Farthing.

The

The valew of Engliſh Coynes.

The value of Engliſh Coines.

A Soueraine is the greateſt engliſh coin, and containeth 2 Roials, or 3 Angels, eyther 9 halfe Crounes, or 4 Crounes and an halfe, that is to ſay, 22 ß 6 d.

Half a Souerain is equall with a Royall.

A Royall containeth an Angell and a halfe, that is to ſay: 11 ß, 3 d.

Halfe a Royall containeth 5 ß,7 d,ob.

A quarter of a Royall containeth 2 ß, 9 d, ob.q̄.

An old Noble, called an Henrie, is worth 2 Crounes, or a Noble and half, that is 10 ß.

Halfe an old Noble is worth 5 ß.

An Angell containeth a Croune and halfe, or 3 halfe Crownes, that is 7 ß 6 d.

Half an Angell is worth 3 ß,9 d.

A Noble called a George, is worth 6 ß 8 d.

Half a Noble is worth 3 ß, 4 d.

A quarter of a Noble (which in the old Statute is called a Farthing) containeth 20 d.

A Crowne containeth 5 ß: and the halfe Croun 2 ß 6 d. Howbeit there is an other Croun of 4 ß,6 d which is knowne by the roſe ſide, for the Roſe hath no Crowne ouer it,as in the other Crowne,but it is enuironed on the 4 quarters with 4 floure deluces,whereby you may beſt know it. But I will returne to ſpeake of the value of the coynes,for I intend not now to deſcribe the formes of them. Now of gold are there no more common coynes.

In ſiluer the greateſt is a Grote, whiche containeth 4 pennies. Thē is there another Grote called an Harp, which goeth for 3 d. Then next is a pennie of 2 d. And then a Dandiprat, worth 3 halfe pence. Nexte it a pennie,then half a pennie,and laſt and leaſt of all a Farthing, whoſe coine is on the one ſide a croſſe,& on the other ſide a purculles. This I tell you, becauſe I ſee manye that cannot know a farthing from a ſmall halfe penie.

Now haue I tolde you all the Engliſhe coynes both of gold and ſiluer, but yet of the three moſt cōmon valewers of money ſpake I nothing, that is to ſay, of pounds, Markes, and ſhillinges, whiche though they haue no Coynes, yet is there no name more in vſe then they : of whiche the ſhilling contayneth 12 pennies or 3 grotes : and the pound 2 old Nobles , 3. George Nobles, or 4 Crownes 9. that is to ſaye, [illegible] a Marke, two George Nobles, that is 13 ſ. 4 d.

Here would I now expreſſe the valewes of ſundrie other coines of diuers countries, but for three cauſes I now refraine. The firſt and chiefeſt is, becauſe they are not currant by the Statutes of this Realme. An other cauſe is, by reaſon they are ſo vncertaine, that they be neuer long at one rate. And againe they are ſo different in ſo many places, that it were matter inough for a great booke, to ſpeake ſufficientlie of them all. Howbeit, yet becauſe you ſhall not be altogither ignoraunt of them, I will ſhewe you the valewes of ſome that are moſte in vſe, and firſt of Fraunce.

Frenche coines.

The moſt common money are Deniers, Soulx and Franks. 12 Deniers make 1 ſ. 20 ſoulx make 1 Frank, ſo that as you ſee, theſe three kindes are like in the rate to penies, ſhillinges, and poundes with vs, but that this is the difference, that their Denier is but the 9 part of our pennie, and ſo their ſoulx (commonly called ſowſes) go 9 to our ſhilling, and 9 of their Frankes to an Engliſhe pounde of money: So that 3 of their

Franks make a noble. And by thoſe 3 may you practiſe how to reduce French mony into Engliſh mony. And as for the reſt of their coynes I will omit till another time, when I intend to ſhewe you the rate of ſundry other kindes of mony.

But now as for the coynes of Flaunders they be ſo chaungeable, that you muſt know thē from time to time, els you cannot reduce them into our money certainely. But yet that you may haue an example of their mony to exerciſe you withall, you ſhall take thoſe that be moſt common as Stiuers both ſingle and double, Grotes Flemmiſh, Carolus, and Gyldens: A Flemiſh Grote is a litle aboue 3 farthings Engliſh. A ſingle Stiuer is 1 d, ob, q. The double Stiuer is 1 d q̃. The ſiluer Carolus ſingle, 2 d q̃. q. c. The double ſtiuer Carolus is 4 d ob, q̃, q. Then is there alſo yͤ Carolus Gyldē which is worth 20 Stiuers And the Flemiſh noble is worth 3 Carolus Gildens, and 12 Styuers.

Flaunders Coynes.

Touching Danſke mony, they haue their ſoulx wherof 20 is a Liuer, which is 2 ß ſterling. They haue alſo their Graſhe whereof 30 makes a Gylderne, which is foure ß.

D. ſter-

ſterling: They haue alſo Dollors, and their common or olde Dollor is 35 Graſhe: new Dollors they haue, whiche be diuers, ſome valued at 24 Graſhe, ſome at 26, and ſome at 30: and thus much I thought good to adde to the Author touching Danſke mony.

But I will let them paſſe now, exhorting you to practiſe to reduce thoſe kinds into Engliſh money, according as I haue ſette forth here following: 2160 deniers, make 240 d: or 20 ß: 3240 deniers, make 360 d, or 30 ß: 8352 deniers, make 928 d, or 2 ℔ 17 ß, 4 d: 2160 ſouls, make 240 ſhillings: and ſo of other in like caſe.

But if you will reduce Flemmiſh money iuſtly, you muſt reduce it firſt into the ſmalleſt parte of Engliſhe money that is in that coine, as for example. If I would reduce 368 double ſtiuers into Engliſh money, conſidering that a double Stiuer containeth 3 d q, you ſhall firſte looke howe many q be in the double Stiuer, and you ſhall finde them 13, therefore multiply the ſumme of the ſtiuers by 13, and then haue you their valew in farthings, which is 4784. Nowe if you diuide that by 4, then will there appeare the number of pence: but better it were to diuide it by 48 (for ſo manye farthings are in 1 ſhilling) and then will the quotient declare the ſumme of the ſhillings.

Likewiſe, if you would reduce any ſumme of ſingle ſtyuers into Engliſhe money, you muſt multiply the ſumme firſt by 12, & then haue you a certaine ſumme, which ſumme if you diuide by 8, then wil amount the ſumme of pence: or if you diuide it by 96, the ſumme of ſhillings will appeare.

But this marke in al diuiſion, when ye do reduce to bring one denomination into an other, if there be any remayner after the diuiſion, that muſt be named by the denomination of the groſſe ſumme that was diuided: as for example: I woulde bring 254 q̃ into pence, therefore I do diuide that 254 by 4, for ſo many farthings make 1 penny, and the quotient is 63, whiche is the ſumme of the pence, and then remaineth yet 2, whiche are farthings ſtill, as one maye proue by diuiding. And this muſt be marked in all Diuiſion, namely when it is done for Reduction.

Yet two wordes more added to the Author

Cõcerning Spaniſh mony, wherof the moſt common mony are Cornados, Marueides Ryalls and Duckets: 6 Cornados make a Marueide, 34 Marueids maketh one Ryall, and 11 Rialls maketh one Duckate, ſo the Ducket containeth 374 Marueids, which to reduce into ſterling mony Engliſh, 24 Cornados maketh our penny: or 5 Marueides & 4 Cornados &c.

VVeights.

A Graine.

A Penny of Troy.

An Ounce.

Thus muche haue I ſayde of Mony, nowe will I ſhew you in like ſorte the diſtinction of weyghts, after the ſtatutes of Englande, where the leaſte portion of weight is commōly a grayne, meaning a grayne of Corne or wheate, dry and gathered out of the middle of the eare. Of theſe graynes in time paſſed 32 wayed iuſt 1 peny of Troy, and then was but 20 pennies in an Ounce. But nowe are there 40 pennies in an Ounce, ſo that there are not fully 14 graynes in one penny.

But now of Ounces after Troy rate (which is the ſtandard of Englande) 12 doe make 1 pound.

Haberde-poiſe vveights.

But commonly there is vſed another weight called Haberdepoiſe, in which 16 ounces make a pounde. Therfore when you would reduce ounces into poundes, you muſt cōſider whether your weyghts be troy weighte or Haberdepoiſe: and if it bée Troy weyght you muſt diuide your ounces by 12, to bring them to poundes: but if it bée Haberdepoiſe, you muſt diuide them by 16. Now againe, there bée greater weights which are called an hundred, halfe a hundred, & a quarterne, and alſo a halfe quarterne &c.

A hundred VVeight.

Scholler. Why? ſo there may be reckened 20 pounde, 40 pounde, 200 pounde and ſuch innumerable.

Maiſter. All theſe are numbers of weyght, but they haue not common weights made to their rate, as the other haue. And agayne, theſe that I did name are not iuſt in number as they ſéeme by their name, for an hundred is not iuſt 100, but is 112 pounde. And ſo the halfe hundred is 56: the quarter 28, and the halfe quarter 14. And theſe be the common weights vſed in moſt things that are ſolde by weight.

Howbeit there are in ſome things other nams, as in wool, 28 poūd is not called a quarterne but a Todde: and 14 pounde is not named half a quarterne but a Stone, and the 7 pound halfe a Stone. Other names bycauſe they differ in many places, and agrée in few,

VVooll weights. Todde. Stone.

I let them paſſe.

Sacke.

But a Sacke of Wooll by the Statutes, is limited to be 26 Stone.

Cheeſe vvaights.

Now in cheeſe, though it be ſolde by the hundreth, and by the Stone in ſome places, yet the very waightes of it are Cloues and Weys. So that a Cloue containeth 8 pound: and a Wey 32 Cloues, which is 256 pound, that is 12 ſcore and 16 pound: And ſo muche weigheth the Wey of Suffolke cheeſe: And the like waight is or ſhoulde be the Barrell of Suffolke Butter.

The Wey of Eſſex Cheeſe containeth 16 ſcore, and 16 pound: And ſo much is alſo the barrell of Eſſex Butter.

Meaſures for liquor.

A Pinte.
Gallon.
Pottell.
Quarte.

Fyrkin.
Tertian.
Kilderkin.
Barrell.

Now of waights are made other meaſures, both for graine and liquor. For a pounde in waight maketh a Pinte in meaſure, ſo that 8 pound (or 8 Pintes) doe make a Gallon: half a Gallon is named a Pottle: and half a Pottel is called a Quarte, which containeth two pintes. Now aboue a Gallon the nexte meaſure is a Firken: then a Tertian, a Kilderkin or halfe Barrell, and a Barrell. And by thoſe meaſures are ſolde commonly, Ale, Beere, Wine, and Oyle, Butter, and Sope, Salmon, Herrings, and Eeles.

Bu

But as theſe be vnlike things, ſo the meaſure of their veſſelles doe differ: for the meaſures of Ale are as followeth. Ale meaſures

Of Ale,	the Fyrken	conteineth	8	gallons
	ÿ Kilderkẽ		16	
	the Barrell		32	

Of Bære	the Fyrken	conteineth	9	gallõs
	ÿ Kilderken		18	
	the Barrell		36	

Sope meaſures. Sope meaſures, both Firken, Kilderken, and Barrell, ſhould be equall to al Ale meaſures. Moreouer the Statutes do limitte the waight of euery of thoſe thrée veſſels being empty.

A Barrell	to weigh empty	26	pounds.
Halfe Barrell		13	
A Firken		6½	

Herring. Herrings alſo be ſolde by the ſame meaſures that Ale and Sope be ſolde by.

Herrings alſo are ſolde by the tale, 120, to the hundred, ten thouſand to the laſte.

Salmon and Eeles. Salmon & Eeles haue a greater meaſure.

Salmon & Eeles	the butte	holdeth	84	Gallons.
	ẏ barrell		42	
	halfe bar.		21	
	the firken		10½	

Howbeit,ſome Statutes did limitte Eele veſſels equall with Herring veſſels.

VVine meaſures.

Nowe as for wine veſſelles ſeldome are ſmaller than Hogges heads, which are of 63 gallons : euery Hogges head is two Barels: yet there are many other wine veſſels, but of them all, ſee this table , and marke the meaſures one to an other.

Of wine and oile.	the Rondlet	holdeth	18½	Gallons.
	the Barrell		31½	
	ẏ hogges hed		63	
	the Tertian		84	
	the Pipe		126	
	the Tonne		252	

Tertians.

But you ſhall marke, that there be other kindes of Tertians : for there be Tertians (that is to ſay) Thirdles of Pipes,of hogges heads,and of Barrels,as wel of other things as of wine.

Butte.

Alſo of Malueſeys,and Sacke, &c.the

halfe

half Tonne is not called a Pipe, but rather a Butte.

And thus much haue I thoughte méete to tell you at this time.

Scholer. And is that alwayes true?

Maiſter. I haue tolde you howe it ſhoulde be, but howe it is I may not ſay: howe they doe differ daily from their iuſt meaſure, that Gagiers can tell you better than I. But I wil let this paſſe now, and ſpeake briefly of the other meaſures.

And as of waightes there did ſpring the liquide meaſures, (whereof I ſpake laſte) ſo of the ſame ſpringeth drye meaſures: as Peckes, Buſhels, Quarters, and ſuche like, whereby are meaſured corne and like grains: alſo ſalt, lime, coales and other like. And this is the order and quantitie of them.

Drie meaſures.

A Pecke is the meaſure of two Gallons. A Pecke.
A Buſhel containeth foure Peckes. A Buſhel.
A Quarter holdeth eight Buſhels. Quarter.
A Wey containeth ſixe quarters. VVey.

Theſe are the commō names & meaſures, but in diuers places there be diuers ſortes.

The buſhel in many places is 2 buſhels: but then is the buſhell there called a Strike. And in ſome places half a quarter is called a

Strike.

a Cornoke. But theſe diuerſities are to many to tel you briefly them al. And again, ſith they are againſt the law and Statutes, I counte them vnmeete to be vſed.

Meaſure to meat lẽgth, breadth, & thicknes. But now remaineth yet an other kinde of Meaſures, wherby men met length & bredth, and thickneſſe, and thoſe are an Ynch, a foote, and ſuch other: whoſe names and quantities this table ſheweth.

An ynch. 3 Graines of barly in lẽgth, make an inch.
12 Ynches make a foote.
Foote. 3 Foote make a yearde.
Yeard. Elle. 3 Foote and 9 ynches make an Elle.
Perch. 5 Yeards and a halfe, make a Perch.

1 Perche in breadth and 40 in length, doe make a Rod of lande, which ſome cal a roode, ſome a yeard land and ſome a Farthendele.

1 Acre. { 2 Farthẽdels, make half an acre of groũd.
4 Farthendels make an Acre.

More 40 Rods in length do make a furlõg: 8, furlongs make a mile whiche conteyneth 320 Perches.

So that an Engliſh mile grounded vppon the Statute is in length 1760 yeardes, 5280 foote, and 63360 ynches.

Somewhat greater then the Italian mile

1000 paces, and 5 foote to a pace.

Here might I tell you many things elſe touching meaſure, and alſo howe to reduce ſtrange meaſures to our meaſures, but bycauſe it can not well bee done wythout the knowledge of Fractions, whiche as yet you haue not learned, I will let them paſſe til an other time, when I ſhal inſtruct you in Geometrie, wherein I ſhould be enforced elſe to repeate the ſame often againe.

Scholer. But yet ſir of the partes of time I pray you tell me ſomewhat.

Maiſter. You know that a naturall daye hath 24 houres, and euery houre hath 60 minutes. It needeth not to tel you, that 7 dayes make a weeke, and 4 weekes make a commõ moneth, and 13 moneths make a yeare, lacking 1 day, & certaine houres, and minutes: But of that I ſhall inſtruct you hereafter.

The partes of time.
A Day.
An houre.
VVeeke.
Moneth.
Yeare.

Here wil I make an end of Reduction for this time, whiche though it be counted no kind ſeueral of Arithmetike, yet you ſee it is no leſſe needefull to be knowen, or eaſier to be done, than of any of the other.

Scholer. Mary ſir, it ſeemeth vnto mee much harder than any other ſort, for it requireth the knowledge of ſo many things: but now ſir when you ſee time, I am readie to learne forth: for as much of Reductiõ as you haue taught me, I remember, but and if I doe at any time forget, I ſhall haue recourſe to the tables which you haue ſet forth for me.

Maſter. So do you, for it wil not be remembred without exerciſe.

And now according to promiſe here followeth the Table which I haue added to this Author for thy vtilitie, intending at the latter ends of my Addition to this Booke, to write of the ordinarie money vſed in moſte partes of Chriſtendome: and their common values currant for traffique in thoſe places, with the manner of their exchaunges, as alſo their vſuall waights and meaſures, which I hope will be as gratefully taken of Gentlemen, merchants, and other my Countreymen in generall as I of good will ſette forth the ſame.

A Table of the names and valuatiō of the moſt vſuall Gold coines throughout Chriſtendome, with their ſeuerall waight of Pence and Grains:and what they are worth of currant mony Engliſhe.

The names and titles of the gold.	The vveight.		The valevve.	
	Pēny.	Grains.	Shil.	Pence.
Royall	4	23	15	0
Halfe Royall	2	11	8	6
Olde Noble	4	9	13	4
Halfe old Noble	2	4	6	8
Angell	3	7	10	0
Halfe Angel	1	15	5	0
Salute	2	5	6	4
2.parts of Salute	1	11	4	2
George Noble	3	0	9	0
half George noble	1	12	4	6
Firſt crowne K.H.	2	9	6	4
Baſe crowne K H.	2	0	5	0
Great Soueraine	10	0	30	0
Souera. K.H beſt	0	0	10	8
Edward Souera.	3	14	10	0
Souera. K.E.	3	14	10	0

Vni-

The names and titles of the gold.	The vveight.		The valevv.	
	Peny	Grains.	Shil.	Pence.
Vnichorne of Scot	2	10	6	6
Elizab. Soueraine	3	14	10	0
Elizab. crowne	1	19	5	0
Scottiſh crowns	2	5	6	0
French Noble	4	16	13	4
Al the ſorts of Frenche crownes	2	5	6	0
Old French cro.	2	5	6	0
Flaunders Rider	2	6	6	6
Gelders Rider	2	2	3	6
Phillips Royal	3	10	10	0
Phillips crowne	2	5	5	0
Collyn Gilden	2	2	4	8
Neſn Andr Gyld:	2	3	5	0
Flanders noble	4	10	12	
Flem. Angel beſt	3	6	9	
Fland. real, or Key	3	10	10	
Carolles, Gylden	1	21	3	6
Flanders Royal	2	6	5	
Saxon, Gylden	2	2	4	8
Flanders crowne	2	5	6	
Phillips Gylde.	2	3	4	2
Golden Lyon	2	16	7	8

The names and titles of the gold	The vveight.		The valevv	
	Pẽny	Grains.	Shil.	Pence.
3.parts of gol. Ly.		21	2	5
2.3 parts gol. Ly	1	19	4	11
Dauids Gylden	2	2	4	
Horne Gylden	1	12	4	11
Old Andre, Gyld.	2	4	4	10
Cruſado long cros.	2	6	6	
Cruſ. ſhort cros	2	6	6	2
Myl rayes	4	20	13	4
Half Mill rayes	2	10	6	8
Portigu. [illegible] ounce	2	16	3 ℔. 8 ß.	
Portigu. i. ounce	2	18		
Golden Caſtilio	2	22	8	10
Ducket of Caſtile				
Ducket of Aragō	2	6	6	6
Hungarie Ducket	2	7	6	4
Double Piſtolate	4	8	11	8
Single Piſtolate	2	4	5	10
Ducket of Valens	2	6		
Ducket of Florēn.	2	5	6	4
Double Ducate	4	11	13	
Single Ducket	2	6	6	6
Dou. duc. of Rome	4	12	12	8

Of ſiluer coines currant in this Realme

The Edward Crowne of 5 ß
The Edward halfe crowne of 2 ß 6 d.
The Edward ſhilling, halfe ſhilling, and the 3 d.
Philip and Maries ſhilling and half ſhilling.
The Marie groat, and Marie 2 d.
Queene Elizabeths ſhillings 6 d. 4 d. 3 d. 2 d. 1 d. 3. ob. and 3 q̃.

Note. It is to be vnderſtoode (gentle Reader) that whereas the waight is called by the name of a penny, it is not meant a penny of ſiluer money, but a penny of Goldſmithes waight, which is 24 Barly cornes drie. And xx. of theſe pence make an ounce: and twelue of theſe ounces make a poũd Troy: So that if a man haue not the waight wherewith to wey any peece that may come to his hand, he may do it with the Barly graines or cornes being drie and taken out of the middle of the eare. Nowe to Progreſſion.

PROGRESSION.

Although untill this day the moſt parte of writers haue defined Progreſſion as a compendious kinde of Addition, yet truelye it is not ſo: for progreſſion (as the verie nature of the worde doth informe any man) is a going forwarde and proceeding in numbers, and that regularlie and orderlie, whoſe place is aptlye choſen to be verie neare, or rather next after the expoſition of the four principal partes of Arithmetike, for in it after a moſte eaſie manner, are all the foure former partes exerciſed and practiſed: and not onelye Addition, as cuſtomablie is done. Whiche cuſtome hathe bene the cauſe, why it hathe ſo ſpeciallye bene named a kinde of Addition, and defined to bee a quicke and briefe Addition of diuerſe ſummes, proceeding by ſome certayne and reaſonable order.

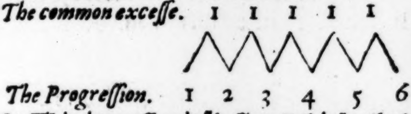
You ſhall alſo vnderſtande, that there are infinite kinds of progreſſions, but for you (as yet) two are ſufficiente to be exerciſed in: of which the one I cal Arithmeticall, and the other Geometricall.

Arithmetical progreſsion.

Arithmetical progreſſion is a rehearſing or placing downe of manye numbers, number after number, in ſuch ſort, that betwéene euerie two nexte numbers rehearſed or placed downe, the difference, diuerſitie, or exceſſe, be equal and alike.

Scho. Syr, I thanke you for that you haue both opened vnto me what Progreſſion is, truly, and alſo why it is here placed. But I pray you with an example make plaine your definition.

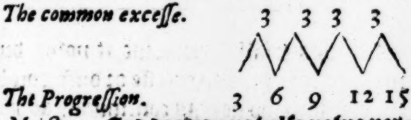
Ma. Examples can not want, ſéeing al reaſonable creatures naturallie vſe the order of one kind of Arithmetical progreſſion (whiche therefore is alſo named Naturall) when ſo euer they diſtinctlie doe counte or number any multitude one by one, ſaying: 1.2.3.4.5.6. wherby the procéeding from number to number, and euerie one ſurmounting and excéeding his fellow next before by a like quantity (which here is 1) declareth the ſame to be Arithmeticall progreſſion. And for the more plainneſſe, I ſet it down in this maner.



Sc. This is moſt euidẽt. And I thinke that I am able to tel you now of any progreſſion Arithmetical propounded, what is that common exceſſe or difference wherby it procéedeth if this order be kept in it.

M. What ſay you of 3.6.9.12.15?

Sc. They excéede eche other by 3. And that may I ſet down in ſuch euident order, as you did your example of Natural progreſſion, in this wiſe.



Maiſter. And doe you not alſo nowe perceiue, that the whole table of Multiplication maye be made by the order of progreſſion Arithmetical: either if you wil begin at the firſt number of any of them on the left hande, and ſo procéede right ouerthwarte: or at any of the firſt numbers of the vpper rowe, and goe directly downward?

Sch. I pray you let me conſider the thing a little, and I wil anſwere you.

1	2	3	4	5	6	7	8	9	10
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
6	12	18	24	30	36	42	48	54	60
7	14	21	28	35	42	49	56	63	70
8	16	24	32	40	48	56	64	72	80
9	18	27	36	45	54	62	72	81	90
10	20	30	40	50	60	70	80	90	100

By this triall I perceiue it nowe verie well: for the common exceſſe or difference betwéene any two next, is continually as much as the firſte number of euerie rowe, either from the lefte hande ouerthwarte taken, or from any of the vppermoſt ouerthwart rows downward.

Ma. Nowe then if of any ſuche progreſſion you woulde ſpéedelie knowe the totall ſumme, muche quicklier thā by common additions rules: firſt tell howe many numbers there are (which numbers here we cal places or parcels) and if they be odde, write theyr ſumme downe by it ſelfe, as in this example, 2,4,6,8,10,12,14, where the numbers are 7, as you may ſée, therefore ſet done 7 in a place alone: then adde togither the firſte number and the laſte, as in this example: adde 2 to 14, and that maketh 16, take halfe of it, and multiplie by the 7, whiche you noted for the nũber of the places, and the ſum that amounteth, is the ſumme of all thoſe figures added togither, as in this example: 8 multiplied by 7 make 56: and that is the ſumme of al the figures.

To knovv the total ſum of an Arithmeticall progreſsion.

Scholer. That will I worke by an other example. I would know how much this ſum is, 5,8,11,14,17,20,23,26,29. I tel the places and they are 9, that I note. Then I putte the firſt number 5, and the laſt 29, togither, and they make 34. I take the halfe of it, that is, 17, and multiplie by 9, and it maketh 153. That you ſaye is the ſumme of all the numbers.

Ma. So ſhal you finde it if you trie it.

Sc. How ſhal I trie it?

Maiſter. By your common addition: for

if you adde all the parcels togither, you ſhall ſee the ſame ſumme amounte, if you dydde worke well. And that manner of Addition trieth all kindes of ſumming anye Progreſſion.

Sc. Then can I ſumme a progreſſion, if the numbers of the parcels be odde. But what if they be euen: as in this example, 1,2,3,4,5,6, 7,8?

Ma. When the number of the parcels is euen, then note that alſo as you did before, and likewiſe adde the firſt ſumme to the laſt, and by the halfe of the number of the places do you multiplie it: as in your example, the parcels are 8, that note I: then adding yͤ firſte ſumme to the laſt, there amounteth 9, that do I multiplie by the halfe of parcels, that is by 4, and it maketh 36, whiche is the ſumme of the 8 parcels.

But if you wil take one rule for theſe both, doe thus. Multiplie the halfe of the one by the other whole, and the ſumme wil amounte al one. For ſometime it chanceth that the number of the parcels be odde, ſo that their halfe can not be taken: and ſometime it chaunceth the Addition of the firſte number and the laſt, to bring forthe an odde number, ſo that the halfe of it can not be taken : but they will neuer be both odde.

Sc. Thē I perceiue this, if there be no more longing to it.

Maiſter. As accuſtomablie it hath bene taughte, thys hath bene the chiefe and onelie exerciſe in Progreſſion vſed. But that you maye perceiue howe diuerſe wayes and to howe great profitte ſo ſimple a thing (as this Arithmetical Progreſſion is) maye be conſidered and vſed, I will here propounde you ſixe propoſitions, of which four of them were inuented by a friende of mine, and neuer before this publiſhed: and the firſte two, were neuer to my knowledge written of, but by thrée men.

Sch. This dothe greatlie encourage me to be attentife vnto your wordes, ſéeing I ſhall not onelye be inſtructed at your handes in the common knowen rules of this excellent art, but beſids that, ſo aboundantlie in other new rules informed, as my verie entrance ſhall ſéeme to paſſe a great many mēs farther ſtudie, and longer continuance. Therefore ſir, I beſéech you, let me know your ſixe propoſitions.

Ma. Theſe they are.

1 To know the laſt number without proceeding by continual addition, til you come vnto it, ſo that the common exceſſe, the firſt number and the number of the places bee knowne.

2 The firſt number of the Progreſsion and the laſte being knowen, with the common exceſſe, to finde the number of the places.

3 The exceſſe being giuen, and the firſte or laſte, to know the quantitie of anye middle number, whoſe place is giuen from the firſt or laſt.

4 The totall ſumme being giuen, and the firſt and laſt, to finde out the number of the places.

5 The total ſum of any Arithmetical progreſsion being giuen, and the firſt and laſt, to finde out the common exceſſe.

6 The totall ſumme being giuen, and the mutual exceſſe, with the number of the places, to giue the firſte or laſte number of the ſame progreſsion.

Many moe conſiderations could I propoũd you in theſe Arithmeticall progreſſions, but theſe are ſufficiẽt to giue you occaſiõ to think, that rules of knowledge and artes are infinitely capable of enlargement.

Scn. Happie were I, if I did but well vnderſtand that which is alreadie inuented and written. And yet in my ſimple fantaſie, theſe thinges offer themſelues (in maner) to be ſtudied for about Progreſſion, therefore I pray you to procede to the rules anſwering to theſe propoſitions.

Mayſter. I will orderlye for euerye of theſe ſixe propoſitions giue you rules: and with euery one an example, vnleſſe yͤ plainneſſe and eaſineſſe need no further exemplefying.

For the Solution of the firſte. Multiplie ,,
the exceſſe by a number leſſe by 1 than the ,,
number of the places, and the offcome ,,
adde to the firſte number, ſo ſhall you ,,
haue the laſte number, whiche is ſoughte ,,
for.

As (for example) if there were ſeuen places in a progreſſion Arithmeticall, whoſe continuall encreaſe, or mutual exceſſe were 4 5 and the firſt number were 5, and I would know what the laſt ſeuenth number is, I multiplie 6, which is 1 leſſe then 7, (the nūber of the places) by 4, thereof commeth 24,

which

whiche I adde to 5, that maketh 29: and that is the laſt number, whiche I deſired to know. And this you maye ſtraighte waye proue, by continuall procæding from 5 till the ſeuenth place, encreaſing euerye one by 4, as thus.

5 9 13 17 21 25 29.

Lo here, the laſt, being alſo the ſeuenth, is 29.

Sc. I perceiue already one gœd propertie in this rule, which in all workes is to be deſired: yͭ is, it wil eaſe one frō great labor, if a progreſſion were propounded of 100 or 200 places, or moe, And alſo it is verye eaſie to worke, and moſt neceſſarie for the totall ſumme finding, in a very long progreſſion.

Mai. The ſecond rule is this. From the
2 “ laſt ſubtract the firſt, the remainer diuide
“ by the common exceſſe, to the Quotient ad
“ 1, and you haue the number of the places,
“ which you wold know: As in this progreſſion.

6 11 16 21 26 31.

If I know only 6 and 31, and that they encreaſe by 5, than according to the rule, from 31 I ſubtract 6, there remaineth 25, which

25 I diuide by 5 (the common exceſſe) the quotient commeth forth 5, to which I ad 1, that maketh 6: and ſo many are the places, as you ſee.

Schol. This rule is ſo eaſie, that I were muche to blame, if I coulde not remember it.

3 Ma. The third propoſition may alwaies thus be ſoluted: Multiplye the exceſſe by a number leſſe by 1. than the diſtaunce of the place is from the firſt or the laſt number giuen: the ofcome ad to the firſt, if the diſtance be reckned from the firſt, and the firſte alſo known or ſubtract from the laſt: if the diſtaunce be from the laſt counted, and yͤ laſte giuen alſo, and that whiche commeth forth, either in that addition to the firſt, or ſubtraction from the laſt, is the number ſought As for example I propound you this progreſſion.

8 15 22 29 36 43 50 57.

And for the apt conſidering yͤ maner of this queſtiõ, I will note ouer euery place his diſtance from the firſt and vnder euery place his diſtance incluſiuely from the laſt, thus.

1	2	3	4	5	6	7	8
8	15	22	29	36	43	50	57
8	7	6	5	4	3	2	1

Now, if that exceſſe whereby this Progreſſion ſtandeth, be knowne to be 7, and the firſt number giuen, being 8, I woulde know what number ſtandeth vnder 4, that is to ſay in the fourth place. I multiplie 7 by 3 (which is leſſe by 1, than the number of the place propounded) that yeeldeth 21, to which I adde 8 (the firſt number) ſo commeth 29: which I ſay to belōg to the fourth place, as ye ſee in the example it alſo doeth: or if in the third place from the laſt, you would know what number in this example ſhould ſtand, the laſt number being knowen to be 57, and the common exceſſe 7, than by 2 (which is leſſe by 1 than the place propounded) I multiplie 7, that giueth 14: whiche I ſubtract from 57, ſo remaineth 43: which appertaineth to the thirde place incluſiuely reckned from the laſt, & ſo my example giueth you.

Scholer. I perceiue right good vſe of this rule: for if I had forgotten what the firſte number were, and remember ſtill but the laſte, the common exceſſe, and the number of the places, then mighte I come by the knowledge of my firſt number againe.

And me thinketh,that it differeth not much from the firſt propoſition ſauing that which you make here a middle number,there was made the laſt:and alſo in this point it differeth,that in it the laſt was only ſought,and no conſideration hadde in numbring the places from the laſt , as here I marke in your numbers noted vnder your progreſſion.

Mayſter. And thinke you not the middle numbers of a Progreſſion ſtanding of a hundred or three hundred places or moe, may as much cumber a man to come to the knowledge of them by continuall encreaſing from the firſt (by the common exceſſe) or abating from the laſt continually (the common exceſſe) as the very final numbers in a ſhorter Progreſſion would doe?

Scholer. Yes ſir, that I thinke righte well,and therefore I am glad of this newe framed propoſition, and the maner of the working of it.

Mayſter. The rule of the fourth is this. 4
Adde the firſt and the laſt together, and by the ofcome diuide the totall ſumme. Double

ble the quotient,and that will be the number of the places.

Scho. Then if in a Progreſſion, whoſe ſum were 207,and the firſt number 12,and the laſt 57, if I adde 57 and 12 together, that maketh 69: and by it I diuide 207, the Quotient will be 3,which I double,and ſo I haue 6,and ſo many muſt be the number of the places,that this progreſſion ſtandeth on.

Mai. Whether it be ſo or no, howe will you trie?

Scho. Halfe 6,whiche is 3,being multiplied by 69,muſt make 207, the totall ſum, if 6 be the number of the places. For ſo the whole worke of your rule in ſumming any Arithmeticall progreſſion did enforme me. I will than multiplie 69 by 3,thus.

$$\begin{array}{r}69\\ 3\\ \hline 207\end{array}$$

It commeth forth iuſtlye.

Maiſt. I muſt muche herein commende your promptnes, both in memorie and in well applying your rule: although in manifeſt wordes it did contayne no ſuche matter.

Scho. Sir, I praye you heare me frame one example more.

Maiſt. I am well pleaſed, ſo that ye be ſhort, for you make me more longer here, then willinglie I would haue bene: but I can not perceaue how I could haue omitted any thing as yet, without your greate lacke thereof.

Sch. If I had receiued 85 poundes of certaine men but of how many I haue forgotten, yet I remember that the firſt gaue mée 7 ℔, and the laſt 27 ℔, and euerye paymente after other did riſe by a like ſum. And the man for whom I receiued this money, conditioned with me, that of euery payment I ſhould haue 12 pence for my labour: nowe vnleſſe I can by arte find the trueth of this caſe, I am like to loſe the moſte parte of my reward.

Maiſter. I perceaue you can hanſomly frame an example, which ſhoulde concerne your owne gayne: I praye you lette me ſée howe you woulde doe Juſtice in this poynte.

Scholer. I adde the firſte and the laſte together that maketh 34: by which I diuide 85, thus. Why how now? Sir, here

$$\begin{array}{ll} 1 & \\ \cancel{1}7 & \\ \cancel{8}\cancel{5} & \left(2\frac{17}{34}\right. \\ \cancel{3}\cancel{4} & \\ 8 & \end{array}$$

is a remnant of 17, in which 34 cannot bee had, ſo that nowe I am in the briars for doubling of my quotient, and farewell then both my Juſtice, and a good lumpe of my gaines.

Ma. Ye are neuer the farther from the matter, though it fall into a fraction. For you ſhall vnderſtand, that the fractiō which of any ſuch worke proceedeth, is euer halfe ☞ of one ſuch, as the vnits of the quotient before are. And that you may trie, if you double that which ſo remaineth, for then it will be equal to your diuiſor, as if ye double 17 (the remnāt) it maketh 34, and your diuiſor alſo was 34, this noteth the remainder to bee half of one.

Sch. Now I am glad of this hard example. For with it I haue a generall rule for the Fraction that maye hap in this worke. So that the quotient being two & a halfe, I double that, & it maketh 5, therefore ſhoulde my gaine be 5 ſhillinges. And to be ſure (by your leaue) I will trie it for I will multiplie halfe of 34, (whiche is the firſte and laſte number ioyned together) by 5 thus. It is moſte true (I ſee)

$$\begin{array}{r} 17 \\ 5 \\ \hline 85 \end{array}$$

that I ſhoulde léeſe nothing by the former working.

Mai. The fift propoſition hath this rule appertaining vnto it: By the fourthe rule finde the number of the places, that béeing done, from the laſt ſubtract the firſte, and the reſidue diuide by a number leſſe by 1, than the number of the places; and the quotient will ſhewe the exceſſe whiche is ſought for. 5

An exaumple hereof ſhal be this: If ye had diſburſed 685 li. to a certaine number of men, you neither can tell howe manye they were ne howe muche the ones money excéeded his next before: but you are ſure that ÿ exceſſe was equal betwéen euery two next: & alſo you remember that the firſte had 19, and the laſt 118 poundes, how woulde you finde both the number of the men & the exceſſe, continually obſerued in the ſucceſſion of their paiments.

Scholer. Your rule doth plainlie bid, firſt to find the number of the places, whiche I will doe according to the fourth rule. I adde 19 and 118 together, thus.

$$\begin{array}{r} 118 \\ 19 \\ \hline 137 \end{array}$$

By this 137, I diuide 685, thus.

```
  8 8
6 8 5 (5
  1 3 7
```

Seing there is no fraction, but a whole nũber, being 5. I double that, and than muſte the number of yͤ places be 10.

Now from yͤ laſt I ſubtracte the firſt, as 19 frõ 118, thus:

```
118
 19
---
 99
```

And ſo remaineth 99.

This 99 I diuide by a number leſſe by 1 than the number of the places, and ſeing the places were 10, I diuide 99 by 9, thus.

```
99 (11
99
```

The quotient is 11, and ſo was the exceſſe, if I haue followed your rule right.

May. You haue wrought euery part of this queſtion both well in order and truely in the practiſe of your rules.

Sch. I will than ſet it downe alſo formablie, ſo that the number of the places, the exceſſe and the total ſum may ſtreighte appeare, as your firſt example ſtode.

The cõmon exceſſe.	11	11	11	11	11	11	11	11	11	11
The Progreſsion.	19	30	41	52	63	74	85	96	107	118

That the places be 10, and that from

the firſt to the laſt the commõ exceſſe is 11, I perceiue moſt euidently, but whether the total ſumme be 685, I haue not yet proued, which I will now doe. I adde 19 and 118 togither that maketh 137: I multiplie that by halfe the number of the places thus.

137
5
685

All thinges agrée moſt exactlye, ſo that I am perfecte ynough in theſe rules, if I forget thẽ not agayne.

Ma. Uſe maketh all things perfect. 6

Your firſt rule is this. By the number of the places diuide the totall ſumme, double the quotient, and that will be the firſt and the laſt ioined in one ſum. Than by a number leſſe by 1, than the number of the places multiplie the exceſſe, that ofcome ſubtracte from the firſt doubled quotient, and the half of the reſidue is the firſt number. The laſte number you may diuerſlye finde out, as by the firſt of our ſixe rules, or by ſubtracting this firſt number from the ſum which here contained both the firſte and the laſt ioyntlye (or thirdly) by continuall adding the exceſſe.

Scho. I pray you make this ſomewhat

more plaine with an example.

Mai. If euery moneth in the yeare (counting them now as thirtéene) you gayned clearely 40 ſhillinges more than you didde the moneth next going before, and at the yeares ende you find the whole gaine 5720 ſhillinges, but yée remember not howe much either the gaine of the firſt moneth, or the laſt was, by this rule it maye be tried out.

Sc. So that here ye ſéeme to applie the 13 monethes to 13 places, yͭ 40 ſhillings euery one more then the other next before it, to be the common exceſſe, and 5720 ß. to be the totall ſumme.

Mai. It is true: by 13, then I diuide 5720 in this maner.

I double this quotient, ſo haue I 880 for the firſte, and the laſt ſumme ioyned togither, by 12, which is leſſe by one than the number of the places, I multiplie 40, (the common exceſſe) ſo commeth 480.

1 1
5720 (440
133
1

40
12

80
40

480

This 480 I ſubtract from 880, ſo remaineth 400: halfe whereof is the firſte number whiche we deſired to know, that is 200.

And as for the laſt number, I can giue you it 3 wayes: As by the firſt of my ſixe rules, I multiplie the exceſſe by a number leſſe by 1 than the number of the places: as 40 by 12, that giueth 480, whiche I adde to the firſt being 200, ſo ſhall the laſte bee 680.

The ſame ſum commeth forth, if yée ſubtract 200 from 880.

And thirdly, if I beginne 200, and ſo procéede, encreaſing by 40, I ſhall at the thirtéenth place haue 680, as thus.

200. 240. 280. 320. 360. 400.
440. 480. 520. 560. 600. 640.
680.

Sc. I thank you moſt hartilye for theſe 6 rules. Now if it be your pleaſure, I would heare and learne ſomwhat of Progreſſion Geometricall.

Ma. There are yet very many rules and propoſitions, which fall into this Arithmeticall progreſſion: but theſe ſhall ſuffiſe for this time.

And in Geometricall progreſſion I will be more briefe, both becauſe I haue bene ſo long in this part of Arithmeticall progreſſion, and alſo for that it woulde require the knowledge of Rootes, and numbers ſurde, (whereof ye haue yet learned nothing) if I ſhoulde frame the like propoſitions in them as I haue done in theſe. Therefore I will only teache you two practiſes, aboute it, and ſo ende the conſiderations & workes of theſe progreſſions. Progreſſion Geometricall is when the numbers increaſe by a like proportion, that is, if the ſecond number containe the firſt, 2, 3, or 4 times, and ſo forth: then the third containeth the ſeconde ſo manye times alſo: and ſo the fourth the third, and the fifth the fourth:

Progreſſion Geometricall.

Wherefore I ſette theſe three examples.

3, 6, 12, 24, 48,
1, 3, 9, 27, 81,
2, 10, 50, 250,

Here in the firſte example you ſee, that euery number containeth the other (that goeth next before him) 2 times: and in the ſeconde example 3 times, in the thirde example 5 times. Now if you will know howe to finde eaſilye the ſumme of any ſuche numbers, do thus. Conſider by what nũber they bée multiplyed, whether by 2,3,4,5, oꝛ anye other, and by the ſame number doe you multiplie the laſt ſumme in the Pꝛogreſſion.

Sc. I pꝛay you woꝛke it by this example, 2,8,32,128,512,2048, whiche I haue framed by pꝛocéeding from 2, and continually multiplying by 4.

Ma. Then muſt I multiplie the laſt ſum (which is 2048) by 4 alſo, and it will bée 8192. Nowe muſt I bate from this ſumme the firſt number of the pꝛogreſſion, whiche here is 2, then reſteth 8190, which ſumme I muſt diuide by 1 leſſe then was the number that I multiplied by . Seing then I multiplied by 4, I muſt diuide by 3, ſo diuiding 8190 by 3, the quotient will be 2730, which is the ſumme of all the Pꝛogreſſion. And now to pꝛoue whether you can doe the ſame, I giue you theſe numbers to adde by this rule, 3, 15, 75, 375, 1875, 9375, 46875.

Sch. I cannot well tell by what number this Pꝛogreſſion doth increaſe.

Maiſter. In any ſuch doubte, doe thus: Diuide the ſeconde number by the firſt, and the quotients will ſhewe you the number that engendreth the Progreſſion.

Sc. Then is that number in this example 5, for ſo many times is 3 in 15.

Mayſter. So is it. Nowe worke as I taught you.

Scholer. The laſte number is 46875, whiche I multiplie be 5, and it yeeldeth 234375, from which I abate the firſt number of the Progreſſion, that is 3, and there reſteth 234372, which I diuide by 4, for ẏ is one leſſe thẽ 5, & the quotient is 58593, which is the whole ſumme of the progreſſion.

An abridgement in progreſsion.

Mai. Now that you know the ſumming of Geometricall Progreſſion, I will ſhewe you a compendious manner eyther to proceede vp, or to finde out the quantitye of a number whoſe diſtance from the firſt may be very greate, whiche to doe by continuall multiplication, would be very tedious, if the numbers be great and the places many.

Sc. Nothing can pleaſure me more then breuitie, if it be playne.

Mayſter. I thinke I am not yet in anye point ſo darke or hard, that you neede to feare any obſcuritie now. The manner is this: ſet downe of your progreſſion foure or fiue of the firſt places, and vnder the firſt put a cipher, vnder the ſeconde 1, vnder the third 2, &c. as if you had a progreſſion encreaſing by a fiue folde quantitie: as here, 2, 10, 50, 250, 1250: then vnder 2 I put a cipher, and vnder 10 the figure of 1: vnder 50, 2: vnder 250, 3: vnder 1250, 4: and ſo forthe if yée will: but to a wiſe and warie worker, a fewe places were ſufficient to procéed by to any number of places in this

2	10	50	250	1250
0	1	2	3	4

ſorte, if anye two of your numbers progreſ- ,,
ſionall be multiplied the one by the other, ,,
and the ofcome diuided by the firſte of your ,,
progreſſion, the quotient is one of your num- ,,
bers progreſſionall, and belonging to ÿ place ,,
of your vnder numbers, that is equal to that ,,
ſumme, that is made of Addition togither of ,,
your two numbers which ſtode vnder theſe ,,
two of your Progreſſionall numbers, that ,,
were multiplied ,,
the one by the other, as in thys example.

2	10	50	250	1250	6250
0	1	2	3	4	5

If I multiplie 10 by 50, thereof commeth

500, whiche I diuide by 2, (the firſte number of the Progreſſion;) and the quotient is 250: whiche 250, muſte ſtande in the thirde place, becauſe the number whiche ſtandeth vnder 10 is 1, and that vnder 50. is 2: and 2 and 1 maketh 3. Therefore I ſay, that 250 belongeth to the thirde place of this progreſſion, as yée ſée alſo here it doeth. Moreouer If I multiplie 50, into it ſelfe, thereof commeth 2500: that 2500 I diuide by 2, the Quotiente is 1500, whiche muſte be ſette in the fourthe place: bicauſe 2 added to himſelfe againe, maketh 4, and in our example 1250 occupieth the fourth place.

$$\begin{array}{r}50\\ 50\\ \hline 00\\ 250\\ \hline 2500\end{array}$$

Sc. Then for that fifth place I multiplie the Progreſſionall numbers ouer 2 and 3, one by the other: and for the ſixt, I multiplie that ouer 3 in it ſelfe. &c.

M. Yée muſt wel remēber that theſe places that we nowe ſpeake of, belong to the vnder numbers, for the true places of the vpper nūbers is euer one place more.

Scholer. That I ſée the reaſon of, bycauſe the vnder numbers begin one after, and againſte the firſt place of my progreſſion ſtandeth a cypher, ſo that the 250 which you ſaide before did belong to the third place, I ſee belongeth to the number of 3 among your vnder numbers, but from the true progreſſions beginning, it is the fourth.

Maiſter You vnderſtand me as I meane. Therefore for your exerciſe of both the rules here giuen for Geometricall progreſſion, I will aſke you a queſtion, muche vſed among the common people, (as they haue a greate many the like) If I woulde ſell you a Horſe, hauing 4 ſhoes, and in euerie ſhoo ſix nailes, with this condition, that you ſhal pay for the firſt naile 1 *ob*. for the ſecond 2 *ob*, for yͤ thirde 4, and for the fourth 8, and ſo forth dubbling vntil the laſt naile. Now I demaunde of you how muche the price of the Horſe woulde amount vnto?

Scholer. Seing the Horſe hathe 4 ſhoes, and in euerie ſhoe 6 nayles, I perceyue here wil bée 24 places. If I coulde nowe haue the laſte number, I woulde quicklie diſpatch this queſtion. I wil therefore with as fewe multiplicatiõs as I can deuiſe, to come to the knowledge of the laſte number of thys progreſſion. In double I ſet forth than a few of my progreſſion, thus.

1	2	4	8	16	32	64
0	1	2	3	4	5	6

If I nowe multiplie the numbers ouer 5 and 6, the one by the other, I ſhall haue the number of the eleuenth place for the vnder numbers, but of the twelfth for the vpper nũbers in which my progreſſion ſtandeth, and then that of yͤ eleuenth place vnder, if I multiplie in it ſelfe, I ſhal haue for the 22 place vnder, but for the 23 of that aboue, whiche I multiplie by that ouer 1 of my nether places, and I ſhall haue the 23 of my nether places, and the 24 of the vpper, which is the number I ſéeke for.

Maiſter. Me thinketh you haue forgotten youre rule for abridging your multiplications: for in it, the ofcome euer of anye multiplication, is to be diuided by the firſte of the progreſſion. And you now ſpeake of no diuiſion.

Scholer. Sir I néede not, as my progreſſiõ beginneth now: for if I ſhould diuide by 1, it maketh no other quotient, then the number is, it doth diuide.

Maiſter. It is verie wel remembred and noted of you, ſo youre worke then according to youre preſcribed manner, which I like well.

```
       64
       32
    -----
      128
      192
    -----
     2048
```

S. I multiply 64 by 32 as here.

And it maketh 2048. whiche is the eleuenth place vnder, but the twelfth aboue, and this, I multiply in it ſelfe, in this manner: And this is the 22 place vnder but the 23 aboue. I multiplie this then by 2, as heere.

```
         2048
         2048
    ---------
        16384
        8192
       0000
      4096
    ---------
      4194304
```

And this of come 8388608, is my foure and twentith place, whiche I haue found now by 3 multiplications.

```
      4194304
            2
    ---------
      8388608
```

Then doe I reſort to the rule of ſumming this Progreſſion, where I conſider that the encreaſe of this ſumme procéedeth by multiplication of 2, and therefore I doe multiplie the laſte ſumme by 2 alſo, and it yéeldeth 16777216. from whiche I abate the firſte nūber which is 1, and then reſteth 16777215. which I ſhoulde diuide by 1 leſſe then I did multiplie: but ſéeing that it is 1, I néede not to diuide it, for 1 (as I haue before ſaide) doth neither multiply nor diuide, therefore I take that ſumme 16777215 for the whole ſumme of the pence, which by Reduction I finde to be 69905 ß, and 7 d, ob: that is 34952 ℔, 10 ß, 7 d, ob.

Maiſter. That is well done, but I thinke you wil buy no horſe of the price.

Scholer. No ſir if I bée wiſe. Yet for my aſſurance wil I take ſo muche paine, as to come to this laſte 8388608 by cõtinual multiplication by 2, as in thys example you maye beholde my worke till I haue done.

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	12
4194304	23
8388608	24

Mayſter. Well, are you not almoſt weary?

Scholler. Well fare my ſhorte rule, for introth it hathe more cunning and more eaſe.

Mayſter. Well, then anſweare me to this queſtion.

A Lorde delivered to a Bricklayer a certayne number of loades of bricke, whereof he willed him to make 12. walles, of ſuch ſorte, that the firſt wall ſhoulde receive 2 thirdeles of the whole number: and the ſeconde 2 thirdles of that that was left. And ſo everie other 2 thirdles of that that remained: and ſo did the bricklayer: And when the 12. walles were made, there remaineth one lode of Bricke.

Nowe I aſke you, howe many loade went to every wall, and howe many loade was in the whole?

Scholer. Why ſir, it is impoſſible for me to tel.

Mayſter. Nay, it is very eaſie, if you marke it well. Marke well that I ſaide, that every wall ſhoulde receive 2 thirdeles of the ſum that was lefte. Nowe take away 2 thirdels from anye ſumme and you muſt needes graunt that that whiche remaineth is 1 thirdle of the ſumme laſte before: example of 9. from which if you take 2 thirdols, there will remaine 3; whiche is one thirdel of 9. Likewaies from 3 bate 2 thirdels: and there will remaine 1.

Scholer. This is true, and nowe I perceiue, that the leaſte wall had but two loade of bricke.

Maiſter. And by the ſame reaſon maye you knowe howe manie loades euerie wall hadde, according as this figure folowing doth ſhew; and likewayes what the whole ſumme of brickes was: for if you make 10 ſummes, multiplying by 3, ſtil frō the laſt remayner, as you maye ſee here on the lefte ſide of the table, there will appeare all the remayners of euerie wall: and if you multiplie the laſt of thoſe 10 ſummes by 3, alſo, then will that be the ſum of the loades whiche were deliuered to the bricklayer.

Againe, if you doe double euery remainer, as you maye ſee at the right ſide of this table, thoſe numbers will ſhowe the ſum of loades that went to eache wall: whereby you maye perceiue, that each wall was 3 times ſo great as the next leſſer.

Sc. Loe, nowe it appeareth eaſie ynoughe. Now ſurely I ſee that Arithmetike is a right excellent arte.

Ma. You will ſay ſo when you knowe more

of the vſe of it : For this is nothing in compariſon to other points that may be wrought by it.

Scholer. Then I beſéech you ſir, ceaſe not to inſtruct me further in this wonderful cunning.

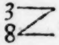
The remainer	1	12	2	*Loades due to*
after euerie wal.	3	11	6	*each wal.*
	9	10	18	
	27	9	54	
	81	8	162	
	243	7	486	
	729	6	1458	
	2187	5	4374	
	6561	4	13122	
	19683	3	39366	
	59049	2	118098	
	177147	1	354294	
Some of the	531441			*Loades deliuered.*

THE GOLDEN Rule.

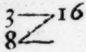
Maiſter.

BY order of the ſcience (as men haue taughte it) there ſhould follow nexte the extraction of Rootes of number, whiche becauſe it is ſomewhat harde for you, yet I will let it paſſe for a while, and will teache you the feate of the rule of Proportions, whiche for his excellencie is called the Golden rule. Whoſe vſe is, by thrée numbers knowen, to finde out an other vnknowen, whiche you deſire to knowe: as thus. If you pay for your borde for thrée moneths 16 ſhillings, howe muche ſhal you pay for 8 moneths.

To know this and all ſuch like queſtions, you ſhall conſider which two of your 3 numbers be of one denominatiō, and ſet thoſe two the one ouer the other, ſo that the vndermoſte be it that the queſtion is aſked of : as in my queſtion 3 and 8 be both of one denomination, for they both be monethes, and becauſe 8 is the number that the queſtion is aſked of, I ſet them one ouer the other, and 8 vndermoſte, thus,



with ſuche a crooked draught of lines. Then doe I ſet the other number whiche is 16, againſte 3, at the right ſide of the line, thus.



And nowe to knowe my queſtion, thys muſt I doe: I muſte multiplie the lowermoſt on the left ſide, by that on the right ſide, and the ſumme that amounteth I muſte diuide by the higheſt, on the left ſide. Or in playner wordes thus: I ſhall multiplie the number of whiche the queſtion is aſked (whyche is called the Thirde number) by the number of an other denomination, (whiche is called the Seconde) and that ſumme that amoũteth muſte I diuide by the ſumme of lyke denomination, whiche is called the Firſte. Then for the knowledge of this queſtion, I multiplie 8 into 16, and there amoũteth 128, whiche I diuide by 3, and it yeeldeth 42 ſhillings, and 2 ß remaineth, whiche I turne into pennies, and they be 24 d, of whiche

The third number.
The ſeconde number.
The firſt number.

the third part is 8 d, ſo the third part of 128 ß is 42 ß, 8 d: which ſumme I write at the right hande of the figure againſt 8, thus.

3 16 ß.
8 42 ß, 8 d.

Hereby I knowe, that if thrée monethes boording doe come to 16 ß, that 8 monethes boording wil come to 42 ß, 8 d: and likewiſe of any other like queſtion.

But here muſt you marke, that the firſte number and the thirde be of one denomination, and alſo the ſeconde and the fourth, the whiche you ſéeke: or elſe be of ſuche denominations, that you in working may bring thē into one. As if a man ſhoulde aſke me thys queſtion.

✱Twelue wéekes iournying coſte me fourtéene French Crownes at 6 ß. the péece, how many poundes is that in one yeare? Here you ſée no two nūbers of one denomination, But yet in working, you may turne them into like denominations, as thus: turne the one yeare into 52 wéekes, and the fourth ſumme will be French Crownes, by the order of the working: Then to knowe this queſtion, multiplie the thirde ſumme 52 by the ſecond 14: and the ſumme will be 728: that diuide by your firſte number 12: and the quotiente will be 60. Crownes: And 8 Crownes remayning: whiche if you turne into ſhillings they will be 48 ſhillings which if you doe diuide by your firſte number 12 the quotiente will be 4: whiche ſignifyeth 4 ß. put theſe 60. French Crownes (which make 18 poundes) with the 4 ſhillings: for the ſumme that anſwereth to the queſtion: And it is the iuſt expẽces of a yeare; And the ſumme wil be thus.

15 — 14
52 — 60—4 ß.

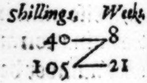
And take this euermore for a general Rule touching this whole Arte, That the doubtful or unknowen number, that you would be reſolued of, ſhall alwayes be ſet in the thirde place, note alſo the firſt number and the thirde, muſt euer be of one nature and denominati-on, or elſe muſt in working be brought to like denomination and then of neceſſitie muſt the other number be in the ſecond place.

Remember alſo, that the place of the firſt number is the higheſt on the left ſide: and the place of the ſeconde right againſt it on ẏ right ſide: the place of the thirde number is vnder the firſte, as by thoſe examples you haue ſeene.

Sc. This I truſt I can doe.

Mayſter. But and if the queſtion be aſked thus: In 8 wéekes I ſpend 40.ß. howe long wil 105 ſhillings ſerue me? Here you ſée that 8 wéekes aunſweres himſelfe, and ſaieth 40 ſhillings. But how long time 105 ſhillings wil ſerue, you know not. Therefore you ſhal ſet 105 in the thirde place, according as I tolde you euen now. And the firſt place muſt allwayes be of the ſame nature or Denomination, that the third is of, which here is 40. Then muſt 8 néeds be that other. Now multiply 105 by 8 and it wil be 840 which if you diuide by 40, it will yéelde 21, whiche is the Fourth number, and ſheweth howe many wéekes 105 ß. will ſerue, if you ſpende 40 ß. in eight wéekes.

The figure of thys queſtion is this: as if you ſhould ſaye: If 40 ß. ſerue for 8 wéekes, 105 ſerue for 21 wéekes.



Other diuerſities there be of working by this rule, but I hadde rather that you woulde learne this one well, than at the beginning to trouble your minde with many formes of working, ſith this way can doe as much as al the other, and hereafter you ſhall learne the other moꝛe conuenientlie.

And foꝛ your further aide and inſtruction to make you better acquainted wyth thys Goldon Rule, I haue here pꝛopened 6 queſtions, and their aunſweres, whiche I thinke moſte conueniente and méete to pꝛeferre the deſirous to perfecte vnderſtanding. The firſte foure are all bꝛaunches of one Queſtion ſpꝛong out of the beſte trée, (foꝛ a young learner to taſte of) that groweth in this Grounde of Artes, foꝛ that no manner of Queſtion in the Rule of 3 what ſo euer it bée, can be pꝛoponed, but it muſte be compꝛehended, vnder the reaſon oꝛ ſtyle of one of theſe foure.

The Queſtions be theſe.

If 15 elles of Cloth coſte 7 ℔.10 ß: what comes, 27 elles too at that pꝛice: Aunſweare. 13 ℔.10 ß.

If 27 elles coſte 13 ℔.10 ß: what are 15 elles woꝛth. Anſwere. 7 ℔. 10 ß.

If 27 elles coſte 13 ℔. 10 ß: howe manye elles ſhal I haue foꝛ 7 ℔. 10 ß. Anſwere 15 elles.

If I ſell 15 elles foꝛ 7 ℔. 10 ß: howe manye elles are to be deliuered foꝛ 13 ℔. 10 ß. Anſwere. 27 elles,

If 8 pound of any thing coſt 16 ß. 6 d. what money is to be receiued foꝛ 49 pounde: Anſwere. 2 ℔ 4 ß. 11 d. 5 ℔ 1.10 d ob q

If 4 ℔. of anye thing coſte 17 d: what money wil 8765 pound of that commoditie coſt. Anſwere. 155 ℔. 4 ß. 3 d. farthing.

Of all which queſtions I omitte the woꝛk of purpoſe, you ſhoulde whet your wit thereby at conuenient leaſure, to climb ech bꝛanch and gather the fruite of them: And doe minde nowe, befoꝛe we make ad ende of this Rule, to giue you ſome Inſtructions of the Backer Rule of 3. whoſe oꝛder is quite contrarye to thys that you haue learned. Foꝛ in thys Rule hitherto euermoꝛe loke how muche the thirde number is greater than the firſte, ſo muche the fourthe number is greater then the ſeconde. And contrarye wayes: loke howe much the firſte ſumme is greater then the thirde, (if it doe chaunce ſo) ſo much is the ſecond ſumme greater then the fourth. But in this Rule, there is a contrary order, as this: That the greater the third ſumme is aboue the firſt, the leſſer the fourth ſum is beneth the ſeconde: and this rule therfore you may call the Backer rule; as in example.

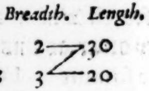
The backer rule.

If I haue boughte 30 yardes of cloth, of 2 yards breadth, and woulde buy canuas of 3 yards brode to line it withall, howe manye yards ſhould I néede?

Queſtion of buying cloth.

Sc. Why, there is none ſo broade.

Mai. I doe not care for that, I doe put this example only for your eaſie vnderſtanding: For if I ſhould put the example in other meaſures, it would be harder to vnderſtand. But nowe to the matter: If you woulde know this queſtion, ſet your numbers as you did before: but you ſhall multiplie now the firſt number by the ſeconde, and that ariſeth thereof, you ſhall diuide by the third: which thing if you doe here, I meane if you multiplie 30 by 2, it will be 60: which ſumme if you diuide by 3, there will appeare 20: whereby I knowe, that if 30 yardes of cloth of two yardes broade, ſhould be lined with canuas of thrée yardes broade, 20 yardes of canuas woulde ſuffice, as this figure ſheweth.



And nowe becauſe yée found fault at my example, how ſay you, perceaue you this?

Sc. Yes ſir, I ſuppoſe.

Mai. Then aunſwere me to this queſtiō: how many elles of canuas of elle breadth, will ſerue to line 20 yardes of Saye, of thrée quarters brode.

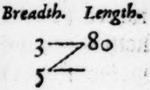
Sc. In good faith ſir, I cannot tell, for I know not how to bring the ſummes to like denominations.

Maiſter. Then wil I tel you: ſith there is mention here of quarters, and againe euerye one of the meaſures both elles and yardes may be parted into quarters, do you parte them ſo both in the breadth and length, and then put forth the queſtion by quarters.

Scholer. Then I ſhall ſay thus. Howe manye quarters of canuas of fiue quarters broade, wil line 80 quarters of 3 quarters brode.

Mayſter. Now aunſwere to the queſtion.

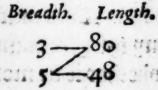
Sc. Firſt I wil ſet them downe in their forme thus, for 5 is ioyned with yͤ queſtion, and is therefore the



third number: then is 3 the number of the ſame denomination, I meane becauſe they be both referred to breadth. Now I multiplie 80 by 3, and it is 240, which I diuide by 5, and it yieldeth 48. Then ſaye I, that 48 quarters of 5 quarters broad, will ſuffice to line 80 quarters of 3 quarters brode.

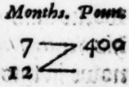
Ma. Turne the quarters againe into elles and yardes.

Sch. Then I ſay, that 9 elles and thrée quarters of a yarde of elle broade, will ſerue to lyne 20 yardes of thrée quarters brodes, as this figure ſheweth.



* M. Now what ſay you to this queſtiō: I lent my friend 400 ℔ for 7 months, now how much money ought he to lēd me again for 12 moneths, to recompence my curteſie ſhewed him. Can you anſwere to this.

Sc. Yes ſir I ſuppoſe, for I will ſet down my nūbers thus



where I multiplie 7 into 400, and it maketh 2800, which I diuide by 12, and it yieldeth 233 ℔ and there is 4 ℔ remaining of my Diuiſion, what ſhall I doe therewith.

Mai. Turne that ſame 4 ℔ into ß and then diuide it by 12 as you did before.

Sc. Well ſir it ſhall be done, ſo haue I 6 ß for my quotient, and yet remaineth 8 ß. vpon my diuiſion.

Ma. You muſt alſo reduce that 8 ß into pence, which maketh 96, and diuide that alſo by your firſt diuiſor.

Sc. So haue I done, and I finde 8 pence for my quotient and nothing is left.

Ma. This muſt you allwayes doe, when any thing remaineth vppon your diuiſion: whether it be mony, weight, meaſure, or any kind of thing whatſoeuer. This rule is ſo profitable for all eſtates of men, that for this rule onlie (if there were no more but it) all men were bound highly to eſteeme Arithmetike.

By this Rule maye a Captaine in warres worke many things, as Maiſter Digges in his *Stratioticos* doth notably declare. Onlie nowe in this my ſimple addition, for a taſte

and

and incouragemēt I wil enlarge yͤ Author with a queſtiō or ij. more, wiſhing you, & euerie my coūtriemen or Gentlemē, whatſoeuer, that by nature be anye thing giuen to *Millitarie* affaires, to be familier and wel acquainted with this Excellente Arte, the whiche he ſhall finde not onely at the Sea, but alſo in the Campe and Fielde ſeruices, aboundantly to aide him, either in fortificatiō, or in paying of Souldiors wages, how differente ſoeuer their paye be: Charges of Ordinaunce pouder, ſhot, Munition and inſtrumentes, whatſoeuer, but now to the queſtion.

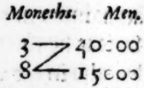
Queſtion of prouiſion touching an armie.

If it ſhould chaunce a Captaine whiche hath 40000 ſouldiers, to be ſo incloſed with his enemie that he could haue no freſh puruiance of vittailes, and that the vittailes which he hath, would ſerue that armie but only 3 moneths, how many men ſhould hee dimiſſe, to make the vittaile to ſuffiſe yͤ reſidue, 8 moneths?

Moneths.	*Men.*
3	40000
8	

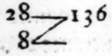
Sc. As you taught me, I ſet the nūbers thus, ſaying: If 3 moneths ſuffiſe 40000, to howe manye will 8 moneths ſuffice?

To know this I multiplie the firſt number 3 into the ſeconde 40000, & it yieldeth 120000, which ſum I diuide by 8, & there wil be in the quotiẽt 15000, which if I doe ſubtract from 40000, the remainer will declare that hée muſte diſmiſſe 15000 as this figure ſheweth



✱ Maiſt. Nowe aunſwere me to this queſtion: If 136 Maſons in a moneth bée able to builde a Forte, to preſerue the Souldiers from the enemie: And ſuche expedition requireth that I woulde haue the ſame finiſhed in eighte dayes, how manye workemen ſaye you is there to bée appointed.

Sch. As you taught me I ſet the numbers thus, ſaying: If 28 daies require 136 Maſons, what number of men by proportion will 8 dayes bring forth.



To know this I multiplie the firſt nũber 28 into 236: And it yieldeth me 3808: whiche I diuide by 8. And my quotiente is 476, which is the iuſt number of Maſons that ſhall ſupplie this worke. And now me think theſe queſtions are very eaſie.

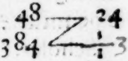
Truely

Ma. Truly if you take delectation herein you ſhall finde this Art not onely eaſie, but wonderfull pleaſant & profitable: Now anſwere me this queſtion, & ſo wil I make an end of this rule, in whole nūbers haſting ỹ ſooner to broken nūbers. For had you ỹ underſtanding of thē perfectly, not only in this Rule, but in all other: the queſtion in ſight might haue ben 10 times more harder to abſolue, & yet as eaſilie & as ſoone wroughte as this.

Sc. Your words doth greatly encorage me to be ſtudious to attaine whole numbers, which me think are wōderfull. But might I once attain to be a practicioner in brokē, I ſhould think my ſelf a happy lad.

M. Now what ſay you to this, If 48 carpenters in 2 dayes can make 200 Staues: eſteeming they work but 12 howers a day: And ſuch neede requireth ỹ 384 carpenters are ſet to the finiſhing of theſe 200 ſtaues, in what time ſay you wil they make thē vp

Scholer. I ſee here that I muſte turne my 2 daies into howers, And ſo doing I ſette my numbers thus.



Saying, if 48 men are

24

24 houres 384 men will make an ende quickely. For it is grounded vpon an olde Prouerbe, many hands make quick ſpéde.

I multiplie 48 into 24, and it amounteth to 1152, whiche I diuide by 384: and my quotient is 3 houres which is my deſire.

Note. I take this for a note worthie the marking either in the Rule of thrée, forwarde, or backwarde, when the two numbers are multiplied togither, the Producte is of the ſame nature, and denomination that the ſecond number is of.

The double Rule. Maiſter. Well, ſithe you perceiue nowe the vſe of this Rule, I will ſhewe other which enſue of the ſame, & firſte the double Rule, which is ſo called, becauſe there is in it double working, by which thing onely it differeth from this.

Sch. Then by an example I ſhal vnderſtand it well ynough.

Of cariage. Ma. So ſhall you, and let this be the example: If the cariage of 100 pound weight 30 miles, doe coſt 12 d. how much will the cariage of 500 weight coſt, being caryed 100 miles?

Scholer. I pray you ſhewe me the working of it.

Mai.

M. You muſt make 2 workings of it: the firſt thus. If 100 pound weight coſt 12 d. how much will 500 lb. coſt? Set your figure thus. And multiplie 500 by 12 and therof amoūteth 6000, which if you diuide by 100, the quotiēt wil be 60, that is the price of 500 for 30 miles.

Weight. *Pence.*
100 — 12
500 —

Thē begin the ſecond worke, ſaying: if 30 miles coſt 60 d, how much wil 100 miles coſt? Set your figure thus. Than multiplie a 100 by 60, wherof amoūteth 6000 which being diuided by 30, will yield 200 d. Than you may ſay, that ſo many penies ſhall coſt the cariage of 500 pound waights 100 miles, after the rate of 12 pence for the 100, caried 30 miles.

Miles. *Pence.*
30 — 60
100 —

Scholer. Now I perceiue it alſo.

Maſ. Theſe and ſuch other like queſtiōs, are to be aunſwered much quicker, at one working by the Rule of 3 compoſed of fiue numbers, whiche here I will not trouble you withall. But at the ende of this Rule will ſhewe you the worke thereof: not onlye of this and the nexte queſtion, but alſo I will there deliuer thrée or 4 other examples, wiſhing you then to make a compariſon the one with the other: And ſo to vſe which way you thinke good.

Sc. Sir I thanke you much for your curteſie, and I long now til this rule be ended, that I ſhall ſee howe I maye behaue my ſelfe with that newe Rule of 5 numbers, for that I haue euer ſince you taughte mee hetherto, in the Golden Rule both forward and backward wroughte but with 3 numbers only.

Queſtion of ſowing.

Ma. Till we haue done with this, lette vs go on forward: and anſwere me to this queſtion: 30 buſhels of wheat ſowed, yielded in one yeare 360: how manye will 80 buſhels yield in 7 yeare.

Sc. Firſt I ſaye, that if 30 buſhels will yield 360 in 1 yeare, then 80 buſhels will yield 960 in 1 yere. Then for the ſeconde worke I ſay: If one yeare yield 960, then 7 yeare will yielde 6720: as theſe two figures doe ſhew.

Seede.	*Encreaſe.*	*Yeares.*	*Encreaſe.*
30	360	1	960
80	960	7	6720

Queſtion of Corn.

But now ſir, if I ſet forth 30 buſhels of corne to another man for 7 yeare, agreeing ſo that he ſhall ſow euerye yeare the whole encreaſe of the corne, and I at the ende of thoſe ſeuen yeares to haue the halfe of the whole increaſe: I would know how many buſhels will there amount to my part ſuppoſing the increaſe to be after yͤ rate of the laſt queſtion, for 30 buſhels in one yeare, 360.

Mai. In ſuch a queſtion you muſt haue ſo manye ſeuerall workinges, as there be yeres, as for example: In the firſt yere 30 buſhels yield 360: then to knowe the yielding of the ſecond yeare, I muſt ſay: If 30 yield 360, how many yieldeth 360? Work by your rule, and you ſhal find 4320, Then ſay for the third yere: if 30 yield 360, how manye will 4320 yielde? you ſhall haue 51840, and ſo euery yeare multiplying the whole encreaſe by 360, and diuiding it by 30, the increaſe of the next yere wil amoūt, as theſe 7 figures (in yͤ next page) do orderly declare: where I haue ſet 7 letters for yͤ 7 yeres, of which the firſt is ſet without art becauſe that is the increaſe whiche you doe preſuppoſe: & the laſt number of eche other doth ſhewe the increaſe of the yeare that it ſtandeth for: which the letters doe declare, ſo that the increaſe of the ſeuen yeare, is 1074954240 buſhels: how manye quarters that is, and alſo how many wayes, you may by Reduction ſoone finde.

a	b
30—360	30 360 360 4320
c	**d**
30 360 4320 51840	30 360 51840 622080

e

30 360
622080 7464960

f

30 360
7464960 89579520

g

30 360
89579520 1074954240

Now with one queſtion more I will proue you. If 6 Mowers doe mowe 45 acres in 5 dayes, how many mowers wil mow 300 acres in 6 dayes?

Sch. If 45 acres doe require 6 mowers,

then 300 acres requireth 40. Now agayn: if 5 dayes require 40 mowers, then 6 dayes needeth but 33 mowers.

Mai. Why do you not make mention of the 2 that remaineth in the laſt diuiſion? for the laſt part of the queſtion is wroughte by the Backer rule, where the firſt number 5, is multiplied into the ſeconde that is 40, whereof amounteth 200, whiche if you diuide by the thirde number 6, the quotiente will be 33, as you ſaid, but then will there remaine 2, which cannot wel be diuided into 6 parts: how be it, you may vnderſtand by the ſirt part of 2, the thirde parte of one mans work, which you muſt put to the 33, or elſe you may ſay, that 33 workemen wil ende all the 300 acres in 6 dayes, ſaue two mens worke for one day, or 2 dayes worke for one man. But ſuch brokē numbers called Fractions, you ſhall hereafter more better perceiue, whē I ſhal wholy inſtruct you of them.

Mai Yet one queſtion more of field matters I will propone, and ſo I will make an end of this double Rule of 3.

Sc. With all my hart ſir I thank you, and I wil diſpatche it as ſoone as I can, becauſe would faine ſee the order of the nexte Rule of 5 numbers.

Maiſt. Then this is my queſtion, If 300 Pioners in 8 houres, will caſt a trenche of 200 Rods: I demaund how manie Labourers wil be able with a like trēche in three houres to entrench a Camp of 3500 Rods.

Sch. I thinke I am nowe in the Backehouſe diche, for I knowe not well whiche way to go about it: And beſides that trulie I think I ſhall neuer come to preferment that way my grouth is ſo ſmall.

Ma. You know not how God may raiſe you hereafter by ſeruice, into the fauour of your Prince, for ẏ auaile of your Countrie, Example, Sir Francis Drake, as worthy a man as euer *England* bred, is not the talleſt man, and yet hath made the greateſt aduēture for the honour of his Prince & Countrie, that euer Engliſh man did.

Sc. Sir, I thanke you for your good encouragement, my mind, though I be little, is as deſirous of knowledge, as any other:
I haue pondred now a little *Booll.* *Men.*
of it, & thus I ſet forthe the 200 300
worke. *Rod.* Z
Saying if 200 Rod require 3400
300

300 men what ſhall 3400 Rod require: I multiplie 3400 by 300: and it yieldeth 1020000: which I diuide by 200 and my quotient is 5100 men.

Then muſt I ſay for my ſecond work, if in 8 houres 5100 men be able to diſcharge it, how many ſhall performe the ſame in 3 houres? now if I ſhould worke by ẏ Golden Rule of proportion forwarde, I ſhould find a leſſe nūber of men, becauſe 3 houres is leſſe then 8 houres: but becauſe reaſō teacheth me that the leſſer the time is, wherin ẏ trench muſt be made, the more Laborers I ought to haue, whereupon I vſe now the backer Rule as in example. And I haue in my quotiēt 5060. So many Pioners muſt I haue, to entrenche the camp in 3 houres.

Ma. You haue anſwered the queſtion very artificiallie: And truelie I commende you for your diligence and apte vnderſtanding: and now according to my promiſe, I will (in whole numbers) giue you a little taſt of the Rule of 3 compounded of 5 numbers.

The Rule of 3 compound of 5 numbers.

This Rule of 3 compoſed is diſtinct for moſt néedefull queſtions into two ſeuerall parts or workinges: And there belongeth vnto it alwayes 5 numbers, wherof in this rule being the firſt part: the ſeconde number and the fift are alwayes of one nature and like denomination, which rule is to be wrought thus: you muſt multiplie the firſt number by the ſeconde: And that ſhalbe your diuiſor: Then again, multiplie the other thrée numbers, the one by the other and their product ſhall be your diuidend.

And now according to my promiſe, wée will firſt work the queſtion of weighte and cariage whiche I deliuered you in ẏ double rule of 3: to be abſolued by this Rule, which was this.

If the cariage of 100 ℔. weight 30 miles coſt coſte 12 d, what wil the carriage of 500 lb. weighte ſtande me in being carried 100 miles.

Then marke wel how theſe numbers ſtand: *Waight, Miles, Pence Waight, Miles.* 100—30—12 — 530—105 multiplie 100 by 30 as thys figure ſheweth: And that number kepe for your diuiſor:

Then multiplie the other 3. numbers the one by the other, and they amount as you ſee to 600000: which you ſhall diuide by 3000: your quotiēt is 200 d: Now you ſee it agreeth with the concluſion of the double Rule.

S. Sir I thanke you moſt hartely: it is euen ſo.

Ma. Yet note this for a generalitie in thys Rule, loke what nature or denominatiō your middle number is: And of the like denomination or nature is alwayes your quotient. Note.

Scholer. Well nowe and it pleaſe you by your patience, I will ſee howe I can ende, the queſtion then nexte following of 30 Buſhels of Wheate ſowed, yelded in one yeare 360, how many then will 80 buſhels yelde in ſeauen yeare: and according to youre reaſons, I ſet my numbers thus:

Which 201600 I diuide by 30: and my quotiente is 6720: buſhels my deſire.

Buſhels, Yere, Buſhels, Buſhels, Yeres,
30—1—360—80—7
1 ——— 30
80 ——— 28800
7 ——— 201600

M Yet one queſtion more I will propound vnto you and ſo leaue this rule, till it pleaſe God hereafter, that I may make you worke it in broken numbers.

What comes the intereſt of 258 ℔ for fiue monethes after the rate of 8 pounde taken in the 100 ℔: for 12 monthes:

Sch Sir as this is a queſtion of gaines So will I warelie worke this queſtion in hope one day to reape ſometing for my paines: and thus I propone it. But I beſeech you if it bee not well ſet downe to ſhewe me myne errour.

Ma. Procéede you haue done verye wel

℔, *months*, ℔ ℔ *months*,
100—12—8-258—5

Sch. Then I doubt not by the grace of God but to ende it: I multiplie 100 by 12 it yeeldeth 1200: and the 3 other numbers multiplied togither produceth 10 20: which I diuide by 1200: and my quotient is 8 poundes. Thē according accoꝛding as you haue taught me heretofoꝛe, I turne the 720 ℔. that is left: into ſhillings: and diuiding it by my firſte number my quotient is 12 ß. So I anſweare that the lone of 258 ℔ foꝛ 5 monthes, after the rate of 8 ℔. in the 100 ℔ foꝛ a yere, comes to 8 ℔ 12 ß.

Ma. You ſay true, I commende your diligence, now beholde the manner of the ſecond part of this rule.

M. In the ſecond part of this rule of 3 compoſed: the third number is like vnto the firſt. And the rule is to be wꝛought as thus : you ſhall now contrarie to the laſt rule multiplie the third number and the fourth togither: and that pꝛoduct ſhal be your deuiſoꝛ: Then multiplie the fift by the ſeconde and the pꝛoducte therof by the firſt: and that is yͤ number that ſhall be diuided. Foꝛ example I pꝛopond this queſtion: foꝛ a pꝛoofe of my laſt queſtion of intereſt. A Merchant hath receiued 8 ℔ 12 ß. foꝛ intereſt foꝛ 5 months terme, which he receiued after the rate of 8 ℔. in the 100 ℔. foꝛ a yere. The queſtion is nowe how much monie was deliuered to raiſe this intereſt: Be-
hold therefoꝛe the ℔. *months* ℔ *months* ℔. ß
maner howe the 100—12—8—5—8-12
queſtion is ſet foꝛth.

Scholer. Sir I perceiue it verie wel: and according to the doctrine whiche you preſcribed for the working thereof: if it pleaſe you nowe it is ſet downe I thinke I can followe the worke.

M. Nay ſtaie a while, and afore you worke marke wel how I deliuer a reaſō, for the perfect vnderſtanding of this rule which is thus: if 8 ℔. in 12 monethes do yéeld me 100 ℔ To take 8 ℔ — 12 ß. for 5 moneths, muſte néedes yéelde a great deale more.

Note.

So vpō the knowledge that I haue in this Arte, The firſt part of this rule is aunſwerable to the rule of 3 forwarde: And this latter part accordeth to the rule 3 backward.

S. Sir I yéelde you moſt hartie thankes for theſe your laſt inſtructions, they haue giuen me great light into theſe two Rules, wherby I maye the better by deliberation conceiue how to vſe them hereafter, whē occaſion ſhall require.

M. You ſay wel, go too now if you wil, and trie your cunning in the queſtion: But thys note take with you by the way, in as much as here is mention made of ſhillings: turne all your mony as you worke, for your more eaſe in worke.

Note.

into ſhillings

Scho-

S. If it pleaſe you to behold me a litle, I wil quicklie end it: for I haue but my firſt: my ſeconde: and my laſt number to be multiplied togither for my diuidende: And my third into my fourth for my diuiſor:

100—12—8—5—8—12

20 20 20
2000 160 172
12 5
4000 800
2000
24000
172
48000
168000
24000
4128000

4128000
8 00

Whiche I diuide by 800. and my quotient is 5160 ſhillings, which in poundes yeeldeth 258 my deſire.

M. I will here for this time in whole numbers ende this rule, and wil inſtructe you in the rules of Felowſhip. You may at your cōuenient leaſure, for your exerciſe worke the ſame, by the rule of 3. at twice: And for your aide and encouragement therein, I ſet down here a proofe how to apply it.

Monthes, A ℔ ß *Pounde*, ℔

5 Z 12 8 8 Z 100

12 Z 412 $\frac{4}{5}$ ß: 412 $\frac{4}{5}$ Z 258 *pounds.*

The Rule of Felowſhip.

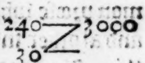
But now wil I ſhew you of the rule of felowſhip or Companie, which hath ſundrie operations, according to the diuers number of the companie. This rule is ſometime without difference of time, and ſometimes there is in it difference of time. Firſte I will ſpeake of that without difference of time, of which let this be an example.

Foure Merchaunts of one companie made a banke of money diuerſlie, for the firſt layde in 50 ℔, the ſeconde 50 ℔, the third 60 ℔, and the fourthe 100 ℔, whiche ſtocke they occupied ſo long, till it was encreaſed to 3000 ℔. Nowe I demaunde of you, what ſhoulde eache man receiue at the parting of this money.

Sch. I perceiue that this rule is like the other, but yet there is a differẽce, which I perceiue not.

Ma. Then wil I ſhew it to you. Firſte by Addition you ſhall bring all the particular ſummes of the Merchaunts into one ſumme, whiche ſhall be the firſt ſumme in your working by the Golden rule, and the whole ſum of the gaines by that ſtocke ſhal be the ſecond ſumme. Now for the third ſum, you ſhall ſet the portion of each mã one after an other, and then work by the Golden rule, and the fourth ſum will ſhewe you each mans gaines: as in example.

The parcels of thoſe foure Merchaunts make in one ſum 240 ℔: ſet that in the firſte place, the gaines in the ſecond, and the firſte mans portion of ſtocke in the [illegible] place thus.



Now multiplie the ſecond by the third, and it will be 90000, whiche you ſhall diuide by

240,and there wil appeare 375 ℔.thus.

240 — 3000
30 — 375

And that the gaines for the firſt man.

Now for the ſecond man, ſet the 50 ℔.that he brought,in the thirde place, and worke as before:and his part will be 625 ℔.as this figure ſheweth.

240 — 3000
50 — 625

Likewaies for the thirde man ſet his mony which was 60 ℔, and his parte of gaines wyll be 150 ℔, as here appeareth

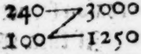
140 — 3000
60 — 750

And ſo for the fourth man, if you ſette his ſumme whiche is 100 ℔, his gaines wil be 1250 ℔, as the prœfe wyll declare.

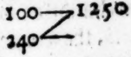
240 — 3000
100 — 1350

Scholer. This I perceiue: but is there any way to examine whether I haue wel done or no?

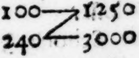
Maiſter. That muſte you doe by one common prœfe which ſerueth to the Golden rule and al other inſuing of the ſame: and that is this: Change the ſtandings of the numbers, and ſet the thirde in the firſt place, the 4 in the ſeconde place, and the firſte in the third place, and they worke by the Golden rule, and if you haue done well, the fourth number now will be the ſame that was the ſeconde before. As for example, I will take the laſt worke whiche was this:



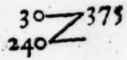
Which to examine I alter as I ſaide, thus:

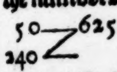


Nowe if I multiply the ſecond number by the thirde, and diuide that that amounteth by the firſte, then will the fourth number be 3000, whiche was the ſeconde before, as you ſee here, whiche is a token, that I haue well done. But as in a ſingle rule one profe thus is ſufficient, ſo in a rule where many operations be, you muſt turne euery of them as I haue done with this one.



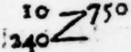
Scholer. Then for the profe of the firſte worke of this rule, I ſhold turn the numbers thus.





And the ſeconde thus.

And for the thirde thus.



And in ech of them if the

workyng were true, the fourth number wyll be ſtill 3000.

Ma. Well, nowe an other example will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for both.

If thre Merchauntes in one ſhippe and of one fellowſhip, had bought marchandiſe, ſo that the firſt had laide out 200 li, the ſeconde 300 li, and the thirde 500 li, and it chaunced by tempeſt that they did caſt ouer boarde into the ſea merchandiſe of the value of 100 pound, howe muche ſhoulde eache man beare in this loſſe?

Scholer. If I ſhal doe in this as you did in the other queſtion, then muſte I ioyne these thre portions togither, 200, 300, and 500, whiche maketh 1000. Then ſaye I, if 1000 loeſe 100, then ſhal 200 loſe 20, and 300 ſhal loſe 30, & 500 ſhall loſe 50, as by theſe thre figures it doth appeare plaine.

1000 — 100
200 — 20

1000 — 100
300 — 30

1000 — 100
500 — 50

Ma. Well ſith now you haue done theſe I

wil propound a queſtiō of more importance, which ſhal make you not only yͤ abler to vnderſtād this Rule, but alſo it wil greatly aide you in the next rule of fellowſhip with time, if ſuch néede be that your money be of diuerſe denominations.

For this may not be forgottē in al ſuch queſtions, if the number be of diuerſe kinds: you muſt by Reductiō bring it into one kind, that is to ſay to the leaſte value that is named in yͤ queſtion. And likewaies ſhall you doe, if the time be of diuerſe kinds, as ſome yeres, ſome monethes, wéekes and dayes, you ſhall make al months, wéeks or days according as yͤ leaſt name of time in yͤ queſtiō is: As for example.

Firſt in diuerſitie of mony. Thrée cōpaniōs bought 2000 ſhéepe, and paide for them 241 ℔. 13 ſ. 4 d. of which ſum one paide 101 ℔, 10 ſ. The ſecond paid 82 ℔. 17 ſ, 10 d. And the third paide 57 ℔, 5 ſ, 6 d: How many ſhéepe muſt each of them haue? Anſwere: The firſt ſhal haue 840. The ſecond 686. And the third 474. And that muſt you worke thus. Queſtion of Sheepe. Solution.

Firſte conſidering that your money is of diuerſe denominations, you ſhall (by Reduction) bring it all into the ſmalleſt denomination whiche is in it, that is to ſaye, pence, and ſo will the total ſum bée 58000. pence.

Now, if you turne eache mans mony into pennies alſo, the firſte mannes ſumme will be 24360 pence: The ſecond mans ſumme 19894. pence. And the third mans mony wil be 13746. pence.

Now to know how many ſhéepe euery mã ſhal haue, let the whole ſum of money that is 58000. pence, in yͤ firſt place & in yͤ ſecõd place ſet the number of ſhéepe, and then orderly in the thirde place ſet eache mans money, and then multiplying the thirde and the ſeconde ſummes togither, and diuiding that that amounteth by the firſte, there will appeare the number of ſhéepe that eche man ought to haue: as theſe thrée figures do ſhewe,

a

58000 — 2000
24360 — 840

b

58000 — 2000
19894 — 686

c

58000 — 2000
13746 — 474

Scholer. Why doe you ſet the mony in the firſt place, ſéeing in the queſtion you ſaye,

2000

2000 ſheepe coſt 58000 d. is not thus, 58000 d. coſt 2000 ſheepe.

Mayſter. You remember, I taught you at the beginning of this Goulden rule, that the firſte and thirde numbers muſt bee of one name, and of like thyngs: and euermore the number that the queſtion is aſked of, muſt bee ſette in the third place. Now is the queſtion playnely this: If foure men bought 2000 ſheepe for 58000 pence, howe many ſheepe ſhall each man haue?

But ſeing in this queſtion there ought more reſpect to be had to the ſumme of mony, than to the ſumme of the perſons, (for in ẏ ſūmes of mony is there proportion toward ẏ ſheepe, and not in the number of perſons) therefore muſt wee turne the queſtion thus.

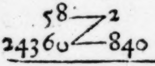
If 18000 pence bought 2000 ſheepe, how many did 24360 d. buy? Agayne, how many did 19894 d. buy? & how many bought 13746 pens.

Scholler. I perciue it reaſonable, and ſo ſhall I doe in all like queſtions.

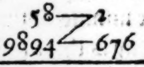
Mayſter. Euen ſo, But for eaſineſſe of the worke marke this: When ſoeuer the firſt and ſecond numbers haue ciphers in the firſt places, you may bothe in the multiplication Note.

and in the diuiſion leaue out thoſe ciphers, ſo that you leaue out like manye out of bothe ſummes, as in this queſtion the firſt number 58000 hath thrée ciphers, and ſo hath the ſeconde that is 2000 : therefore caſte awaye their ciphers, and ſo will the firſt number bée 58, and the ſecond 2: ſet them in their places, and worke according to the rule, and you ſhal perceiue that it wil be al one, ſauing that this is the ſhorter and eaſier way, as theſe thrée figures do ſhew.

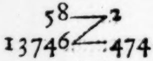
a



b



c



And this you ſée is both eaſier, and alſo the more certaine waye to know the anſwere to this queſtion.

S. Truth it is as you ſay: but ſir, me ſéemeth I might aſke a further queſtion here, not onlie how manye ſhéepe eche man ſhould haue, but alſo what euerie ſhéepe coſt.

Maiſter. That queſtion doeth not onelye be-

belong to this rule, but may alſo be diſcuſſed by Diuiſion, eſpecially if the queſtions number be one onelie: as thus. Diuide the totall ſumme 58000 pence, by 2000 (or 2 58 by omitting the ciphers) and the quotient will be 29 pence, that is 2 ß, 5 d, howbeit, by this rule you maye doe it, and beſte when the number of the queſtion doth exceede 1: as if I ſhoulde aſke this queſtion, 2000 ſheepe coſte 58000 d, howe much did 20 coſte? Then ſhal I ſet my figure thus.

[illegible]

And doing after the rule, there wil amounte 580 pence, that is 2 lb 8 ß 4 d the price of one ſcore: But if you wil uſe that eaſie waye that I did teach you, you may change the firſte and ſeconde number thus.

[illegible]

Thus doe you perceiue the uſe of the rule without time. And that you may as well perceiue the ſame with diuerſitie of time; I propoſe this example.

The rule of felowſhip with time.

Foure Merchants made a common ſtocke, which at the yeares ende was encreaſed to 35145 lb. Nowe to knowe what ſhall be ech mans portion of gaines, you muſte know [illegible] eche

Queſtion of a banke.

eache mans ſtock and time of continuance.

The firſt man of theſe foure laide in 669.℔. which hee did take from the ſtock agayne, at the end of 10.moneths. The ſecond man layd in 810.℔.for 8.moneths. The thirde layde in 900.℔.for 7.moneths. And the fourth layd in 1040.℔.for 12.moneths.

This queſtion ſhall you examin as you did the other before, ſauing that where as in the ☞ third place of the figure you did ſet eche mans ſumme alone, here you ſhall ſet the ſame being multiplyed by the number of their time:& likewiſe in the firſte place of the figure, you ſhal ſet that number which amoũteth of their whole ſummes ſo multiplyed by their time,& added into one whole ſumme as thus.

The firſt mans ſomme is 669 ℔. which I multiplye by 10.(that was the number of his time) and it maketh 6690 The ſecond mans ſumme 810.℔.multiplyed by 8, (which was his time) make 6480. The third mans ſum 900.℔.multiplied by 7(for that was his time yeldeth 6300. The fourth mans ſumme was 1040 ℔ and his time 12.multiply the one by the other,and and it will be 12480.

Theſe foure ſummes thus multiplyed by their time,muſt be ſet orderly in yͤ third place of the figure: and in the firſt place muſt bee ſet the whole ſumme of all foure, whiche is 31950, and the gaine muſt be in the ſecond place, which is 35145. Now to end the queſtion, I ſay firſte: If 31950 did get 35145, what did 6690 get? Anſwere, 7359 ℔, as by this figure appeareth.

a

31950 — 35145
6690 — 7359

Likewiſe the ſecond man had to his part 7128 ℔, the third muſt haue 6930 ℔ And the fourth mã ſhal haue for his part 13728 ℔ as theſe figures doe partly declare.

b

31950 — 35145
6480 — 7128

c

31950 — 35145
6300 — 6930

d

31950 — 35145
12480 — 13728

Scholer. This I like verye well: but what proofe is there of this worke?

Mai. The ſame that I taughte you for the other. Howbeit, there is vſed both for this worke and the other alſo this manner of proofe, to adde all the portions togither,

Another proofe.

T.v. and

and if they agrée to the whole ſumme, then ſéemeth it well done: but this is no ſure rule.

Sch. Yet will I proue it in this example. The foure parcels are theſe, which if I adde togither, there will amount 35145, and that was the whole ſumme: ſo is this rule true here.

7359
7128
6930
13728
―――――
35145

Maiſter. And ſo will it be ſtill when the worke is truely done.

Note the imperfection of this kinde of proofe.

But if you liſt to ſée it proued falſe, take 10000 ℔ from the fourth man, and put it to any of the other 3, and then be yée ſure that you haue not done well, and yet wil the proofe allow it, for the Addition will ſtill be all one.

Sc. It muſt néedes be ſo: but what haue I now to learne?

Maiſt. There are many other excellent partes behinde, of which I will not, as nowe, make mention because that without the knowledge of Fractions, they cannot be duely taught, and much leſſe underſtanded. Therfore wil I propoſe to you two or thrée queſtions more, whereby you may practiſe the better the feate of the rule of felowſhip, (that thereby you may better perceiue yͤ vſe of all other) & ſo make an end for this time.

There is in a Cathedral Church 20 Cannons, and 30 Vicars, thoſe maye ſpende by yere 2600 ℔. but euery Cannon muſt haue to his part 5 times ſo much as euery Vicar hath: howe muche is euery mans portion ſay you?

Queſtion of Canons

Sc. I pray you make the aunſwere your ſelfe, ſo ſhall I perceiue beſt the meanes to aunſwere to ſuch other like.

Mai. In this queſtion you muſt doe as in thoſe that haue diuerſitie of time, for here is diuerſitie of portions: Therefore ſhall you multiplye the number of the perſons by their difference of portion: (as you did in the other by time) Then muſt you multiplie the 20 (which is the number of Cannons) by 5, (for that is the number of their portion) ſo will it be 100: Then 30, (that is the number of Vicars) by 1, (that is the number of their portion) and it will be 30: put thoſe two ſummes togither, and they make 130: then ſay thus: If 130 ſpēd 2600 ℔, what maye 100 ſpende? The rule ſheweth 2000 ℔.

Againe for Vicars: If 130 ſpende 2600

pound, what may 30 ſpend? Anſwer 600 ℔ as theſe figures ſhew.

130	2600	130	2600
100	2000	30	600

But if euery Cãnon ſhould haue ſo often times 4 ℔ as the Vicar ſhould haue 3 ℔ thẽ ſhould I multiplie 20 by 4, (that were 80) and 30 by 3 (that were 90) and then both were 170. Then ſhould the figures be ſette thus.

	℔.	ß. d.		℔	ß. d.
170	2600		170	2600	
80	1223,	10, 7,	90	1376,	9, 5,

But this ſort is to hard for you, by reaſõ of the Fractions, therefore I will let it reſt to that place. And by this rule you ſee what the 20 Cannons may ſpend, which ſumme if you diuide by 20, you ſhall ſee eche Cannons portion: and ſo of the Vicars, if you diuide their ſumme by 30, the quotient wil declare euery Vicars portion.

The ſecond Dialogue.

The accompting by Counters.

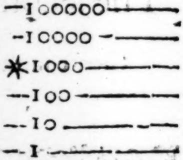
Mayſter.

NOw that you haue learned the commō kindes of Arithmetike with yͤ pen, you ſhall ſee the ſame art in Counters: whiche feate doth not onely ſerue foꝛ them that cannot wꝛite and reade, but alſo foꝛ them that can doe both, but haue not at ſome times their pen oꝛ tables readie with them.

This ſoꝛt is in two foꝛmes commonlye: The one by lines, and the other withoute lines. In that that hath lines, the lines doe ſtande foꝛ the oꝛder of places: and in that that hath no lines, there muſt be ſet in their ſteade ſo many counters as ſhall neede, foꝛ eche line one, and they ſhall ſupplie yͤ ſteade of the lines.

Sc. By examples I ſhould better perceiue your meaning.

Mai. For example of the Lynes, loe hęere you ſee ſixe lines, which ſtande for ſixe places, ſo that the nethermoſte ſtandeth for the firſte place, and the next aboue it for the ſeconde, and ſo vpwarde, til you come to the higheſt, which is the ſixt line, and ſtandeth for the ſixt place.



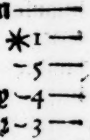
Now what is the value of euery place or line you may perceiue by the figures which I haue ſet on them, whiche is according as you learned before in Numeration of Fifigures by the pen: for, the firſte place is the place of vnits or ones, and euerye Counter ſet in that line betokeneth but one: and the ſecond line is the place of 10, for euery coũter there ſtandeth for 10. The thirde lyne the place of hundreds, yͤ fourth of thouſãds, and ſo forth.

Sc. Sir, I doe perceiue that the ſame order is here of lines, as was in the other figures by places, ſo that you ſhall not nęede longer to ſtande about Numeration, except there be any other difference.

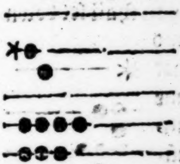
M. If you doe vnderſtande it, then how will you ſet 1543?

Sc. Thus as I ſuppoſe.

Ma. You haue ſet the places truly but your figures be not meete for



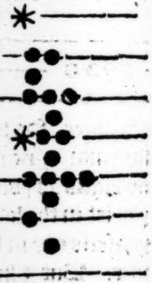
this vſe: for the meeteſt figure in this behalf, is the figure of a Counter round, as you ſee here, where I haue expreſſed that ſame ſumme.



Scholer. So that you haue not one figure for 2. nor 3, nor 4 and ſo forth, but as many digits as you haue, ſo many Counters you ſet in the loweſt line: and for euery 10 you ſette one in the ſeconde line: and ſo of other. But I knowe not by what reaſon you ſet that one counter for 500 betweene two lines.

Maiſt. You ſhall remember this; that whenſoeuer you neede to ſette downe 5, 50, or 500, or 5000, or ſo forth any number whoſe Numerator is 5, you ſhall ſette one counter for it in the next ſpace aboue yͤ line that it hath his denomination of: as in this example of that 500, becauſe the numerator is 5, it muſt be ſet in a voyde ſpace: and bicauſe yͤ denominator is hundred, I know that his place is the voyde ſpace next aboue hundreds, that is to ſay, aboue the thirde line.

And farther you ſhall marke, that in all working by this ſorte, if you ſhall ſet down any ſumme betwéene 4 and 10, for the firſt part of that number you ſhal ſet downe 5, and then ſo many Counters more, as there reſt nũbers aboue 5. And this is true both of digits and articles. And for example I will ſet downe this ſumme 287965, whiche ſumme if you marke well, you néede none other examples for to learne the Numeration of this forme,



But this ſhall you marke, that as you didde in the other kindes of Arithmetike, ſette a pricke in the places of thouſandes, in this worke you ſhal ſet a Starre, as you ſée before.

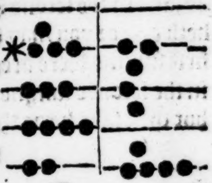
Scholer. Then I perceiue Numeration: but I pray you how ſhall I doe in this art to adde two ſummes or more togither.

ADDITION.

Maiſter.

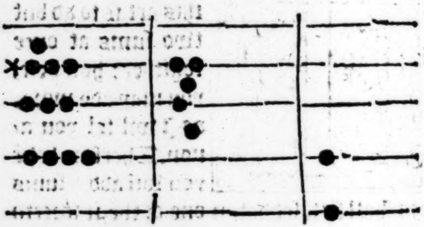
THe eaſieſt waye in this art, is to ad but two ſums at once togither: howbeit, you may add more, as I wil tel you anon. Therfore whē you wil add 2 ſums you ſhall firſt ſet down one of thē, it forceth not which, and then by it draw a line croſſe the other lines. And afterward ſet down the other ſum, ſo that that line may be betwéen them: as if you woulde add 2659 to 8342, you muſt ſet your ſums as you ſée here.

Addition of 2 ſummes.

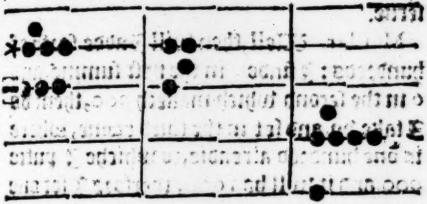


And then if you liſt, you maye adde the one to the other in the ſame place: or elſe you may ad them both togither in a new place: which way, becauſe it is moſt playneſt, I will ſhew you firſt.

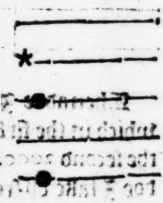
Therefore will I beginne at the vnits, which in the firſt ſume is but 2, and in the ſecond ſumme 9, that maketh 11. Thoſe do I take vp, and for them I ſette 11 in the new roome, thus.



Then doe I take vp all the Articles vnder a hundred, which in the firſt ſumme are 40, and in the ſecond ſumme 50, that maketh 90: or you may ſay better, that in the firſt ſumme there are 4 articles of 10, and in the ſeconde ſumme 5, whiche maketh 9, but then take heede that you ſet thẽ in their right lines as you ſee here.



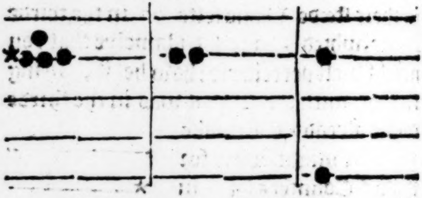
Where I haue taken away 40 from the firſte ſumme, and 50 from the ſeconde, and in their ſteede I haue ſette 90 in the thirde roome, which I haue ſet plainelye that you might well perceiue it: how be it, ſeeing that 90 with the 10 that was in the thirde roome already, doth make 100, I might better for thoſe 6 Counters ſet 1 in the thirde line, thus.



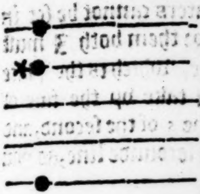
For it is all in one ſum as you maye ſee, but it is beſt neuer to ſette fyue counters in any line, for that maye be done with one Counter in a higher place.

Scholer. I iudge that good reaſon, for manye are vnnéedefull where one will ſerue.

Maiſter. Well, then will I adde forth of hundreds: I finde 3 in the firſt ſumme, and 6 in the ſecond, which maketh 900, them do I take vp, and ſet in the third roome, where is one hundred alreadie, to whiche I putte 900, and it will be 1000, therfore I ſet one counter in the fourth line for them all, as you ſée here.



Then adde I the thouſandes together, which in the firſt ſumme are 8000, and in the ſecond 2000, that maketh 10000: them doe I take vp from thoſe two places, and for them I ſet one counter in the fifte line, and then appeareth as you ſée to be 11001, for

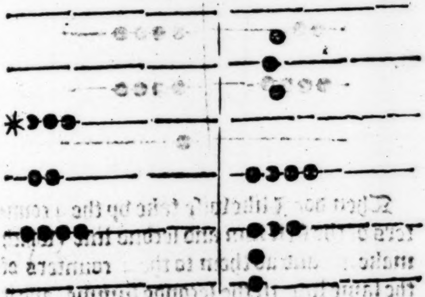


for ſo manye doth amount of the Addition of 8342 to 2659.

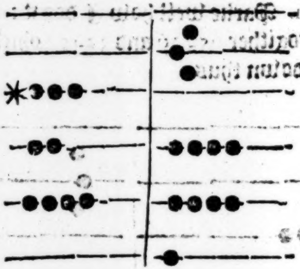
To add ſummes togither.

Scholer. Syr, this I doe perceiue: but howe ſhall I ſette one ſume to an other, not chaunging them to a third place?

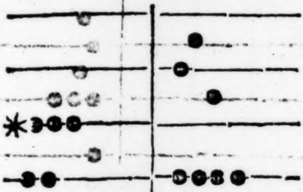
Mai. Marke well how I doe it: I will adde togither 65436 and 3245, which firſte I ſet down thus.



Then do I begin with the ſmalleſt, which in the firſt ſume is 5, that do I take vp, and would put to the other 5 in the ſecond ſum, ſauing that two Counters cannot be ſet in a voyd place of 5 but for them both I muſt ſet 1 in the ſecond lyne, which is the place of 10: therefore I take vp the fiue of the firſt ſumme, and the 5 of the ſecond, and for them I ſet 1 in the ſecounde line, as you ſee here.

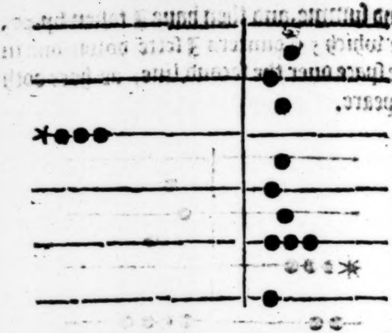


Then doe I likewiſe take vp the 4 counters of the firſt ſum and ſecond line (which make 40) and ad them to the 4 counters of the ſame line, in the ſeconde ſumme, and it maketh 80, but as I ſayde, I maye not conuenientlye ſet aboue 4 counters in one line, therefore to thoſe 4 that I toke vp in the firſt ſumme, I take one alſo of the ſecond ſumme, and then haue I taken vp 50, for which 5 Counters I ſette downe one in the ſpace ouer the ſecond line, as here doth appeare.

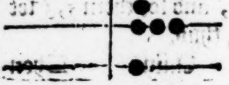


And then is there 80, as well with thoſe 4 counters, as if you had ſet downe the other 4 alſo.

Now do I take the 200 in the firſt ſum, and adde them to the 40 in the ſecond ſum, and it maketh 600, therefore I take vp the 2 counters in the firſt ſumme, and 3 of thē in the ſecond ſumme, and for them 5, I ſet 1 in the ſpace aboue, thus.



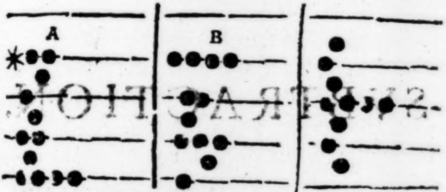
Then I take the 3000 in the firſt ſūme, vnto which there are none in the ſeconde ſumme agréeing, therefore I doe onelye remoue thoſe thrée Counters from the firſt ſum into the ſeconde, as here doth appeare.



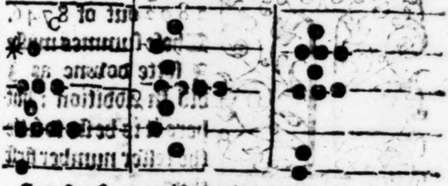
And ſo you ſee the whole ſumme that amounteth of the Addition of 65436 with 3245, to bee 68681.

And if you haue marked theſe two examples well, you neede no further inſtruction in Addition of 2 onely ſummes: but if you haue more than two ſummes to adde, you may do them thus.

Firſte adde two of them, and then adde the thirde and the fourth, or more if there be ſo many: as if I woulde adde 2679 with 4286 and 1391. Firſt I adde the two firſt ſummes thus.



And then I adde the third therto thus.



And ſo of more, if you haue them.

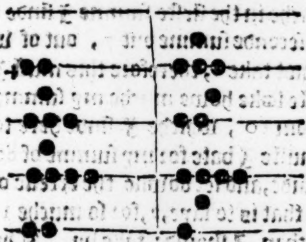
Scho. Nowe I thinke beſte that you paſſe forthe to Subtraction, except there be anye wayes to examine this manner of Addition, then I thinke that were good to be knowen next.

A proofe. Maiſter. There is the ſame proofe here that is in the other Addition by the penne, I meane Subtraction, for that onelie is a ſure waye: but conſidering that Subtraction muſte be firſt knowen, I will firſt teach you the arte of Subtraction, and that by this example.

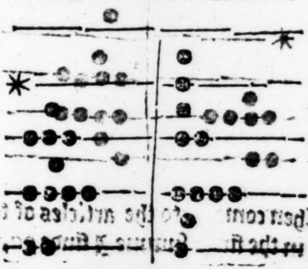
SVBTRACTION.

I Woulde ſubtract 2892 out of 8746. Theſe ſummes muſte I ſette downe as I did in Addition: but here it is beſte to ſette the leſſer number firſt thus.

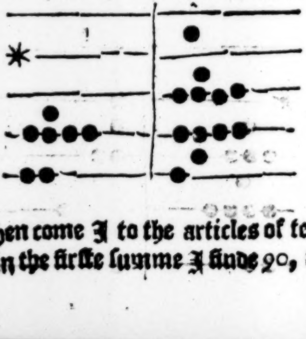
Then



Then ſhal I beginne to ſubtract the greateſt numbers firſt (contrarie to the vſe of the pen) that is the thouſandes in this example: therefore I finde amongſt the thouſands 2, for which I withdrawe ſo manye from the ſecond ſumme (where are 8) and ſo remayneth there 6, as this example ſheweth.

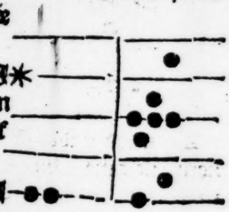


Then do I likewayes with the hundreds, of whiche in the firſte ſumme I finde 8, and in the ſeconde ſumme but 7, out of whyche I can not take 8, therefore this muſt I doe: I muſte looke howe muche my ſomme differeth from 10, whiche I finde here to be 2, then muſte I bate for my ſumme of 800, one thouſande, and ſet downe the exceſſe of hundreds, that is to ſaye, 2, for ſo muche 1000 is more than I ſhoulde take vp. Therefore from the firſte ſumme I take that 800, and from the ſeconde ſumme (where are 6000) I take vppe one thouſande, and leaue 5000, but then ſet I downe the 200, vnto the 700 that are there alreadie, and make them 900, thus.



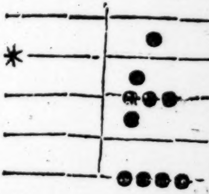
Then come I to the articles of tennes, where in the firſte ſumme I finde 90, and in the

the ſeconde ſumme but onely 40. Now conſidering that 90 can be bated from 40, I loke howe muche that 90 doth differ from the next ſumme aboue it, that is 100, or elſe (which is all to one effecte) I loke howe much 9 doeth differ from 10, and I finde it to be 1, then in the ſteade of that 90, I doe take from the ſeconde ſumme 100: but conſidering that is 10 to muche, I ſet downe 1 in the nexte line beneth for it, as you ſee here.



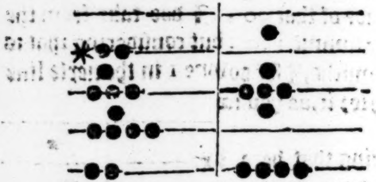
Sauing that here I haue ſet one counter in the ſpace, in ſteade of 5, in the next line.

And thus haue I ſubtract al, ſaue two whiche I muſte bate from the 6 in the ſecond ſumme and there will remaine 4, thus.

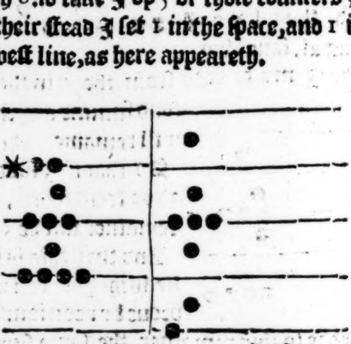


So that if I ſubtract 2892 from 8746, the remainer wil be 5854 And that this is truly wrought you may proue by addition: for if you adde to this remainer the ſame ſumme that you did ſubtrate, then will the former ſumme 8746, amount againe.

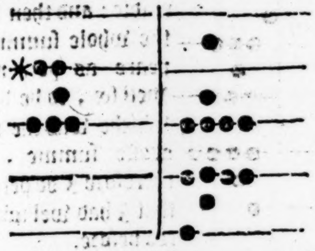
Scho. That will I proue: and firſt I ſet the ſum that was Subtracted, which was 2892, and then the remainer 5854, thus.



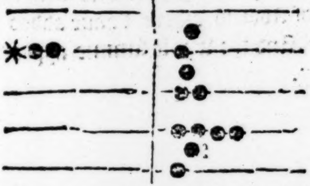
Then doe I adde the firſt 2 to 4, which maketh 6: ſo take I vp 5 of thoſe counters, and in their ſtead I ſet 1 in the ſpace, and 1 in the loweſt line, as here appeareth.



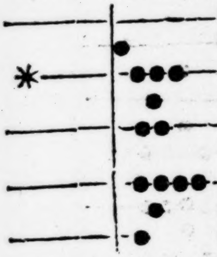
Then doe I adde the 90 next aboue to the 50, and it maketh 140, therefore I take vppe thoſe 6 counters, and for them I ſet 1, to the hundreds in the thirde line, and foure in the ſecond line thus.



Then do I come to the hundreds, of which I finde 8 in the firſt ſum, & 9 in the ſeconde, yͤ maketh 1700: therefore I take vp thoſe 9 counters, & in their ſteade I ſet 1 in yͤ fourth line, and 1 in the ſpace next beneath, and 2 in the third line as you ſee here.



Then is there leſte in the firſte ſumme but onelie 2000, whiche I ſhall take vppe from thence, and ſet in the ſame line in the ſeconde ſumme, to the one that is there alreadie: and then wyll the whole ſumme appeare as you maye well ſee, to be 8746, whiche was the firſte groſſe ſumme, and therefore I do perceiue that I had wel ſubtracted before.

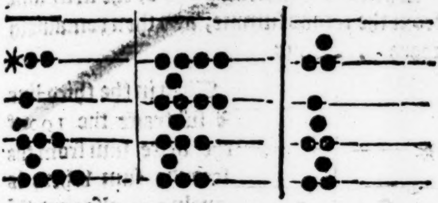


And thus you may ſee, howe Subtraction may be tried by Addition.

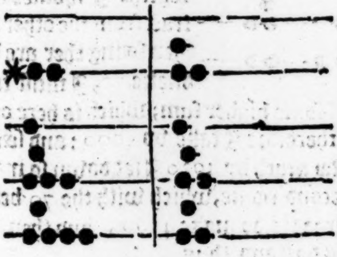
Scho. I perceiue the ſame order here wyth Counters, that I learned before in figures.

Ma. Then let me ſee how you can trie Addition by Subtraction.

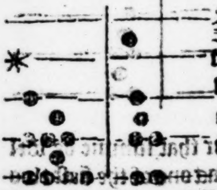
Scholer. Firſte I will ſette forth thys example of Additiõ, where I haue added 2189, to 4988. And the whole ſumme appeareth to be 7177.



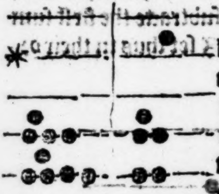
Nowe to trie whether that ſumme be wel added or no, I wil ſubtract one of the firſt two ſummes from the thirde, and if I haue well done, yͤ remainer wil be like that other ſum, as for example. I wil ſubtracte the firſt ſum from the thirde, which I ſet thus in their order.



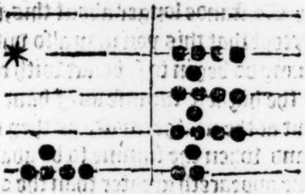
Then doe I ſubtract 2000 of the firſte ſum from the ſeconde ſumme, and then remaineth there 5000, thus.



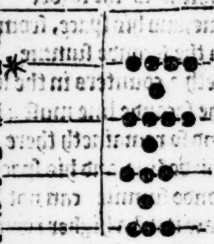
Then in the third line I ſubtracte the 100 of the firſte ſum from the ſeconde ſum where is onely 100 alſo: and thẽ in the third line reſteth nothing, as you maye ſee in the example following.



Then the in ſeconde line with his ſpace vnder him, I finde 80, whiche I ſhoulde ſubtract from the other ſũ: then ſeeing ther are but onely 70, I muſt take it out of ſome higher ſum, which is here only 5000: therefore I take vp 5000: and ſeeing that is too much by 4920, I ſet downe ſo many in the ſecond roome, which with the 70 being there alreadie, do make 4990, and then the ſummes do ſtand thus.



Yet remaineth there in the firſte ſum, 9 to be abated from the ſecond ſum, wherein that place of units doth appeare only 7: then muſt I bate a higher ſumme, that is to ſaye 10. but ſeeing that 10 is more than 9 (whiche I ſhoulde abate) by 1, therefore ſhall I take up one counter from the ſecond, and ſet downe the ſame in the firſte or loweſt line, as you ſee here.

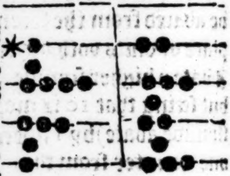


And ſo haue I ended this worke, and the ſumme appeareth to be the ſame whyche was the ſeconde ſumme of mine Addition, and therefore I perceiue I haue wel done.

May To ſtande longer about this, it is but folly, except that this you may alſo vnderſtãd, that many do begin to ſubtract with counters not at the higheſt ſumme as I haue taughte you, but at the neathermoſt, as they do vſe to adde: and when the ſumme to be abated in anye line appeareth greater than the other, thẽ do they borrow one of the next higher roome, as for example.

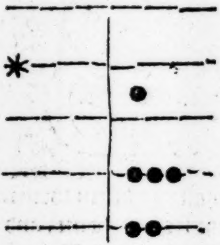
nother
ay of ad-
tion.

If I ſhoulde abate 1846 from 2378, they ſet the ſummes thus.



Firſt they take 6, whiche is the lower line, and his ſpace, from 8 in the ſame roomes in the ſeconde ſumme, and yet there remayneth 2 counters in the loweſt line. Then in the ſeconde line muſt 4 be ſubtracted from 7, and ſo remaineth there 3. Then 800 in the thirde line, and his ſpace, from 300 of the ſeconde ſumme can not be, therefore doe they bate it frõ a higher roome, that is from 1000; and becauſe that 1000 is too muche by 200, therfore muſt I ſet downe 200 in the thirde line, after I haue taken vp 1000 from the fourth line. Then is there yet 1000 in the fourth line of the firſt ſum, whiche if I withdrawe from the ſeconde ſum, then doth al the figures ſtand in order, thus.

So that (as you ſee) it differeth not greatly whether you beginne ſubtraction at the higher lines, or at the lower.



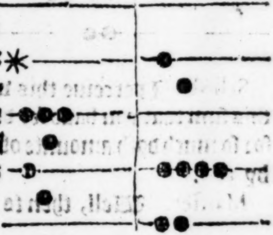
Howe be it, as ſome men like yͤ one way beſte, ſo ſome like the other: therefore you nowe knowing bothe, maye vſe whyche you liſte.

MVLTIPLICATION.

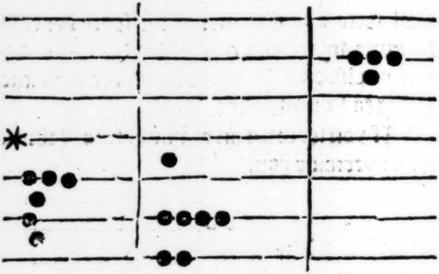
BUt nowe touchyng Multiplication: you ſhall ſette your numbers in two roomes (as you did in thoſe other kindes) but ſo ẏ the multiplier be ſet in the firſt roome, then ſhall you begin with the higheſt numbers of the ſeconde roome, and multiplie them firſte, after this ſort. Take ẏ ouermoſt line in your ☞ firſt working, as if it were the loweſt line, ſetting on it ſome moueable marke (as you liſt) and looke how many counters be in him, take them vp, and for them ſette downe the whole multiplier ſo many times as you tooke vp counters: reckning (I ſay) that line for the Unites. And when you haue done with the higheſt number, then come to the nexte lyne beneath, and do euen ſo with it, and ſo with the nexte, till you haue done all. And if there be anye number in a ſpace, then for it ſhall you take the multiplier 5 times: and then muſte you reckon that line for the Unites, whiche is next beneath that ſpace. Or elſe after a ſhorter waye, you ſhall take onelie halfe the multiplier, but thē ſhal you take the line next aboue that ſpace for the line of Unites. But in ſuche working, if by chaunce your multiplyer be an odde number, ſo that you can not take the halfe of it iuſtlye, then muſte you take the greater halfe, and ſet downe that, as if that it were the iuſte halfe: and further you ſhall ſette one Counter in the ſpace beneath that line, which you recken for the line of Units, or elſe onely remoue forwarde the ſame that is to be multiplied.

S. If you ſet forth an exāple hereto, I think I ſhal perceiue you.

Maaſter. Take this example: I woulde multiplie 1542 by 265, therfore I ſet the numbers thus.

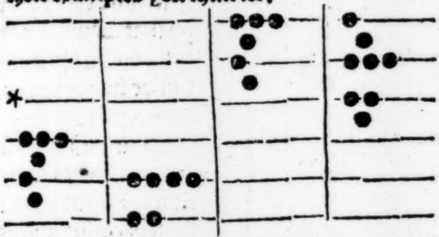


Then firſte I beginne at the 1000 in the higheſt roome, as if it were the firſt place, and I take it vp, ſetting downe foꝛ it ſo often (that is once) the multiplyer, which is 365, thus as you ſee here: where, foꝛ the one counter taken vp from the fourth line, I haue ſet down other 6, which make yͤ ſum of the multiplier, reckening that fourth line as if it were the firſt, which thing I haue marked by the hand ſet at the beginning of the ſame.

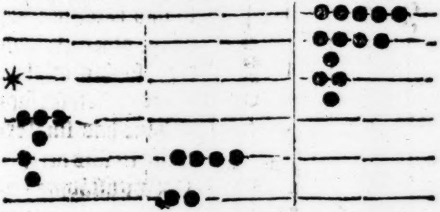


Scholer. I perceiue this well, foꝛ in déede this ſum that you haue ſet down is 365000: foꝛ ſo much doth amounte of 1000, multiplied by 365.

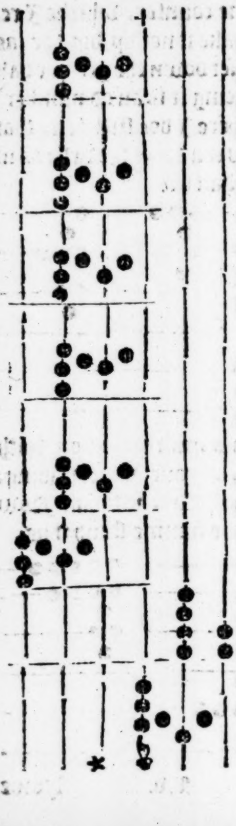
Maiſter. Well, then to goe foꝛth, in the next ſpace I finde one counter, whiche I remoue forward, but take it not vp, but doe (as in ſuch caſe I muſt) ſet down the greater half of my multiplier (ſeeing it is an od number) which is 182, and here I doe ſtill lette that fourth place ſtand, as if it were the firſt: as in theſe examples you ſhall ſee.



Where I haue ſet this multiplication with other, but for the eaſe of your vnderſtãding, I haue ſet a little line betweene them. Now ſhould they both in one ſumme ſtand thus.



An other forme of Multiplica tion.



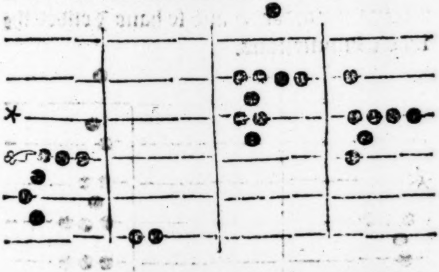
Howbeit, an other fourme to multiplye ſuch coūters in ſpace, is this: Firſte to remoue ỹ finger to the line next beneth that ſpace, and thē to take vp that Counter, and to ſet down the multiplier fiue times: as here you ſee.

Which ſummes if you do adde togither into one ſumme, you ſhall perceiue that it will be the ſame that appeareth of the other working before, ſo that both ſorts are to one intente: but as the other is ſhorter, ſo this is playner to reaſon for ſuch as haue had ſmal exerciſe in this arte.

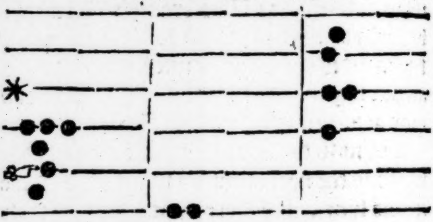
Notwithſtāding you

may adde them in your mynde before you ſet them downe: as in this example you might haue ſayde, 5 times 300 is 1500, and 5 times 60 is 300: alſo 5 times 5 is 25, which all put togither, doe make 1825, whiche you may at one time ſet downe if you liſt.

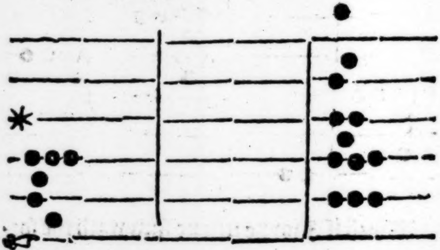
But now to go forth, I muſt remoue the hande to the next counters whiche are in the ſecond lyne, and there muſt I take vp thoſe 4 counters ſetting downe for them my multiplier 4 times ſeuerally, or elſe I maye gather that whole ſum in my mynde firſte, and then ſet it downe: as to ſay, 4 times 300 is 1200: 4 times 60 are 240: and 4 times 5 make 20, that is in all 1460, that ſhall I ſet downe alſo, as here you ſee.



Which if I ioyne in one ſum with the former numbers it will appeare thus,



Then to ende this Multiplication, I remoue the finger to the loweſt line, where are only 2, them do I take vp, and in their ſtæde doe I ſet downe twice 365, that is 730, for which I ſet one in the ſpace aboue the thirde line for 500, and 2 more in the thirde lyne with that one that is there alreadie, and the reſt in their order, and ſo haue I ended the whole ſumme, thus,



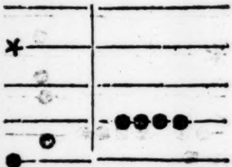
Whereby you ſée, that 1542 (whiche is the number of yeares ſith Chꝛiſte his incarnation) being multiplyed by 365 (whiche is the number of dayes in one yeare) doth amounte vnto 562830, which declareth the nūber of dayes ſith Chꝛiſtes incarnation vnto the end of 1542 yeares, (beſide 385 dayes and 12 houres foꝛ leape yeares.)

The ſumme of the daies ſith Chriſts incarnation

Sc. Now will I pꝛoue by an other example, as this: 40 labourers (after 6 d. the day foꝛ eche man) haue wꝛoughte 28 dayes: I would know what their wages doth amoūt vnto.

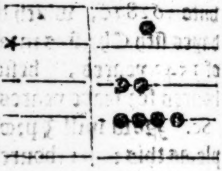
In this caſe muſt I woꝛke doublye: firſt I muſt multiplie the number of the labourers by the wages of a man foꝛ one daye, ſo will the charge of one day amount.

Then ſecōdarily ſhal I multiplie the charge of one daye by the whole number of dayes, and ſo wil ẏ whole ſum appear: firſt therfoꝛe I ſhal ſet the ſūmes thus:



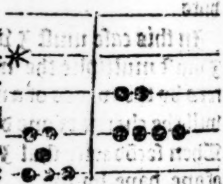
Where in the firſte place is the Multiplier (that is 1 dayes wages foꝛ one man) & in the ſeconde ſpace is ſet the nu̅ber of ẏ workme̅ to be multiplied.

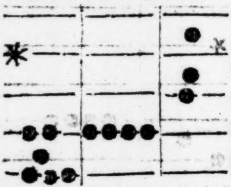
Then ſaye I 60 times 4 (reckoning that ſecond line of the line of Units) maketh 24, for which ſumme I ſhould ſet 2 counters in the third line, and 4 in the ſecond therefore doe I ſet 2 in the third line, and let the 4 ſtand ſtill in the ſecond line thus.



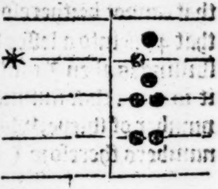
So appeareth the whole dayes wages to be 240 d. ẏ is 20 ſ.

Then doe I multiplye agayne the ſame ſumme by the nu̅ber of dayes, and firſte I ſet the nu̅bers thus, The̅ becauſe ther are counters in diuerſe lines, I ſhall begin with the higheſt, and take them vp, ſetting for them the multiplier ſo many times as I toke vp counters, that is twiſe, then wil ẏ ſum ſtād thus.





Then come I to the ſecond line, and take vp thoſe 4 Counters, ſetting for thē the multiplier foure times, ſo wil the whole ſumme appeare thus.



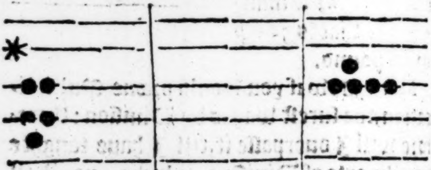
So is the whole wages of 40 workemen for 28 dayes (after 6 d. eche daye for a man) 6720 d. that is 560 ß or 28 pound.

Ma. Now if you would proue Multiplication, the ſureſt way is by Diuiſion: therefore will I ouerpaſſe it, till I haue taught you the arte of Diuiſion, whiche, you ſhall worke thus.

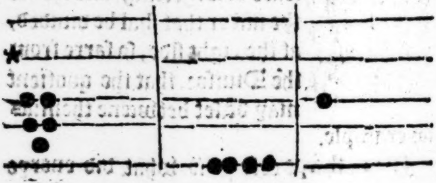
DIVISION.

First ſet downe the diuiſor, for feare of forgetting, and thē ſet the nūber that ſhal be diuided, at the right ſide, ſo farre from the Diuiſor, that the quotient may be ſet betwéene them: as for example.

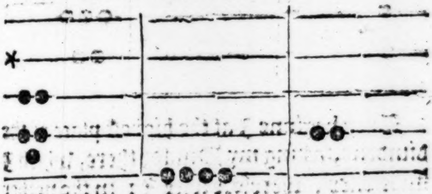
If 225 ſhéepe coſt 45 ℔. what did euerye ſhéepe coſt? To know this, I ſhoulde diuide the whole ſumme that is 45 ℔, by 225, but that cannot be: therefore muſt I firſte reduce that 45 ℔ into a leſſer denomination, as into ſhillinges, then I multiplie 45 by 20, and it is 900: that ſumme ſhall I diuide by the number of ſhéepe, which is 225, theſe two numbers therefore I ſet thus.



Then begin I at the higheſt lyne of the diuident, and ſéeke how often I maye haue the diuiſour therein, and that maye I doe foure times: then ſaye I, foure times 2 are 8, which if I take from 9, there reſteth but 1, thus.



And becauſe I founde the diuiſor 4 times in the diuident, I haue ſet as you ſee, 4 in the middle roome, which is the place of the quotient: but now muſt I take the reſt of ỹ diuiſour as often out of the remayner, therefore come I to the ſeconde line of the diuiſor, ſaying: 2 foure times make 8, take 8 from 10, and there reſteth 2, thus.

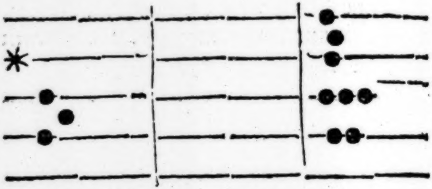


Then come I to the loweſt number which is 5, and multiplie it 4 times, ſo is it 20, that take I from 20, & there remayneth nothing: ſo that I ſee my quotient to be 4, whiche are in valewe ſhillings, for ſo was the diuident: and therby I know ỹ if 225 Sheepe did coſt 45 ℔, euery ſheepe coſt 4 ß.

Sch. This can I doe, as you ſhall perceiue by this exãple. If 160 ſouldiours do ſpẽd euery moneth 68 ℔, what ſpendeth ech man?

Example of vvages

Firſt becauſe I cannot diuide the 68, by 160, therefore I will turne the ℔ into pennies by multiplicatiõ, ſo ſhal there be 16320 d. Now muſt I diuide this ſumme by the number of ſouldiors, therefore I ſet them in order thus.

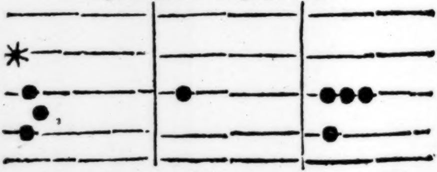


Then beginne I at the higheſt place of the diuidend, ſeeking my Diuiſor there, which I finde once, therefore ſette I 1 in the nether line.

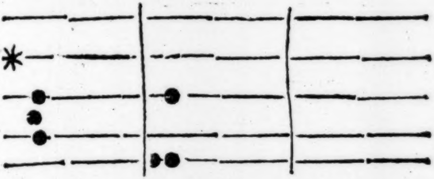
Ma. Not in the nether line of the whole ſumme, but in the nether lyne of that worke which is the third line.

Sc. So ſtandeth it with reaſon.

Ma. Then thus doe they ſtand.



Then ſéke I agayne the reſt, how often I may finde my diuiſour: and I ſée that in the 300 I mighte finde 100 thrée tymes, but then the 60 will not be ſo often found in 20, therefore I take 2 for my quotient: thē take I 100 twice frō 300, and there reſteth 100, out of which with the 20 (that maketh 120) I may take 60 alſo twice, and then ſtandeth the numbers thus.

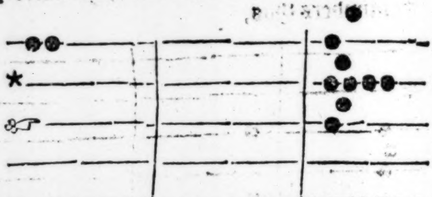


Where I haue ſet the quotient 2 in the loweſt line: So is euery Souldiors portion 102 d. that is 8 ß, 6 d.

Ma. But yet becauſe you ſhall iuſtlye perceiue the reaſon of Diuiſion, it ſhall bée good that you doe ſet your diuiſor ſtil agaynſt thoſe numbers from which you doe take it, as by this example I will declare.

If the purchaſe of 20 acres of ground did coſt 290 pound, what did one acre coſt? Example of purchaſe.

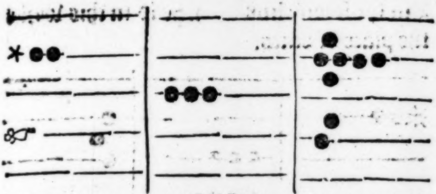
Firſt will I turne the poundes into pennies, ſo will there be 69600 pence. Then in ſetting downe theſe numbers, I ſhal do thus. Firſt ſet the diuidend on the right hand as it ought, and then the diuiſor on the lefte hande agaynſt thoſe numbers from which I intend to take him firſte, as here you ſee, where I haue ſet the diuiſor two lines higher than is his owne place.



Scho. This is like the order of Diuiſion by the pen.

Ma. Truth you ſay, and now muſt I ſette the quotient of this worke in the thirde line, for that is the lyne of vnits in reſpecte to the diuiſor in this worke.

Then I ſeeke how often the diuiſor maye be found in the diuident, and that I fynd 3 tymes, then ſet I 3 in the third lyne for the quotient and take awaye that 60000 from the diuidend, and farther I do ſet the diuiſor one line lower, as you ſee here.



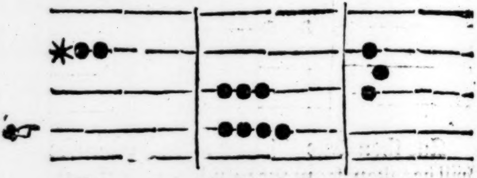
And then ſeeke I howe often the diuiſor will be taken from the number agaynſte it, which will be 4 times and 1 remaining.

Scho. But what if it chaunce that when the diuiſor is ſo remoued, it cannot be ones taken out of the diuident againſt it?

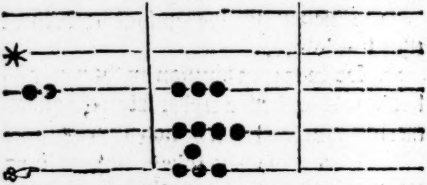
Ma. Then muſte the diuiſour be ſet in an other line lower.

Sc. So was it in diuiſion by the pen, and therefore was there a cipher ſet in the quotient: but how ſhall that be noted here?

Ma. Here needeth no token, for the lines doe repreſent the places: onely looke that you ſet your quotient in that place whiche ſtandeth for vnits in reſpect of yͤ diuiſor: but now to returne to the example. I finde the diuiſor 4 times in the diuident, and 1 remaining, for 4 times 2 make 8, which I take frō 9, & there reſteth 1, as this figure following ſheweth; and in the middle ſpace for the quotient I ſet 4 in the ſecond line [illegible]he is in this worke the place of vnits.



Then remoue I the diuiſour to the next lower line, & ſeeke how often I may haue it in the diuident, which I may doe here 8 tymes iuſt, and nothing remain, as in this fourme.



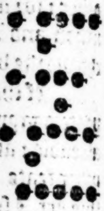
Where you may ſee, that the whole quotient is 348 d., that is 29 ſ., whereby I knowe that ſo much coſt the purchaſe of one acre.

Sc. Now reſteth the proues of Multiplication, and alſo of Diuiſion.

Ma. Their beſt proues are eche one by the other: for multiplication is proued by Diuiſion, and Diuiſion by Multiplication, as in the worke by the pen you learned.

Sc. If that be all, you ſhall not neede to repeate agayn that that was ſufficiẽtly taught alreadie: and except you will teache me any other feats, here maye you make an ende of this art, I ſuppoſe.

Ma. So will I doe as touching whole number: and as for brokẽ number, I will not trouble your wit with it, till you haue practiſed this ſo well, that you be full perfect, ſo that you neede not to doubt in any point that I haue taught you, and then maye I boldlye inſtruct you in the arte of Fractions or Broken number: wherein I will alſo ſhew you the reaſons of al that you haue now learned. But yet before I make an end, I will ſhew you the order of common caſting, wherin are both pennies, ſhillings, and poundes, proceeding by no grounded reaſon, but onelye by a receyued forme, and that diuerſlye of diuerſe men: for the Marchantes vſe one forme, and Auditours an other.



BUt firſt for Marchantes fourme, marke this example here, in whiche I haue expreſſed this ſum 198 ℔. 19 ß 11 d. So that you maye ſee that the loweſt line ſerueth for pennies, the nexte aboue for ſhillings, the third for pounds, and the fourth for ſcores of poundes.

And further you may ſee, that the ſpace betweene d. and ß may receiue but one counter (as al other ſpaces likewiſe do) and that one ſtandeth in that place for 6 d.

Likewiſe betweene the ſhillinges and the poundes, one counter ſtandeth for 10 ß.

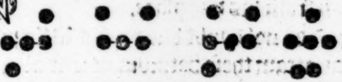
And betweene the poundes and 20 ℔. one counter ſtandeth for 10 ℔.

But beſide thoſe you maye ſee at the lefte ſide of ſhillinges, that one counter ſtandeth alone, and betokeneth 5 ß.

So agaynſt the poundes, that one counter ſtandeth for 5 ℔. And againſt the 20 pounds, the one counter ſtandeth for 5 ſcore pounds, that is 100 pounde, ſo that euery ſide cou̅ter is 5 times ſo muche as one of them againſte which he ſtandeth.

Auditours Accompt.

NOwe for the accompte of Auditours, take this example.



Auditours accompt.

Where I haue expreſſed the ſame ſumme 198 ℔ 19 ſ 11 d.

But here you ſee the pence ſtand towardes the right hande, and the other increaſing orderlie toward the left hande.

Againe you maye ſee that Auditours wyll make 2 lines (yea and more) for pence, ſhillings, and all other values, if their ſummes extende thereto. Alſo you ſee that they ſet one counter at the right ende of each rowe, which ſo ſet there, ſtandeth for 5 of that rowe: and on the lefte corner of the rowe it ſtandeth for 10 of the ſame row.

But now if you would adde other ſubtract after any of both thoſe ſortes, if you marke the order of the other ſeate which I taught you, you may eaſilie doe ẏ ſame here without

much

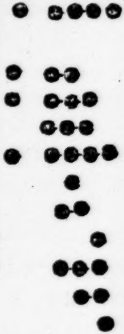
muche teaching : for in Addition you muſte firſte ſette down one ſumme, and to the ſame ſet the other orderlie, and in like maner if you haue manye : but in Subtraction you muſte ſet downe firſt the greateſt ſumme, and from it muſte you abate the other, euerie denomination from his due place.

Sc. I do not doubt but with a little practiſe I ſhal attain theſe both: but how ſhal I multiplie and diuide after theſe formes?

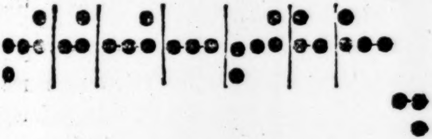
M. You can not duelie doe anye of both by theſe ſortes, therefore in ſuch caſe you muſte reſort to your other artes.

S. Sir, yet I ſee not by theſe ſortes howe to expreſſe hundreds, if they exceede one hundred, neither yet thouſands.

Maiſter. They that vſe ſuche accomptes that it exceede 200 in one ſumme, they ſette not 5 at the left hande of the ſcores of pounds, but they ſet all the hundreds in an other farther rowe, and 500 at the lefte hande thereof, and the thouſandes they ſet in a farther rowe yet, and at the lefte ſide thereof they ſette the 5000, and in the ſpace ouer they ſette the 10000, and in a higher rowe 20000, whiche all I haue expreſſed in this example, whyche is 97869 ℔, 12 ß, 9 d, ob, q̃. Ninetye ſeauen thouſande,eighte hundred, thre ſcore & nyne pounde,twelue ſhillings and nine pence halfpeny farthing, for I had not tolde you before, where,neither how you ſhould ſet down farthings,which(as you ſee here) muſt be ſette in a voide ſpace ſideling beneath the pens:for q̄ one counter,for ob,2 counters:for ob,q̄,3 counters:and more there can not be: for 4 farthings make 1 d, which muſt be ſet in his due place.



And if you deſire the ſame ſumme after Auditours maner:Lo here it is.



But in this thing you ſhall take this for ſufficient,and the reſt you ſhal obſerue as you may ſee by the working of ech ſort: for the diuerſe wits of men haue inuented diuerſe and ſundrie wayes,almoſt vnnumerable.

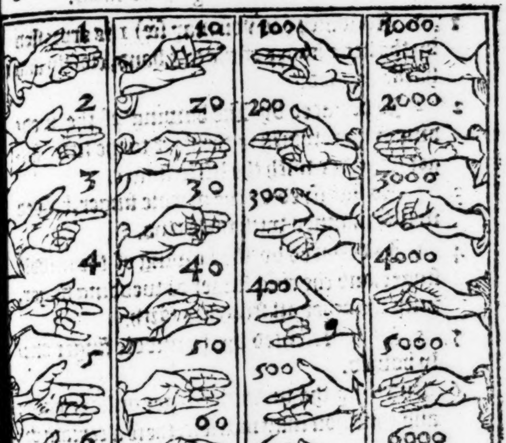
THE ARTE OF NVMbring on the hande.

But one feate I ſhall teach you, which not onely for ẏ ſtrangeneſſe and ſecreteneſſe is muche pleaſant, but alſo for the good commoditie of it, right worthy to be wel marked.

This feate hath bin vſed aboue 2000 yeres at the leaſt, and yet was it neuer commonlie knowen, eſpeciallie in Engliſhe it was neuer taught yet. This is the arte of nũbring on the hand, with diuerſe geſtures of ẏ fingers, expreſſing anye ſum conceiued in the minde. And firſt to beginne.

If you will expreſſe anye ſum vnder 100, you ſhall expreſſe it with your left hand: and from 100 vnto 1000, you ſhal expreſſe it with your right hand, as here orderly by this Table folowing you maye perceiue.

Here followeth the Table of the Art of the hande.


1
2
3
4
5
6
7
8
9
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000

1 1 In which (as you may ſee) 1 is expreſſed by the little finger of the left hand, cloſely and harde crooked.

2 2 Is declared by like bowing of the wedding finger (which is the next to the little finger) togither with the little finger.

3 3 Is ſignifyed by the middle finger, bowed in like manner with theſe two.

4 4 Is declared by the bowing of the middle finger, and the ring finger or wedding finger, with the other all ſtretched forth.

5 5 Is repreſented by the middle finger onely bowed.

6 And 6 by the wedding finger onlie crooked: and thus you may mark in theſe a certain order. But now 7, 8, and 9, are expreſſed wyth the bowing of the ſame fingers, as are 1, 2, & 3, but after another forme.

7 For 7 is declared by the bowing of ẏ little finger as is 1, ſaue that for 1 the finger is claſped in, hard and round, but for to expreſſe 7, you ſhall bow the middle ioynt of the little finger onelye, and holds the other ioyntes ſtraight.

Sch. If you wil giue me leaue to expreſſe it after my rude maner, thus I vnderſtād your meaning: that one is expreſſed by crooking in the

the little finger,like the head of a Biſhoppes bagle : and 7 is declared by the ſame finger bowed like a gibbet.

M So I perceiue you vnderſtand it.

Then to expreſſe 8, you ſhall bowe after 8
the ſame maner both the little finger,and the
ring finger.

And if you bowe likewiſe with them the 9
middle finger,then doth it betoken 9.

Nowe to expreſſe 10, you ſhal bowe your 10
forefinger rounde, and ſet the end of it on the
higheſt iointe of the thumbe.

And for to expreſſe 20, you muſte ſet youre 20
fingers ſtraight,and the end of your thumb to
the partition of the formoſt & middle finger.

30 Is repreſented by the ioyning togither 30
of the heads of the foremoſt finger & the thũb.

40 Is declared by ſetting of the thumbe 40
croſſewaies on the formoſt finger.

50 Is ſignified by right ſtretching forth of 50
the fingers iointly and applying of yͤ thumbs
ende to the partition of the middle finger,and
the ring finger or wedding finger.

60 Is formed by bending of the thumbe 60
crooked,and croſſing it with the forefinger.

70 Is expreſſed by the bowing of the fore- 70
moſt finger, and ſetting the ende of the thũbe
be-

betwéene the 2 formoſt or higheſt ioints of it.

80 80 Is expreſſed by ſetting of the foremoſte finger croſſewayes on the thumbe, ſo that 80 differeth thus frō 40: for that 80, the forefinger is ſet croſſewayes on the thumbe and for 40 the thumb is ſet croſſe ouer the forefinger.

90 90 Is ſignified by bending the forefinger, and ſetting the end of it in yͤ innermoſt ioint of the thumbe, that is euen at the foote of it. And thus are al the nūbers ended vnder 100.

Sc. In déed theſe be al the numbers from 1 to 10, & then all the tenthes within 100, but

11, 12, this teacheth me not how to expreſſe 11, 12,

13, 21, 13, &c. 21, 22, 23, &c. and ſuch like.

22, 23, M. You can little vnderſtand, if you cā not doe that without teaching. What is 11? is it not 10 and 1? then expreſſe 10 as you were taught and 1 alſo, that is 11: and for 12 expreſſe 10 and 2: for 22, ſet 20 and 2: and ſo for 68, you muſt make 60, and thereto 8: and ſo of al other ſortes.

But now if you wold repreſent 100, either any number aboue it, you muſt doe that with the right hand, after this maner.

100. You muſt expreſſe 100 in the righte hande with the little finger, ſo bowed as you did expreſſe 1 in the left hand.

And as you expreſſed 2 in the lefte hande,
the ſame faſhion in the right hande doeth
declare 200. 200

The fourme of 3 in the right hand ſtandeth
for 300. 300

The forme of 4 for 400. 400

Likewiſe the forme of 5, for 500. 500

The forme of 6, for 600. And to be ſhorte: 600
looke how you did expreſſe ſingle vnities and
tenthes in the left hande, ſo muſt you expreſſe
vnities and tenthes of hundreds, in the right
hand.

Scholer. I vnderſtande you thus: that if I
woulde repreſente 900, I muſte ſo forme the 900
fingers of my right hande to expreſſe 9. And
as in my lefte hande I expreſſe 10, ſo in my
right hand muſt I expreſſe a 1000. 1000

And ſo the forme of euerie tenth in the lefte
hand, ſerueth to expreſſe the number of thouſands,
ſo the ſumme of 40 ſtandeth for 4000. 4000

The ſumme of 80, for 8000. 8000

And the forme of 90 (which is the greateſt)
for 9000, and aboue that I can not expreſſe 9000
any number.

Maiſter. No, not with one finger, howe
be it, with diuerſe fingers you maye expreſſe
9999. and al at one time, & that lacketh but

1 of 10000. So that vnder ten thouſande you maye by your fingers expreſſe anye ſumme. And this ſhall ſuffiſe for Numeration on the fingers. And as for Addition, Subtraction, Multiplication, and Diuiſion (whiche yet were neuer taught by any man as farre as I doe knowe) I will inſtruct you after the treatiſe of Fractions: and now for this time fare well, and looke that you ceaſſe not to practiſe that you haue learned.

Sc. Sir, with moſt hartie minde I thanke you, both for your good learning and alſo your good counſel, which (God willing) I truſte to follow.

FINIS.

¶THE SECONDE part of the Arithmetike touching Fractions, brieflie ſet forth.

Scholer.

Albeit I perceiue your manifolde buſineſſe doth ſo occupie, or rather oppreſſe you, that you can not as yet cõpletelie end that treatiſe of Fractions Arithmeticall, whyche you haue prepared, wherein not onelie ſundrie workes of Geometrie, Muſicke, and Aſtronomie be largelie ſet forthe, but alſo diuers concluſions and naturall workes, touching mixtures of metals, and compoſitions of medicines, with other ſtraunge examples, yet in the mean ſeaſon I can not ſtay my earneſt deſire , but importunelie craue of you ſome briefe preparation towarde the vſe of Fractions, whereby at the leaſte I maye be able to vnderſtande the common workes of them,and the vulgare vſe of thoſe rules,whiche without them can not wel be wrought.

Maiſter. If my leaſure were as great as my will is good,you ſhoulde not neede to vſe anye importunate crauing, for yͤ attaining of that thing, whereby I maye be perſwaded that I ſhal anye waies profite the common wealth, or helpe the honeſt ſtudies of anye good members in the ſame : wherefore, while myne attendaunce will permitte me to walke and talke , I am well willing to helpe you as I may.

VVhat a Fraction is.

Therefore firſte to beginne with explication of this name Fraction, what take you it to be?

Scholer. Marie ſir, I thinke a Fraction (as I haue heard it often named) to be a broken number, that is to ſay, to be no whole nūber, but a part of a number.

Mayſter A Fraction in deede is a broken number, and ſo conſequentlie, the part of another number : but that muſte be vnderſtanded of ſuche an other number, as can not be diuided into any other partes thã Fractions: for although I may take the third part of 60, or the fourth part of it, and ſo of other partes diuerſlie, yet theſe partes be not properlie, nor ought not to be called Fractions, bycauſe they maye be expreſſed by whole numbers: for the third part of it is 20: the fourth part is 15: the twelfth part is 5, and ſo forth of other parts, which all be whole numbers.

VVhat a Fraction is properlie.

Wherefore properlie a Fraction expreſſeth the partes or part onelie of an vnit, that is to ſaye, that the number which is the whole or entire ſumme of anye Fraction, may not be greater than one: and therefore it followeth, that no one Fraction alone can be ſo gret, that it ſhall make 1, as by examples I will declare as ſoone as I haue taught you to knowe the forme howe a Fraction is expreſſed or repreſented in writing.

NVMERATION.

BUt firſt to begin with the expreſſing of a Fraction, which is the numeration of it, you muſt vnderſtand that a Fraction is repreſented by 2 numbers, ſet one ouer the other, and a line drawen betwéene them as thus, $\frac{1}{3}$. $\frac{3}{4}$. $\frac{4}{5}$ $\frac{10}{17}$ which foure Fractions you muſte pronounce thus: $\frac{1}{3}$ one thirde parte: $\frac{3}{4}$ thrée quarters: $\frac{2}{5}$, two fifte partes: $\frac{10}{17}$, tenne ſeauentéene partes

Scho. I vnderſtande the forme of theyr expreſſion and pronunciation, but their meaning or valuation ſéemeth more obſcure: yet I think that by the two firſt Fractions I vnderſtand the valuation of the two later Fractions, and ſo conſequentlie of other.

M. Value them then, that I may perceyue your taking of them

Scholer. $\frac{2}{5}$ betokeneth two fifte partes, that is to ſay, if one be diuided into 5 parts, that Fraction doeth expreſſe ij. of thoſe fifthe

parts:

partes: $\frac{10}{17}$ doth ſignifie, that if one be diuided into xvij. partes, I muſt take tenne of them. And this I gather of the two firſte examples: for $\frac{1}{3}$, that is one thirde parte, doth eaſily declare, that if anye one thing be diuided into three partes, I muſte take but one of them: ſo $\frac{3}{4}$ that is three quarters, doeth declare that one being diuided into four quarters, I muſt take (for this Fractiō) three of thoſe quarters.

If there be no more difficultie in their Numeration, thē I pray you go forward to their Addition and Subtraction, and ſo to the other kinds of workes for I vnderſtande that the ſame kinds of workes be in Fractions, that be in whole numbers.

Maiſter. There are the ſame kyndes of workes in bothe, albeit the order of them is diuerſe, as I will anone declare: but yet more in Numeration before we leaue it. You muſte vnderſtande, that thoſe two numbers whiche expreſſe a Fraction, haue ſeuerall names. The ouermoſt whiche is aboue the line, is called the Numerator, and the other beneath the lyne, is called the Denominator.

Numerator, and Denominator.

Scholer. And what is the reaſon of theyr diuerſe names? For in mine opinion both bee

Numeratoꝛs, ſeeing both they do expꝛeſſe the numeration of the Fraction.

Maiſter. You are deceiued: foꝛ one onelye (whiche is the ouermoſte) doeth expꝛeſſe the Numeration: and the denominatour doeth declare the number of partes into whiche the vnit is diuided, as in this example, when I ſay: Diuide a pounde weighte of Golde betweene foure men, ſo that the firſte man ſhal haue $\frac{2}{15}$ the ſeconde $\frac{3}{15}$ the thirde $\frac{4}{15}$, and the fourth $\frac{6}{15}$.

Now do you perceiue yͤ by the denominatoꝛ (whiche is one in al foure Fractions) it is intended, that the pounde waight ſhoulde be diuided into ſo manye partes, I meane 15, and by the foure ſeuerall numeratoꝛs is limitted the diuerſe poꝛtion that each man ſhold haue, that is, that whē the whole is parted into 15, the firſte man ſhall haue 2 of thoſe 15 partes: the ſecond man thꝛee of them: the third man 4: and the fourth man 6. And ſo may you ſee yͤ ſeueral offices (as it were) of thoſe two numbers, I meane of the Numeratoꝛ and the denominatoꝛ.

And hereby you perceiue, that a man can haue no moꝛe parts of any thing than it was diuided into, nether yet aptlie ſo many: ſo that it were vnaptly ſayd: You ſhall haue $\frac{15}{15}$. that is xv fifteene partes of any thing, ſeeing it were better ſayde: You ſhal haue the whole thing.

Sc. So doth it appeare reaſonablye: for the labour is vaine, to diuide anye thing, and than to applie the Diuiſion to no vſe. And much leſſe reaſonable were it to ſay $\frac{16}{15}$: for if the whole be diuided into 15 parts only, it is not poſſible to take 16 of them, that is to ſay, more than altogither.

Maiſter. This is true touching the proper and apte vſe of the name of a Fraction: yet improperlye, and after a vulgare acceptation (for eaſineſſe in worke) both thoſe formes be called Fractions, becauſe they be writtẽ like fractions, although they be none in deede for $\frac{15}{15}$, and generally all ſuche other: where the Numerator and Denominator be equal, are not Fractions: but the whole thing, with all his partes. And ſo $\frac{16}{12}$ is not to be called a fraction, but a mixt number, of a whole number and a Fraction: for it is as muche, as 1 $\frac{4}{12}$, that is one whole one, and 4 twelue partes, as ſhall be declared in Reduction. Therefore they doe abuſe the names, that call them Fractions, where the Numerator is either equall or greater than the Denominator.

Sc. But is there any needefull cauſe why they ſhould ſo abuſe the name?

Mai. There is cauſe why they ſhal ſometimes, for eaſineſſe in worke, write ſome nũbers after that ſorte, like fractions: but they needed not to call them fractions, but as they be whole numbers or mixt numbers (that is whole numbers with Fractions) expreſſed like fractions.

Now muſt you vnderſtande; that as no fraction properly can be greater than 1, ſo in ſmalneſſe vnder one the nature of Fractions doeth extende infinitelye: as the nature of whole numbers is to increaſe aboue one infinitelye, ſo that not onely one, may be diuided into infinite Fractions or partes, but alſo euerye Fraction maye be diuided into infinite Fractions or partes, whiche commonlye bee called Fractions of Fractions, and they be expreſſed diuerſlye: As for example, $\frac{3}{4} . \frac{2}{3} . \frac{1}{2}$, that is three quarters of two third partes, of one halfe parte. Whereby is ſignifyed, that if one be diuided into two halfes, and the one halfe into three partes, and two of thoſe thrée partes, be diuided ioyntlye into foure quarters, this Fraction of Fractions doeth repreſente thrée of thoſe quarters.

Scholer. I praye you lette me proue by an example in common money, whether I doe rightlye vnderſtand you or no. One Crowne, which I take for an vnit, doth contains 60 pennies, therefore the halfe of it is thirtye pence: $\frac{2}{3}$ of that halfe is 20 pence, whereof $\frac{3}{4}$ is fiftéene pence, ſo then 15 pence is $\frac{3}{4}.\frac{2}{3}.\frac{1}{2}$ of a Crowne. And ſo 3 pence is $\frac{3}{4}.\frac{2}{3}.\frac{1}{2}.$ of a ſhilling.

Mayſter. You perceyue this well ynough, but how happened that you founde no doubte in the forme of writing theſe Fractions, ſéeing the two latter Fractions haue no line betwéene their numbers, as the firſt hath?

Sc. Becauſe I had forgotten (as Scholers oft times doe) that that was tolde me before: but I praye you, expreſſe the reaſon thereof.

Mai. This forme is but voluntarye, and therefore hath none other reaſon than the will of the diuiſer, which forme many do follow. Some other doe make lines betwéene euery

uery Fraction, and adde wordes of diſtinction, after this ſorte, $\frac{3}{4}$ of $\frac{2}{3}$, of $\frac{1}{2}$, which forme is good alſo.

Some other expreſſe them thus in ſlope forme, to diſtincte them from ſeuerall Fractions of one whole number, for if they were ☞ ſet in one right line thus, $\frac{3}{4}$ $\frac{2}{3}$ $\frac{1}{2}$. then oughte it to be pronounced, thrée quarters, and two thirde partes and an halfe, which maketh almoſt two whole vnits, lacking but one xij. part. And ſo is it nothing agréeable with the other Fraction of Fractions, wherefore it is a great ouerſight in certayne learned men, which doe expreſſe them ſo confuſedly with ſuch ſeuerall Fractions, that a man can not know the one from the other.

$\frac{3}{4}$
$\frac{2}{3}$
$\frac{1}{2}$

Therefore ſome men (as Stifelius) doe expreſſe without a line numbers of proportion, being applied to Addition or Subtraction: becauſe they muſt be takẽ as two, where the line in Fractions maketh them to be taken for one: for of the Numeratour and Denominatour is made one number.

Scholer. Then I perceiue there be thrée ſeuerall varieties in Fractions: Firſt when one only Fractiõ is ſet for one nũber, as $\frac{4}{5}$, yͭ is

Three ſeuerall varieties.

is foure fifth parts. The ſecond, is whē there be ſet two or more ſeuerall Fractions of one number, as $\frac{4}{9}$ $\frac{2}{5}$, that is iiij. ninth partes, and two fift parts. The third ſort is Fractions of Fractions, as $\frac{4}{9}$ $\frac{2}{5}$, that is 4 ninth parts of two fifth parts.

Ma. You haue ſaid well, if you vnderſtand well your own wordes.

Scho. If it ſhall pleaſe you, I will by an example in the parts of an old Engliſhe Angell expreſſe my meaning.

Mai. Let me heare you.

Scholer. The olde Engliſhe Angell did containe 7 ſhillings 6 d, that is 90 d. Nowe $\frac{4}{5}$ of it, is 72 d. And of the ſame 90 pence, if I take $\frac{4}{9}$ and $\frac{2}{5}$, that is foure ninth partes, and 2 fifth partes, $\frac{4}{9}$ is 40, and $\frac{2}{5}$ is 36, which both make 76: but if I take $\frac{4}{9}$ of $\frac{2}{5}$, that is foure ninth partes of two fift partes, ſeeing $\frac{2}{5}$ is but 36, then $\frac{4}{9}$ of 36 will yield but 16: for $\frac{1}{9}$ of 36, is but 4, and that taken foure times maketh 16.

Maiſter. This is plainely expreſſed, and truely and hereby (I doubte not) but you doe perceiue, ẏ as great a differēce as is betweene 16 & 76, ſo much differēce is betweene theſe two Fractions $\frac{4}{9}$ and $\frac{2}{5}$: and $\frac{4}{9}$ of $\frac{2}{5}$.

And now that you vnderſtande theſe varieties, I will procéede to the reſt of the workes: firſt admoniſhing you, that there is an other order to be followed in Fractions than there was in whole numbers, for in whole numbers this was the order: Numeration, Addition, Subtraction, Multiplication, Diuiſion, and Reduction, but in Fractions (to followe the ſame aptneſſe in procéeding from the eaſieſt workes to the harder) we muſt vſe this order of the workes: Numeration, Multiplication, Diuiſion, Reduction, Addition, and Subtraction.

Scho. That Multiplication and Diuiſion ſhoulde goe togither, and Subtraction to followe Addition, naturall order doeth perſwade: but why Multiplication ſhoulde be firſt in order here next to Numeration, and Reduction in the middle, I deſire to vnderſtand the reaſon

May. As in the Arte of whole numbers order woulde reaſonablye beginne with the eaſieſt, and ſo goe forwarde by degrées to the hardeſt, euen ſo reaſon teacheth in Fractions the like order. And conſidering that Addition or Subtraction of Fractions can very ſeldom be wrought without multiplication and Reduction:

duction : and contrariwayes, Multiplication and Reductiō may be wrought without this forme of Addition or Subtraction. Therfore was it orderly required, that Multiplication and Reduction ſhoulde goe before Addition and Subtraction. And the ſame reaſon ſerueth for the placing of Multiplication before Reduction.

Sch. Then if Multiplication be the eaſieſt, I pray you declare the forme of it firſt by rule, and then by example.

May. Your example is good.

MVLTIPLICATION.

Therefore when anye two Fractions be proponed to be multiplied togither, yͤ Numerator of the one muſt be multiplied by the Numerator of the other : and the ſumme that amounteth therof, muſt be ſet for a new numerator : likewiſe the Denominatour of the one muſt be multiplied by the Denomi-

nator

maker of the other, and that that amounteth, ſhal be ſet for the common Denominator: & this new third Fraction expreſſeth the Product of the multiplication of the two firſt fractions proponed, whereof take this example, $\frac{3}{5}$ multiplied by $\frac{5}{12}$, doth make $\frac{15}{60}$.

Scholer. I perceiue then, that 3 being the Numeratour of the firſt Fraction, is multiplied by 5, being the Numeratour of the ſeconde Fraction, whereof amounteth 15, the Numeratour of the thirde Fraction. And ſo likewiſe, 5 being Denominatour of the firſt Fraction, is multiplied by 12 the Denominator of the ſecond fraction, whereof amounteth 60 the newe Denominator: ſo that I perceiue howe the worke is done, but I doe not perceiue howe $\frac{15}{60}$ is greater than $\frac{3}{5}$: For if I ſhall vſe my former maner of examination by the partes of ſome Coyne, I ſee that $\frac{3}{5}$ of a Crowne is 36 d., and $\frac{5}{12}$ of a Crowne, is 25 d., whereof the one multiplyed by the other, doth make 900 d., whiche is 15 Crownes: but by your multiplication there amounteth $\frac{15}{60}$, which is but 15 d., and that is much leſſe then anye of both the firſte Fractions.

Mayſt. That difference is betwéen multiplication in whole numbers, and multiplication in broken numbers that in whole nūbers the ſumme that amounteth, is greater than both the other whereof it came: but in Fractions it is contrary waies: for the ſum that amounteth is leſſer thā any of the other two fractions, whereof it came.

Sc. I deſire much to vnderſtande the reaſon thereof.

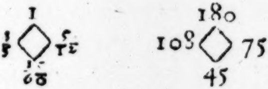
Ma. Although I purpoſed to reſerue the reaſons of workes Arithmeticall for the perfecte Booke of Arithmetike, yet I will ſhew you this, becauſe of the ſtraungeneſſe of the worke.

You ſee in whole numbers, that of two numbers being multiplied together, is made the third number: which third number doth beare the ſame proportion to the nūber multiplyed, that the multiplier doth beare to an vnit. And ſo in Fractions, the third number which amounteth of multiplication, beareth the ſame proportion to eache of the two firſte fractions, that the other of thoſe two fractiōs doth beare to an vnit.

Scholer. Sir I vnderſtande your wordes thus: when 40 is multiplied by 12, there doth amount 480, whiche 480 doeth contayne 40 ſo many times in it, as twelue doth containe vnits, that is to ſay: twelue times. And ſo it appeareth. that 480 doeth contain twelue ſo many times alſo, as 40 doeth containe Vnites, that is 40 times. But now I ſee not how the thirde number in this exāple of Fractions can contain any of the two former (as it happened in whole nūbers) ſeeing it is leſſer than eyther of them.

Mai. No maruaile if you cannot ſee that thing which is not poſſible to be ſeene of anye man, how the third number in multiplication of Fractions ſhould be greater than anye of the two former Fractions, but yet this may you ſee (which I ſayde) that the thirde number in Fractions ſo multiplyed, doeth beare the ſame proportion to any of the two former fractions, that the other of thoſe 2 fractions doth beare to an vnite, as in your example $\frac{3}{5}$ being multiplied by $\frac{5}{12}$, doeth make $\frac{15}{60}$. Now ſaye I, that $\frac{15}{60}$ doeth beare the ſame proportion to $\frac{3}{5}$, that $\frac{5}{12}$ doth beare to an vnit, as you may in your owne forme of examination by coyne trie it. For in an olde Angell are 180 halfe pence, whiche I ſet for the intire vnit whoſe parts (according to the Fractions aforeſayde) are theſe, for $\frac{15}{60}$ ſette 45 ob.

for $\frac{3}{5}$ take 108 *ob* and for $\frac{5}{12}$ put 75 *ob* Nowe doeth 45 beare the ſame proportion to 108 that 75 doeth beare to 180: for 45 is $\frac{5}{12}$ of 108, and ſo is 75 alſo $\frac{5}{12}$ of 180. And for eaſier applying of ech compariſon, conſider this forme of ſetting all theſe nūbers before your eyes, where the ſecōd demōſtration towards your right hand is anſwerable to the firſte in euerye proponed part, where for $\frac{3}{5}$ (of 180) ſtands his value 108: for $\frac{5}{12}$ ſtands 75: and for $\frac{15}{60}$ is 45.



But theſe reaſons may be better reſerued till another time, whē the knowledge of proportions in due order ſhall be taught. Yet in the meane ſeaſon I will ſhewe you howe it commeth to paſſe that in Fractions the third ſumme muſt needes be leſſer then any of the other two.

Conſider thus, that when a Fraction is proponed, as in the former example $\frac{3}{5}$, if it bee multiplied by more thā 1, it will make more than one entire number. As if I multiplie $\frac{3}{5}$ by 5, that is to ſaye, if I take it 5 tymes, it will make thrée entire vnits: example in a Crowne, $\frac{3}{5}$ of it maketh 3 ß, which if I take fiue times, it will amounte to 15 ſhillinges, that is thrée intire Crownes: ſo if I take the ſame $\frac{3}{5}$ but twice, it will yielde 6 ß, that is one entire Crowne and $\frac{1}{5}$. Now if I take it but once, it cannot be more thã it was before, ☞ that is, 3 ß. And if I take it leſſe then once, it cannot be ſo much as it was before. Then ſéeing that a Fraction is leſſe than one, if I multiplie a Fraction by another Fraction, it followeth that I doe take that firſt Fraction leſſe than once, and therfore the ſumme that amounteth, muſt néedes be leſſe than the firſt fraction.

Sc. Sir, I thanke you much for this reaſon, And I truſte I doe perceiue the thing, as by example of this ſame Fraction $\frac{3}{5}$ I wil expreſſe. If I take $\frac{3}{5}$ of a Crowne once, that is to ſay, if I multiplie $\frac{3}{5}$ by 1, it will be as it was before, but 3 ß: ſo if I doe multiplie it by $\frac{1}{2}$, that is, if I take it but half one time, then will it be but halfe ſo muche: likewiſe if I multiplie it by $\frac{1}{3}$, that is, if I take but the thirde part of once, it will yielde but 12 pence, that is the thirde part of the firſt Fraction.

And ſo to make an ende. If I take it but the twelfth part of once, that is, if I doe multiplie it by $\frac{1}{12}$, it will yielde but the twelfth part of the firſte Fraction, which is but three pence. And it followeth that if $\frac{1}{12}$ make 3 pence, then $\frac{5}{12}$ muſt needes make fiue tymes ſo muche, that is 15 pence, which was the ſumme that hath giuen the occaſion of all this doubt.

Maiſter. Then I perceiue you haue ſufficient vnderſtanding in this ſorte of multiplication for this time, wherefore I will omitte that I might ſaye more of Multiplication, till we come to reduction, and will paſſe to the other workes, and firſte to Diuiſion, whoſe place followeth Multiplication, both by naturall order, and alſo in eaſineſſe of worke.

DIVISION.

When ſo euer two fractions bee proponed, that the one ſhoulde be diuided by the other, I muſte ſette downe firſte the Fraction that ſhall be diuided (which is called the Diuidend) and then after it the other, whiche is the Diuiſor. Then ſhall I multiplie the numerator of the diuidend by the denominator of the diuiſour, and that which amounteth, I muſt put for a new numerator. Agayne, I ſhall multiplie the denominator of the diuidend by the numerator of the diuiſor, and the number that amounteth thereof, I muſt put for the new denominator. And this third fraction is the quotient of the ſayd diuiſion.

Scholer. This ſeemeth eaſie in forme, as by example, thus: If I would diuide $\frac{5}{8}$ by $\frac{2}{6}$, firſt I muſt multiplie 5 (being the numerator of the diuidend) by 6, whiche is the denominator of the Diuiſor, and thereof riſeth 30: then I multiplie 8 (being the denominator of the diuidend) by 2, being Numerator in the diuiſor, and ſo riſeth 16, the whiche I muſte make in a thirde Fraction, thus $\frac{30}{16}$.

Ma. Me ſéemeth you are quicker in vnderſtanding nowe, than you were when I taught you the arte of whole numbers: but that is no maruell, for the more knowledge that any man getteth, the readier ſhal he find his wit, and quicker in vnderſtanding: but yet of 2 thinges I will admoniſh you, which you mighte haue obſerued here for eaſe of worke and lightneſſe of vnderſtanding the nature of the Quotient.

Whenſouer you diuide one Fraction by an other, either they be both equall togither eyther elſe the one is greater than the other: if they be equall, their quotient ſhal be ſuch, that the numerator and the denominator of it ſhall be equall alſo. And if ẏ 2 firſte fractions be vnequall, their quotient ſhall declare the ſame by the vnequalitie of the numeratour and denominatour, as in theſe examples following ſhall appeare.

Firſt of equall Fractions: $\frac{4}{9}$ and $\frac{12}{27}$ bée equall togither; and if the one be diuided by the other, the quotient will be $\frac{108}{108}$, as you may perceiue by that rule aforeſayde.

Now in the vnequal Fractions, as $\frac{4}{9}$ and $\frac{3}{10}$ the quotient wil be $\frac{40}{27}$: where the Numerator is greater than the denominator.

Sch. I ſée it is ſo, but I ſée not the reaſon why it ſhould be ſo.

Note hovve to knovve the proportiō betvveen 2 numbers.

Maſter. The reaſon is this, when anye Fraction is diuided by an other, the quotient declareth what proportion the diuidend beareth to the diuiſor. So $\frac{1}{2}$ diuided by $\frac{1}{4}$, maketh 2, which muſt be ſounded, not two, but twice; declaring that $\frac{1}{4}$ is contained twice in $\frac{1}{2}$.

And note this, that the Numerator in the Quotient, repreſenteth the Diuidend, & the Denominator repreſenteth the diuiſor. And this is alwayes true, whether the greater fraction be diuided by the leſſer, or the leſſer by the greater. But this propoſition will not be exactly knowne, till you haue learned the art of proportions: notwithſtāding ſom what of it will I declare in the next rule of Reduction. But now for the eaſie remembrance of the Quotiēt in diuiſion, as ſoone as you haue ſet downe your two Fractions, the one againſte the other, then make a ſtreighte line for the quotiente: and as ſoone as you haue multiplyed the Numerator of the diuidende, by the Denominator of the diuiſor, ſette the number that amounteth, ouer the ſaid line, and then multiply the other two numbers, & ſet their totall vnder the ſame line.

Scholer. I perceiue you woulde not haue me truſt to memorie til I wer better expert, leaſt oftentimes I happen by miſſe remembraunce to be abuſed. This example I take for that declaration.

If I woulde diuide $\frac{2}{3}$ by $\frac{3}{4}$ I muſt ſet the numbers one againſt the other, $\frac{2}{3}$ by $\frac{3}{4}$ (as here vnder appeare) & then make an other line for the Quotient in ſome good diſtaunce, where I maye ſet the numbers of the Quotient, as ſoone as any of them is multiplied: So then as ſoone as I haue multiplied 2 by 4, which maketh 8, I ſhal ſet that 8 ouer that line thus. $\frac{8}{}$ And then multiplye 3 by 3, whiche yeeldeth 9: and that 9 muſte I ſet vnder the ſame line, and then will the whole quotient appeare thus $\frac{8}{9}$. Whereby appeareth (as I remember your wordes) that $\frac{2}{3}$ is in propor- Note.

tion to $\frac{3}{4}$, as 8 is to 9: but howe maye I perceiue that?

Mayſter. Althoughe you ſhall better perceiue it by the rule of Reduction, yet this example may be declared in common coine, as in a common ſhilling of xij. pence, of whiche $\frac{2}{3}$ maketh 8 d., and $\frac{3}{4}$ dothe make 9 pence, and ſo you maye eaſily ſee that their proportions doe agree. And if you had taken this example before, when you toke the example of , and $\frac{5}{6}$, your Quotiēt would appere (as this doth) muche eaſier to underſtande, whereas that Quotient being $\frac{18}{16}$, is not an eaſie proportion for you to perceiue, being yet little acquainted with proportions: whereof to giue you ſome taſte, I will enter to the rule of Reduction; in which alſo I wil declare other workes, both of Multiplication and alſo of Diuiſion, whiche nowe I muſte for a time omitte, as things that do neede the help of Reduction.

REDV-

REDVCTION.

Therefore wil I nowe declare the diuerſities of Reduction of Fractions, whiche commonly haue fiue varieties.

Fiue varieties of Reduction.

1. Firſt, when there be, ſundrie Fractions of one entire Unitie, they muſte be reduced to one denomination, and alſo into one Fraction.

2 Secondarily, when ther be proponed fractions of fractions, they muſt be reduced likewiſe into one Fraction, for other wayes they can not bée brought into one Denomination.

3 Thirdly, when an Improper fraction is proponed, that is to ſay, a fraction in forme, which indeed is greater thā an Unitie, it muſt be reduced into apt form, expreſſing ẏ Unitie or Unities of it, & the proper fraction diſtinctly. And ſome times alſo it ſhall be needefull to conuerte ſuch a mixte number of Unities,

with Fractions into the forme of a Fraction, that is into an Improper Fraction, which 2 formes I eſteeme but as one, bicauſe they worke on one kind of number.

4 Fourthlie, there happeneth ſometimes Fractions to be written in greate numbers, whiche mighte be written in leſſer numbers, therefore is there a meane to reduce ſuche greate numbers into their ſmalleſt tearmes.

5 Fiftelie, when anye Fraction betokeneth the partes of a whole thing, whyche hath by common partition certaine partes, but none of like Denomination wyth that Fraction, then maye you reduce the ſayde Fraction into an other, whoſe Denomination ſhall expreſſe the common partes of that whole thing.

Scholer. Thys diſtinction in doctrine delighteth me much, but more with hope than preſente fruite, for as yet I doe not vnderſtande ſcarſelie the varieties, and much leſſe the practiſe and vſe of their workes.

Mayſter. Reduction is an orderlie alteration of numbers out of one forme into an other, whiche is neuer done orderlye but for ſome needefull vſe, as in euery of the ſaide 5 ſeuerall varieties I wil diſtinctly declare.

The firſt ſort of Reduction.

Firſte therfore, when two or more ſeuerall Fractions of anye Unite be proponed, as for example, $\frac{3}{1}$ and $\frac{4}{6}$: becauſe it is harde to tell what proportion of the entier number thoſe two Fractions doe expreſſe, therefore was Reduction diuiſed, to bée a meane whereby theſe ſeuerall Fractions mighte be broughte into one Denomination and Fraction.

And in theſe Fractions this is the arte for bringing them to one denomination.

Hovv to reduce fractions of diuerſe denominations into one Denomination.

Multiplie firſte the Denominators togither, and the totall thereof you ſhall ſette twice downe vnder two ſeuerall lines for two newe Denominators, or rather for one common Denominator: Then multiplye the Numeratour of the firſte Fraction, by the Denominator of the ſeconde, and ſette the totall thereof for the Numeratour ouer the firſte line. Likewiſe multiplie the Numerator of the ſeconde Fraction by the Denominator of the firſt, and ſet that totall ouer the ſeconde line for the Numerator of that Fraction, and ſo are thoſe two firſt Fractions of ſeuerall denominations, brought to one De-

Denomination.

Sch. If I vnderſtand you, as I thinke I do, my example ſhal declare the ſame. The Fractions which you proponed were theſe, $\frac{3}{16}$ and $\frac{4}{6}$ whoſe Denominators (being 16 and 6) I multiplie togither, and there amounteth 96, which I ſet vnder lines, thus. $\frac{}{96}$: $\frac{}{96}$:
Then I multiplie the Numerator of the firſt Fraction by the Denominator of the ſecond, ſaying: 3 into 6 maketh 18, that ſet I ouer the firſt line for a newe Numeratour, and it wil be thus. $\frac{18}{96}$

Likewaies I multiplie the Numeratour of the ſecond Fraction by the Denominatour of the firſt: ſaying: 4 times 16 maketh 64, that I ſet for ẏ ſecond Numeratour, and the Fraction wil appeare thus. $\frac{64}{96}$
So that both Fractions broughte to one Denomination, muſt ſtand thus: $\frac{18}{96}$ and $\frac{6}{94}$

Ma. You haue done wel.

Scholer. I beſeech you, let me examine it after my accuſtomed forme, by common parts of coine.

Ma. Go to.

Scholer. A new Angell accompted at eight ſhillings, contayneth 96 pence, whereof $\frac{1}{16}$

that is the xvj. parte, is ſixe pence, and $\frac{3}{16}$ is 18 pence, that is $\frac{18}{96}$. Againe $\frac{1}{6}$ of the ſame Angel, is 16 pence, ſo that $\frac{4}{6}$ maketh 64 d, that is $\frac{64}{96}$. And ſo I finde the ſummes to agree with the other before.

Maiſter. So haue you nowe the Arte to bring ſuch two Fractions into one Denomination. And if there be more than ij, then muſt you multiplie al the Denominators togither, and ſet the totall therof ſo many times down as there be Fractions, and then to get for ech one a newe Numerator. Multiplie the Numerator of the firſte, by the Denominator of the ſecond, and the total therof multiplie by ẏ denominator of the third, and ſo foorth if there be more. Likewiſe multiplie the Numerator of the ſecond, by the denominator of the firſt, and the total thereof by the Denominator of the thirde. And in the ſame ſorte multiply the Numerator of the third into ẏ Denominator of the firſt: & the totall thereof into the Denominator of the ſeconde, and ſo foorth, if there were mo. So theſe 3 Fractions $\frac{2}{5}$ $\frac{3}{4}$ $\frac{2}{3}$ doeth make by Reduction theſe other 3. Fractions of one Denomination $\frac{24}{60}$ $\frac{45}{60}$ $\frac{40}{60}$. All whiche you may bring into one Fraction by adding

Note the Reduction of three Fractions or more to one.

adding the Numeratours togither, and putting that totall for the common Numerator, reſeruing ſtill that ſame common Denominator, And thoſe 3 Fractions make one Improper Fraction thus. $\frac{109}{60}$

Scho. All this I perceiue, and alſo that thys laſte Fraction is more than an Unitie, and therefore you did call it an Improper Fraction.

M There be certaine other formes of working in this reduction, whiche I will briefly touch alſo, to giue you an occaſion to exerciſe your wit therin.

The firſt varietie of this Reduction.

The firſte varietie is this. When you haue made and written downe youre common Denominator (as I haue taught before) then to get a Numerator for the firſt, do thus. Diuide the common Denominator by the Denominator of the firſt Fraction, and the quotiente multiplied by the Numerator of the ſame, yéeldeth a new Numerator for the firſt new Fraction. So likewiſe doe with the ſecond and the third, and with all the reſidue if there be more.

Scholer. That will I proue in your laſte example of theſe 3 Fractions $\frac{4}{5}$. $\frac{3}{4}$. $\frac{2}{3}$. When the Denominators be multiplyed, they make

60 foꝛ 5 into 4 maketh 20, and 20 by 3 yeldeth 60, that I ſette downe 3 times, thus. $\frac{}{60}$:$\frac{}{60}$:$\frac{}{60}$: then to haue a Numeratoꝛ foꝛ the firſte, I muſte diuide 60 by 5, (the Denominatoꝛ of the firſte) & the quotient is 12, which I muſte multiplye by 2 (the Numeratoꝛ of the firſte) & that maketh 24, and ſo haue I foꝛ the firſt Fraction $\frac{24}{60}$.

Likewyſe foꝛ the ſeconde fraction: I diuide 60 by 4, and there commeth 15, whiche I multiplye by 3, and ſo haue I 45 and the ſeconde fraction $\frac{45}{60}$. Then foꝛ the third in like ſoꝛt will come $\frac{40}{60}$.

The ſecond varietie.

Mayſter. An other way is this. If it happen ſo that the leſſer Denominatoꝛ can by any multiplication make the greater, thẽ note the multiplier, and by it multiplie ẏ Numeratour ouer that leſſer Denominatour, and foꝛ the leſſer Denominatour put the greater, as thus in theſe two Fractions, $\frac{3}{12}$ and $\frac{2}{3}$ thꝛee being the leſſer Denominatour multiplyed by 4, will make 12, whyche is the greater Denominatour: therefoꝛe by the ſame 4, I do multiplie 2, whiche is the Numeratour ouer 3, and that maketh 8: vnder which I doe put 12 being the greater Denominatour, whiche is alſo made by Multiplication of 4 into 3, ſo ſo haue I theſe ij. Fractiōs $\frac{5}{12}$, $\frac{8}{12}$: thus ſhortlye reduced without altering the one Fraction.

S. This I vnderſtande.

The thirde varietie.

M. Then marke this thirde waye: If the denominators doe not happen ſo, that one by Multiplication may make the other, thē loke whether they both maye be partes of anye other one number, as in $\frac{5}{12}$ and $\frac{7}{18}$, althoughe the leſſer taken but twice be to great to make 18, yet they both may be parts vnto 36: therfore loke howe manie times 12 is in 36, and that quotient being multiplyed by the numeratour ouer 12, the totall ſhall be putte in ſteade of the Numerator ouer 12, & for 12 put 36, thus, $\frac{15}{36}$. So likewiſe looke howe often is 18 in 36, and becauſe it is twice, therefore by 2 multiplie 7, whiche is ouer 18, and it wyll be 14, ſet that for the numerator, and in ſtead of 18 put 36, and then ſhall youre Fractions reduced, ſtande thus, $\frac{15}{36}$ $\frac{14}{36}$ in ſteade of $\frac{5}{12}$ and $\frac{7}{18}$.

Proofe.

¶ And if you will proue whether you haue wrought well or no, that maye be proued by Reduction of them agayne to their former denominations, whiche arte ſhall be taughte in the fourth kinde of Reduction, where greater termes of Fractions be reduced into ſmaller in number, but no ſmaller in proportiō. ☞ And if in ſuche Reduction the ſame termes or nūbers come again that were before, then is the worke good, elſe not.

S. Sir, I heare your wordes, but I doe not vnderſtand many of thē, which it may pleaſe you to declare.

Ma. With a good will, when conueniente place ſerueth, but that muſt be in the ſaid iiij. kinde of Reduction. In the meane ſeaſon I wil declare the ſeconde forme of Reduction, which teacheth howe to reduce Fractions of Fractions into one Fraction, and ſo to one Denomination.

Reductiō of fractions of fraction into one fraction and denomination.

When fractions of fractions be proponed, you ſhall multiplie the Numeratours of each into other, and ſet the total for the new Numerator, and then multiplie all the Denominatours likewaies, and take their total for the new denominator, and ſo are they ſpeedily reduced.

Sc. If that be al then I vnderſtand it alreadie, as by this example I wil declare. Theſe be the Fractions, $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ which I wold reduce to one denomination.

Therfore begin I with the Numeratours,

and multiplie them all togither, ſaying: 3 into 2 maketh 6, and 6 by 6 maketh 36, which multiplyed by 7, yéeldeth 252, that I ſet ouer a line for the Numerator, thus:

$$\frac{252}{}$$

Then I multiplie the denominatours, 4 by 3 maketh 12, & that by 7 bringeth 84, which multiplyed by 9, yéeldeth 756, the new denominatour. And ſo the whole reduced fraction is this,

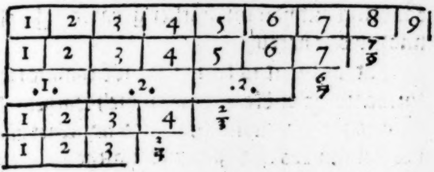
$$\frac{252}{756}$$

which is to hard a Fraction for me to vnderſtand yet.

Ma. You thinke ſo, and no maruaile, but anone you ſhall learne to iudge it eaſilie, for this Fraction is no more in déede then $\frac{1}{3}$, although it be in greater tearmes, and therefore more ſtranger and more obſcure.

And this ſuffiseth for this Reduction, ſaue that I will ſhew you by a figure of meaſure, the iuſte rate and reaſon of this kinde of Fractions, and alſo the due vnderſtanding of the Reduction.

The entier meaſure parted into 9.



Here you ſée the longeſt meaſure, (whyche ſtandeth for the whole and entier quantitie) firſte parted into 9 Diuiſions, whereof 7 are ſeuered by the ſecond meaſure: and thereof againe are parted out 6. And that 6 being diſtinct into 3 partes, 2 of them are parted by the fourth meaſure, of which fourth meaſure, being diuided into 4 partes, the loweſt meaſure doth containe $\frac{3}{4}$, ſo that the ſame $\frac{3}{4}$ muſte be named, not $\frac{3}{4}$ of the whole meaſure, but in déede is $\frac{3}{4}$ of $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{9}$ or as I woulde rather expreſſe it, $\frac{3}{4}\,\frac{2}{3}\,\frac{6}{7}\,\frac{7}{9}$.

Scholer. This example is ſo ſenſible, that I can not chooſe but ſée it. And furthermore, I ſée alſo, that the ſame fraction is equall to $\frac{3}{9}$ of the entier meaſure, as the lines whiche runne vp and downe doe expreſſelie ſet forth.

Alſo I ſee here, that $\frac{2}{3}.\frac{6}{7}.\frac{7}{9}$ is equall to $\frac{4}{9}$ And further yet, that $\frac{6}{7}.\frac{7}{9}$ is equal to $\frac{6}{9}$.

Ma. I am gladde that you ſee it ſo wel, not doubting but you wil gather greater light of knowledge hereby.

The third forme of Reduction.

But nowe it is time that we come to the thirde forme of Reduction, which teacheth of Improper Fractions, that is to ſay, Mixt nũbers of Unites and Fractions, although they appeare like Fractions, as this $\frac{26}{5}$, whyche doeth include 5 Unites wholly, and $\frac{1}{5}$ ouer. Wherfore firſt you ſhall know them, by that the Numeratour is greater than the Denominatour.

Scho. In deede ſir that appeareth reaſonable, that if the Numerator doe expreſſe more partes to be taken of any vnitie then the Denominator doth ſignifie that vnitie to be diuided into, it muſt needes follow that ſuch a fraction importeth more than the whole, that is to ſaye, the whole with certaine partes ouer. But what Reduction is there in it?

Ma. There be two ſeuerall kindes of Reduction, concerning ſuche Fractions. Sometimes it ſhal be needsfull to conuert thoſe fractions into the Unites and the proper fraction that wil remaine And ſometimes contrarie waies, wayes it ſhal be mete to reduce mixt nūbers, that is vnities, written with fractions into the forme of one ſimple fraction, & ſo be there two wayes.

Sch. What is the meane of the firſt waye to turne improper fractions into Unities wyth their proper Fractions?

M. That is thus. Your Numerator beeing greater than the Denominator, muſte be diuided by the ſame denominator, & the quotiēt therof expreſſeth the vnities, yͤ remainer ſhal be put for the Numerator of the fraction that reſteth, and the denominator muſt be yͤ ſame that was before.

Reductiō of improper fractions into vnits vvith their proper Fractions.

Sc. For example, I take $\frac{17}{5}$. And diuiding 17 by 5, the quotient wil be 3, and there will remaine 2.

Ma. That muſt you write thus, 3 $\frac{2}{5}$ where (you ſee) I haue written , without any line, as entier numbers ought to be writtē, and the 2 that remained, I haue ſet ouer the former denominator with a line, as a proper fractiō. And this number doth ſignifie now 3 Units, and $\frac{2}{5}$ of one.

Scho Then if I would by Unites here vnderſtand crownes, ſo it were 3 crownes, and $\frac{2}{5}$, that is 2 ß.

M. Euen ſo, and therfore $\frac{17}{5}$ did ſignifie the ſame. But this happeneth ſometimes, that when the Reduction is ſo wrought, there remayneth nothing. And then it is not a mixte number, but a ſimple entier number, repreſented like a Fraction.

Scholer As $\frac{9}{3}$ wil make 3 iuſte, and $\frac{18}{3}$ wil make euen 6. This I wil remember. But nowe, what is the ſeconde forme of Reduction, that you ſpake of for theſe ſortes of Fractions?

Reductiō of whole numbers, either alone, or ioined with fractiōs into fractions.

M. Whenſoeuer you haue any of theſe two ſorts of numbers, that is to ſaye, whole numbers without Fractions, or whole numbers with fractions, and you would turne thē into the forme of a fraction you muſt multiply the whole number by that denominatour, which you wil haue to remaine ſtil, and to the total thereof adde the numeratour which you haue alreadie, and al that ſhall you ſet for the new numerator, keeping ſtill the former denominatour: as if you haue 6 $\frac{3}{4}$ which you woulde conuert into an Improper Fractiō you muſt multiplie 6 by 4, whereof commeth 24, and thereto adde the numeratour which is 3, and ſo haue you 27 for the numerator, and 4 ſtill for the Denominator.

Scholer.

Sc. Then is $\frac{27}{4}$ equall to 6 $\frac{3}{4}$.

Ma. Euen iuſt, and ſo backward (as appeareth by the former Reductiō) 6 $\frac{3}{4}$ maketh $\frac{27}{4}$ And thus one of theſe Reductions may be the proofe of the others worke.

Sc. This I perceiue, but now if you wold turne whole numbers without fractions into any fraction, I ſee not how that maye bee done, becauſe there is no denominatour to make the multiplication by.

Ma. That was well marked : but this you know , that no man intendeth to turne any whole number into a Fraction, but hee hath in his mind that denominator by which ☞ the Multiplication muſt be made : for the proofe whereof I ſette downe 7, whiche is a whole number. And if you will haue this nūber conuerted into any certayne fraction, will me to doe it.

Scholer. I pray you reduce 7 into a Fraction.

Ma. Then you care not what the fraction be, ſo it be ſome fraction.

Sc. No, I paſſe not for the ſort of the Fraction.

Ma. Then how can you thinke that you require me to doe any thing certayne, when you leaue me to doe as I liſt? and ſeeing you ſtande at that ſtaye, whether thinke you that I muſt firſte intende in mynde what fraction I will make of it, before I can doe it indeede?

Sc. Elſe you ſhould doe ignorauntly.

Ma. Then I will limit my ſelfe (ſeeing you will not) to turne it into quarters. And therefore I multiplie 7 by 4 (whiche is the denomination of quarters) and there amoũteth 28 to be ſet for the Numeratour, and the 4 muſt be ſet for the Denominator, and the fraction will be thus, $\frac{28}{4}$.

Sch. Indeede I perceiue this to be reaſonable, for without much triall I vnderſtand that $\frac{28}{4}$ of any thing doeth make 7. And ſo then if I would turne 8 into fift parts, it wil make $\frac{40}{5}$ which is all one with 8. for eyghte crownes turned into fift partes (that is, into ſhillinges) will make 40 ſhillinges, that is $\frac{40}{5}$ of a crowne.

Ma. Seing you vnderſtande nowe theſe three kinds of Reduction, I will declare vnto you the fourth kinde, that is when Fractions be written in greater Tearmes than they neede, howe they maye be broughte to leſſer Tearmes.

The fourth kind of Reduction.

Scholer To write anye thing in greater Tearmes than needeth, ſeemeth to be a fault, and ſo this Rule ſeemeth to amende that fault.

Ma. It were a fault to do any thing without neede which after muſt be redreſſed: but in this caſe it is not ſo: neither did I ſay abſolutely (as you doe) that it needeth not to expreſſe thoſe Fractions in ſo greate tearmes, but that the fractions doe not neede, I meane for their value to be vnderſtanded: but yet it maye be needefull for the eaſe of thoſe works whereto they be applyed, as for example: In the firſte kinde of Reduction this was youre owne example: $\frac{3}{16}$ and $\frac{4}{6}$, whiche when you would reduce, you were fayne to turne them firſt into one Denomination, & ſo appeared they thus, $\frac{18}{96}$, and $\frac{64}{96}$, where the Fractions (for their owne vnderſtanding) needed not to be turned out of ſmaller tearmes into greater, but yet the eaſineſſe of working needed it.

Sch. Sir, I vnderſtand nowe, not onelye the difference of this neede (for the Fractions mighte better be vnderſtanded as Fractions ſeuerall, eche in his value, when they were in leſſer tearmes, although they coulde not

Tearmes of fractions.

not ſo well be reduced) but alſo I vnderſtand what you meane by greater Tearmes and leſſer Tearmes, whereof before I was in doubt, for I ſee you call the Numeratour and denominatour, the Termes of the Fraction.

Reduction of Fractions into their ſmalleſt tearmes.

Maiſter. I am glad you vnderſtande it ſo well. Now then when you woulde value any fractions (becauſe that maye beſt be done when the Tearms are ſmalleſt) you ſhal reduce them to the ſmalleſt that you can, which thing you may doe thus: Diuide the greateſt of any ſuch two Termes by the leſſer, and if any thing remayne by that remainer, diuide the laſt Diuiſour: and if any thing remayne now, by that diuide the laſt diuiſour (whiche was before the remainer of the firſt Diuiſiō) and ſo continue ſtill, till nothing do remayne in the Diuiſion: and then marke your laſte Diuiſour, for it is the number that will eaſilye reduce your Fraction, if you diuide both the Numeratour and the Denominatour by the ſame number, and put for the Numeratour the qnotient of his Diuiſion, and for the Denominatour alſo his quotient, that riſeth by his Diuiſion.

Scho. I take for example $\frac{18}{96}$, and becauſe

96 is the greateſt number, I diuide it by 18, and the quotient is 5, and there reſteth 6: what ſhall I doe with this quotient?

Maiſter. Nothing in this worke, but nowe ſeeing there remaineth ſomewhat, by that remainer muſt you diuide the laſt diuiſor.

Sc. If I ſhal diuide 18 (which was the laſt diuiſour by 6, that was the remayner, ſo is the quotient 3, and nothing reſteth.

Ma. As for the quotient I omitte him yet, but becauſe there doth remain nothing therfore is 6 (which was your laſt diuiſour) that number by which you may reduce the fraction proponed,

Sch. Then as you taught me, I muſte diuide the Numerator 18 by 6, and the quotient is 3, which I muſt put for the numerator ouer a line, thus. $\frac{3}{\ }$ and then by the ſayde 6, muſt I diuide alſo the denominator 96, and the quotient will be 16, whiche I muſt take for the Denominator, and ſo is the fraction $\frac{1}{16}$. And ſo me thinketh this Rule doth proue ỹ work of the firſt Reduction.

Maiſt. That is true, if the firſt Reduction were made of Fractions in their leaſte tearmes, and elſe not, withoute ſome helpe, as the ſecond number in that place will declare.

Sc. The ſecond nūber was $\frac{4}{6}$, which was turned into $\frac{64}{96}$, by that rule. Nowe if I ſhall by this Rule reduce it againe into the leaſte tearmes, I muſt diuide 96 by 64, and there reſteth 32, by which 32 I diuide 64, & there remayneth nothing, wherefore I muſt take that 32 for the Diuiſour, to reduce the ſayde Fraction. Then doe I diuide 64 by 32, and the Quotient is 2, whiche I ſette for my Numeratour. Againe, I diuide 96 by 32, and the Quotient will be 3, and ſo haue I but $\frac{2}{3}$

Ma. Muſe not at the matter, for you haue done well ynough: but you thinke you haue not the fraction that you looked for, that is $\frac{4}{6}$, yet haue you one equall to it, as by the parts of a ſhilling you may proue.

Sc. Truth it is, for ech of them will bring forth 8 pence, ſo that $\frac{8}{12}$ and $\frac{4}{6}$ and $\frac{2}{3}$, be all thrée equall. And nowe I perceiue that becauſe $\frac{4}{6}$ was not written in the leaſte Tearmes that it mighte be, therefore this Reduction broughte forthe not it, but that other whiche is written in the leaſte tearmes. Now vnderſtande I this rule wel. But is there anye other waye to worke this Reduction.

Maſter. Yes, but firſte note this, that if you finde no ſuche diuiſor, to reduce the Fraction till you come to 1, becauſe 1 doeth make no diuiſion, therefore that Fraction is alreadie in his leaſt tearmes, as by $\frac{71}{100}$ you may proue and ſo of $\frac{81}{98}$ and many other like. But now for your better ayde to find the due proportion in leaſt tearmes, with more eaſe for a yong learner, you ſhall mediate or take the halfe of the Numerator, and alſo the denominator as long as you may vpon a line alwayes parting them with a daſhe of your penne as you worke, which maye eaſilye be done, if the numbers be euen: as 2, 4, 6, 8, or 10: But if they be odde (though it be but in one of them) then muſt you abbreuiate them by 3, by 5, by 7, &c.

An other vvay for to vvorke Reduction.

And becauſe Examples doe moſt inſtruct, I haue in the Page following, ſet downe the maner of 2 or 3, whoſe laſt number at the ende of the lyne, ſheweth the leaſt tearme or valuation of that Fraction.

288	144	72	36	18	9	3	1
576	288	144	72	36	18	6	2

28	14	7	1
112	56	28	4

Abbreuiated firſt by 5, then by 293.

1465	293	1
4395	879	3

Sc. Sir I thanke you much, this is verye eaſie, and good for a yong learner.

Ma. So it is, but yet notwithſtanding, if you can without that diuiſion by memorie eſpie the greateſt number that maye diuide exactly both Tearmes of your Fraction proponed, then neede you not to vſe that diuiſion, as in this fraction $\frac{60}{96}$ I ſee that 12 is the greateſt number that can diuide them both: and therefore without any work, by memorie onely, I turne that into $\frac{5}{8}$, but this abilitie in knowledge is gotten by exerciſe.

Yet one other waye of eaſie Reduction in this kinde there is, when your Fraction hath any Cipers in yͤ firſt places of both tearmes, then may you by caſting away the ciphers, make a breefe Reduction, as thus $\frac{100}{400}$, here take away the Ciphers, and it wilbe $\frac{1}{4}$, which is the ſame in value with $\frac{100}{400}$.

Scholer. And ſo if I haue $\frac{400}{850}$ it will be $\frac{4}{85}$.

Ma. You are deceiued, for you take away more ciphers from the Numerator, than you doe take from the denominator, whiche you may not doe. ☞

Sc. I confeſſe my fault, whiche came of too much haſte, I was more gladder of the rule than wiſe in vſing it: but now I vnderſtād it I truſt.

The fift kind of Reduction

Ma. Then may I go in hand with the fift or laſt kinde of Reduction, which teacheth how to turne any fraction propoſed into any other denomination that you liſt: or into any parts of common coynes, waightes, or meaſures, or ſuch like.

For declaration whereof, firſt you ſhall mark whether your fraction be a ſimple fraction, either elſe a fraction of ſundrie parts, I meane of more tearmes than 2. And if your fraction be a fraction of Fractions, or otherwaies compounde, you muſt reduce it to one ſimple fraction. And then marke well the denomination of that other fraction, into which you would turne this, for by that denominatour you muſt multiplie the Numeratour of your firſt fraction, and the totall Producte thereof ſhall you diuide by the denominator of your firſt fraction, and that quotient ſhall be the numeratour to the denominatour proponed: as for example, I haue this fractiõ $\frac{3}{5}$, which I would turne into tenth partes, therfore I multiplie this 10 by 3, that is the Numeratour of my fraction, and there riſeth 30, which I diuide by 5, and the Quotient is 6, which muſt be the Numerator to 10, and ſo $\frac{3}{5}$ will be $\frac{6}{10}$.

Sc. This is eaſie ynough to doe.

Ma. Then ſhall you ſee an other example of the ſame fraction that is not ſo eaſie: as if I would turne $\frac{3}{5}$ into viij. partes, proue you that worke.

Sc. I muſt multiplie 8 by 3, and there amounteth 24, which I diuide by 5, and the quotient is 4, then is the new fraction $\frac{4}{8}$.

Ma. And ſee you nothing doubtfull in this worke?

Sc. I ſee that when 24 was diuided by 5, there remayned 4, whiche I did not paſſe of, becauſe ye ſpeake nothing of any remayner, but only of the quotient.

Ma. By likelihoode you remember what I ſayd to you in diuiſion of whole numbers, that you ſhoulde not paſſe of the remayner there, but only note it as a ſumme that could not be diuided without knowledge of Fractions. Wherefore now marke this, that in all Diuiſions of whole numbers, when there is any remainer, you ſhall ſet it ouer a line as a Numeratour, and ſet the Diuiſour for the Denominatour, and that Fraction doeth make the Diuiſion complete, and is parte of the quotient: as if I woulde diuide 48 by 5, the quotient will be 9 $\frac{3}{5}$: ſo in your former worke when 24 was diuided by 5, the quotient ſhould be 4 $\frac{4}{5}$, and ſo the new Fraction ſhoulde bée thus: that $\frac{4\frac{4}{5}}{8}$ is $\frac{4}{8}$ of the entire number, and $\frac{4}{5}$ of $\frac{4}{8}$: which you may proue be example of ſome coyne.

Sc. Then I take a Crowne, whoſe $\frac{1}{5}$ is 2 ß. Now if I would proue whether that 2 ß bee 4 and $\frac{4}{5}$ of $\frac{4}{8}$ I ſhall haue a combrous worke to doe.

Maiſter. In déede for whole pennies your example is troubleſome: yet tourning the Crowne into halfe pennies, it is eaſie ynough.

Sc. Now will I proue it.

Ma. Firſt let me tell you an eaſie way how to finde any number that will eaſily be diuided into ſuch parts as you deſire, which way is this. Set down the parts that you deſire, and then by one of them multiplie all the other, the totall whereof ſhall containe all the partes propoſed, as if I would haue a number that may be diuided into 4, 5, 6, and 7 partes, by 4 multiplie 5, and there riſeth 20: then multiplie 20 by 6, & it will make 120: which multiplied by 7, will yield 840: and ſo of any other numbers.

Sc. Then in our former example where is mention but of 5 partes and 8 partes, I ſhall only multiplie 5 by 8, which maketh 40, and that number will ſerue.

Ma. So will it.

Sc. Then what is $\frac{3}{5}$ of 40?

Ma. Proue by the ſame rule whiche you confeſſe eaſie ynough: 3 times 40, is 120, which being diuided by 5, maketh [illegible] $\frac{\quad\frac{3}{5}}{8}$
24 and that is iuſt.

Now to know whether it be equall to 24, firſte I ſee by the ſame rule, that $\frac{1}{2}$ is 20, and

$\frac{1}{8}$ is 5, of which 5, I muſt take $\frac{4}{5}$: and that by the ſame Rule is 4. So that I ſée now, that is equal to $\frac{3}{5}$.

$$\frac{4\ \frac{4}{5}}{8}$$

Ma. And by the waye note this forme of fraction how it is written, that is to ſay, both the Numerator and his Fraction aboue the lyne, although I know it may be written otherwyſe, as thus: $\frac{4}{8}$ and $\frac{1}{5}$ of $\frac{4}{8}$, but I accoumpte the other waye more apte a greate deale.

✳ Scho. Now I pray you ſir let me proue how this $\frac{4}{8}$ and $\frac{1}{5}$ of $\frac{4}{8}$ of a Crown be equal to $\frac{3}{5}$ of a Crown our firſt proponed fractiõ, that hath brought forth theſe hard fractions to my thinking yet.

Firſt I ſée that $\frac{3}{5}$ of a crowne is 3 ß: which is 36 d, or 72 halfe pence. Now if I can find that this fraction $\frac{4}{8}$. and $\frac{1}{5}$ of $\frac{4}{8}$ be equall unto 3 ß: then am I fully anſwered.

Becauſe I cannot take $\frac{4}{8}$ of a Crowne, I turne the crown into half pence, as you wild me, which makes 120, which I diuide by 8, my quotiẽt is 15, which takẽ 4 times makes 60: now reſteth me to haue $\frac{1}{5}$ of the $\frac{4}{8}$, wherof $\frac{1}{8}$ is 15, that 15 being parted into 5 parts, ŷ quotiẽt is 3, which takẽ 4 times makes 12, which with my 60 before, amounteth to 72: which are then equall to $\frac{1}{5}$ my deſire.

Ma. And ſo may you expreſſe by an other way then is before mentioned, all Fractions of Fractions, as thus: $\frac{1}{4}$
That is $\frac{1}{4}$ of $\frac{5}{8}$, and ſo of other, $\frac{5}{8}$
but I remit theſe fourmes to the arbitrement of euery wiſe artes man, to vſe as he thinketh moſt apt and readie.

But now one example more for this rule, and then ſhall we ende it. If I haue $\frac{7}{15}$ of a Soueraine (accounting the Soueraigne 20 ſhillinges) how many ſhillinges is that $\frac{7}{15}$?

Sc. I muſt multiplie 7 by 20, and that maketh 140, which I ſhall diuide by 15, and the Quotient will be 9 $\frac{5}{15}$: or elſe in leſſer tearmes, $\frac{1}{3}$.

Ma. That is 9 ß, and one third parte of a ſhilling, that is 4 d., as by this ſame rule you may proue. And this for this time ſhall ſuffiſe for Reduction, ſaue that I muſt now repeate a little touching Multiplication and Diuiſion, and ſo go forward.

In Multiplication it happeneth ſometyme that there be whole numbers to be multiplied with Fractiõs: And maye be in two ſortes, for eyther the whole number is ſeuerall from the Fraction, and is the Multiplyer, or elſe, the whole number is ioyned with one, or both of the Fractions, and ſo maketh a mixt number thereof. If it be in the firſt ſorte, then néedeth there no Reduction, but onelye multiplie the Numerator of the Fraction by that whole number, and the totall thereof ſet for the new Numerator.

Reduction of whole numbers into Fractions.

Scho. I vnderſtand you thus. If I haue $\frac{6}{21}$, to be multiplied by 16, then muſt I multiplie that 16 with 6, which is the Numerator, whereof commeth 96, and that muſt I ſet for the new Numerator, kéeping ſtill 23 for the Denominator, and ſo the Fraction will be $\frac{96}{23}$ that is $4\frac{4}{23}$.

Ma. And in this ſorte of worke you maye abbridge the labour, thus. If it happen the denominator to be ſuch a number, as maye euenly be diuided by the ſayd whole number propoſed, then diuide it thereby, and ſette the quotient of that Diuiſion for the former denominator: but reſerue ſtill the Numerator, and ſo is the Multiplication ended.

Scho. Then I fayne this example, $\frac{7}{20}$ to be multiplied by 5. And becauſe 5 will iuſtly diuide 20, therefore I take the quotient of that diuiſion whiche is 4, and ſette in ſteade of 20, and ſo the Fraction will be $\frac{7}{4}$, that is $1\frac{3}{4}$.

Mayſter. Whiche is all one with $\frac{35}{20}$, that woulde haue followed of the other ſort of worke.

Sch. I perceiue it very well.

Hovv to multiplie mixt numbers.

Maiſter. Nowe then for the other ſorte where the number is mixt, take this waye: firſte to reduce the ſayd whole number, and fraction into one fraction Improper (as I ſhewed you in Reduction) and then multiplie them togither, as if they were proper fractions.

Sc. 13 $\frac{3}{5}$ being ſet to be multiplied by $\frac{5}{8}$, firſt I muſte reduce the mixte number, as appeareth in the margent, by multiplying 13 by 5, and that maketh 65, wherto I muſt adde the Numerator 3, and ſo the fraction will be $\frac{68}{5}$, whiche nowe I ſhal multiplie after the accuſtomed forme, and it wil be $\frac{340}{40}$.

$$\frac{13\frac{3}{5}}{\frac{68}{5}}$$

Ma You haue done well: and ſo maye you ſee, that although moſte part of the formes of Multiplication maye be wroughte without Reduction, yet ſome can not, as namelye mixed numbers.

And yet one note more will I tell you of Multiplication, before we leaue it: That is, when ſo euer you woulde multiplye anye Fraction by 2, whiche commonlie is called Duplation, you maye doe it not onlie by doubling the Numeratour, but alſo by parting the Denominator into halfe, if he be euen.

Scholer. Then if I woulde double $\frac{5}{12}$, I may chooſe whether I wil make it, $\frac{10}{12}$, or elſe $\frac{5}{6}$. And in deede I ſee that all is one, but that the diuiding of the Denominatour ſeemeth the better waye to make ſmaller tearmes of the Fraction, and ſo they ſhall neede the leſſe Reduction.

M. It is ſo: and nowe I ſhall not neede to tell you that Multiplication is proued by Diuiſion, and Diuiſion likewayes by multiplication, but the like workes that I ſhewed you in Multiplication, will I ſhewe you in Diuiſion alſo.

DIVISION.

Diuiſion to diuide a whole nũber by a Fraction.

When any whole number ſhall be diuided by a Fraction, you muſt multiplie ẏ ſaid whole number wyth the Denominator of the fraction, and ſette the totall thereof for the newe Numerator, and for the Denominator, let the Numerator of the Fraction.

Scholer. Then 20 diuided by $\frac{3}{4}$ wil make $\frac{60}{3}$. as here appeareth: $\frac{20}{1}$ by $\frac{3}{4}$ $\frac{60}{3}$

To diuide a Fraction by a whole number.

Mayſter. Euen ſo: But if you woulde diuide the fraction by the whole number, thẽ multiplie the Denominatour by the ſame whole number, and ſette the totall for the Denominatour, without chaunging the Numerator.

Scholer. Then to diuide $\frac{20}{23}$ by 4, it wyll by $\frac{20}{92}$. As here appeareth in this example. $\frac{20}{23}$ by $\frac{4}{1}$ $\frac{20}{92}$

M. You ſay well. And by the ſame example you giue me occaſion to remember an other briefe way to do the ſame: for if you had diuided the ſayde Numerator by 4, and ſette the quotient for the Numerator, keeping ſtil the olde Denominator, it woulde haue bene not only as well done, but alſo in a fraction of leſſer termes.

Another briefe vvay

Scholer. I geſſe it to be euen ſo, by a lyke worke that you taughte me in Multiplication. And for proofe thereof $\frac{20}{23}$ being the diuidend, and 4 the Deuiſour, I diuide the Numeratour 20 by 4, and the quotient is 5, whiche I ſette for 20 ouer 23, thus $\frac{5}{23}$. And I ſee that it is all one with $\frac{20}{92}$, as by diuiding or abbreuing both theſe tearmes by 4, and ſo reducing them to their leaſte Denomination, I maye eaſilie proue: as appeareth in this example: $\frac{20}{92}$ — $\frac{5}{23}$

Ma.

xt numbers. M. You cõceiue it wel. And if there bée mixt numbers (either one or both) you muſt firſt reduce that mixte number into an improper Fraction. And then worke as you haue learned.

S. That was ſufficientlie taught in Multiplication. Therfore I pray you go forward to ſome other thing.

Ma. Then take this note yet for diuiſion. If the Denominatours be like, then diuide the Numeratours as if it were in whole nũbers, and the quotient whether it be Fractiõ, whole number, or mixt, is a good quotient for that Diuiſion. And generallie if one of the Numerators may iuſtlie diuide the other, by that quotient multiplie the Denominator of the leſſer Numerator, and ſet it that doeth amounte in the roome of ẏ ſame denominator, and then for a Numerator to it, ſet the denominator of the other fraction.

Scho. Then if I woulde diuide $\frac{3}{4}$ by $\frac{12}{17}$ I ſée that 3 will diuide 12, and the quotient wil be 4, by whiche I muſt multiplie the other 4 that is the Denominator vnder 3, and then it is 16, whiche I ſet for the Denominator 4, and ouer it in ſteade of the 3, I muſt ſet 17, the other Denominator, and ſo is it thus, $\frac{17}{16}$.

Maiſter. And ſo is $\frac{17}{16}$ in ſteade of $\frac{51}{48}$, which woulde haue riſen by the common worke: as here appeareth:

$\frac{3}{4}$ by $\frac{17}{12}$

$\frac{51}{48}$

And now for Mediation (which is to diuide by 2) marke this: If the Numeratour be euen, ſet the halfe of it in his place without the Diuiſour, and ſo haue you done: and if the Numeratour be not euen, then double the denominator.

Sch. That is if I would mediate $\frac{6}{11}$ I may make the quotient $\frac{3}{11}$. And if I woulde mediate $\frac{7}{11}$ I muſt make it $\frac{7}{22}$.

Mayſter. Nowe truſte I that you haue ſufficiente knowledge in Reduction, Multiplication, and Diuiſion: and therefore will I goe in hande wyth Addition and Subtraction, whyche nowe will appeare eaſie ynough.

ADDITION.

To adde Fractions of one denomination.

VHenſoeuer you haue any fractions to be added, you muſt conſider whether they be of one Denomination or not. And if they be of one denomination, then adde the Numerators togither, and ſet ẏ that amounteth, for the Numerator ouer the common Denominator, and ſo you haue done. The reaſon is, bicauſe that ſuche differ little in Addition or Subtraction from the worke of vulgare denominations, where the Denominators bee no numbers: as 3 pence and 5 pence, make 8 pence, where ẏ denomination is not altered.

To ad fractions of diuerſe denominations.

But & if the fractiõs be not of one denomination, or any of them be mixt of whole nũbers and fractions, then muſt you firſt reduce them to one denomination, and after adde them. And if they be manye, then adde firſt two of them, and to the ſum that doth amount of the Addition, add the third, and then the fourth, & ſo forth, if you haue ſo many.

Scho.

Sc. This ſeemeth eaſie ynough, now that I haue alreadie learned to multiplie and to reduce, without which two, I could neuer haue wrought this. And therefore now I ſee good reaſon, why you did place Multiplication and Reduction before Addition.

Maiſter. It is wel conſidered, but yet refuſe not to expreſſe your underſtanding of it, by an example.

Sch. Then would I adde firſt $\frac{7}{18}$ with $\frac{5}{18}$, and becauſe the Denominators are like (and ſo needeth no Reduction) I adde 7 to 5, which maketh 12, and then is my ſum $\frac{12}{18}$, that is in ſmaller numbers $\frac{2}{3}$.

And if I haue many numbers to be added, as here $\frac{34}{85}$ $\frac{9}{10}$, firſte I muſt reduce them (bycauſe they haue diuerſe denominatours) into one denomination , and then will they be thus.

$$\frac{150}{400} \qquad \frac{320}{400} \qquad \frac{360}{400}$$

or in leſſe tearmes $\frac{15}{40}$ $\frac{32}{40}$ $\frac{36}{40}$, whiche by Addition do make $\frac{83}{40}$, that is 2 $\frac{3}{40}$.

M Now may we go to ſubtraction.

SVBTRACTION.

Subtraction hathe the ſame precepts that addition hadde, for if the Denominatours bee like, then muſte you ſubtracte the one numeratour from the other, and the reſt is to be ſet ouer the common denominator, and ſo your Subtraction is ended : but and if you haue manye Fractions to be ſubtracted out of many, then muſt you reduce them to one denomination, and into two ſeueral Fractions, that is, all that muſte be ſubtracted into one Fraction , and the reſidue into an other Fraction , and then worke as I ſayde before.

Scholer. For the firſt example I take $\frac{15}{12}$ to be ſubtracted out of $\frac{17}{12}$. and the reſte will be $\frac{2}{12}$ or $\frac{1}{6}$.

For an other example I take $\frac{3}{4}$ to be ſubtracted out of $\frac{7}{8}$, which I muſt reduce, and it wil be thus, $\frac{24}{32}$ and $\frac{28}{32}$.

Then doe I ſubtracte 24 out of 28, and

there reſteth 4, which I ſet ouer the common Denominator for a Remayner, thus, $\frac{4}{32}$, that is $\frac{1}{8}$.

Now for the thirde example, I take $\frac{3}{4}$ and $\frac{5}{6}$ to be ſubtracted from $\frac{7}{8}$ and $\frac{9}{10}$. And bycauſe their denominators be diuers, I do reduce thē thus, $\frac{1440}{1920}$, $\frac{1600}{1920}$, $\frac{1680}{1920}$, $\frac{1728}{1920}$.

Then do I adde the two firſt, & they make $\frac{3040}{1920}$. Alſo I adde the two laſte, and they yeeld $\frac{3408}{1620}$ Then doe I ſubtract 3040 out of 3408, and there reſteth 368, ſo is the remainer $\frac{368}{1920}$ that is in ſmaller Tearmes, $\frac{23}{120}$. And thus haue I done with Subtraction, excepte you haue any more to teach me.

Mai. Proue one example more of two Fractions of diuers denominations.

Sch. I take theſe two Fractions, $\frac{7}{8}$ and $\frac{9}{24}$, which being reduced, wil ſtād thus, $\frac{168}{192}$, & $\frac{72}{192}$ Now would I ſubtract 168 out of 72, but I can not.

Ma. Then maye you perceiue that you miſtooke the Fractions: for you cā neuer ſubtract the greater out of the leſſer, althoughe you may adde, multiplie or diuide the greater with the leſſer. And albeit that $\frac{7}{8}$ hathe bothe hys tearmes leſſer than $\frac{9}{24}$, yet is $\frac{9}{24}$ the leſſer Fraction: for generallie if you multiplie ŷ

he greateſt
f tvvo fra-
ions.

Numerators and denominators of two fractions croſſe ways, that Fraction is the greateſt, of whoſe numerator commeth the greateſt ſumme, as in this example: 7 multiplied by 24, maketh 168 : and 9 being multiplyed by 8, yeldeth but 72 therefore is the firſt Fraction $\frac{7}{8}$ the greateſt of theſe two, ſo can you not ſubtract it out of a leſſer Fraction.

But and you ſhould ſubtract a fraction out of a whole number what would you do?

Sc Marrie I woulde reduce the whole nũber into a Fraction of the ſame Denomination that my Fraction is, and then worke by Subtraction.

M. So may you doe, but it is eaſier much, if your fraction be a proper fraction, that is to ſay, leſſe then an Unite, to take an Unite frõ the whole number, and then turne it into an Improper Fraction, and ſo worke your ſubtraction. As if I woulde ſubtracte $\frac{3}{5}$ from 4, I maye take one from 4, and turne it into $\frac{5}{5}$, from whiche if I bate $\frac{7}{6}$, there will remayne 3 $\frac{2}{5}$. And if the firſte fraction be an Improper Fraction, then may I take ſo manye Unites from the whole number, that they may make an improper fraction greater then that firſte, and then worke by Subtraction: As if there be proponed $\frac{10}{3}$ to be ſubtracted frō 6, becauſe $\frac{10}{3}$ is more than 3, and not ſo much as 4. I muſte take 4 from 6, and turne them into thirdes thus. $\frac{12}{3}$ then abate $\frac{10}{3}$, and there reſteth $\frac{2}{3}$ ſo the whole remainer is $2\frac{2}{3}$. Or, elſe you may at your pleaſure take $3\frac{1}{3}$ whiche is $\frac{10}{3}$ from 6 whole: Then ſet one vnder 6. as thus $\frac{6}{1}$: And then to reduce thoſe 2 fractions into one Denomination as here appeareth:

$\frac{10}{3}$ from $\frac{6}{1}$

$$\begin{array}{ccc} & 8 & \\ \frac{10}{3} & \times & \frac{18}{3} \\ & 3 & \end{array}$$

Then $\frac{10}{3}$ from $\frac{18}{3}$ reſteth $\frac{8}{3}$: which maketh $2\frac{2}{3}$ youre deſire:

And thus wil I make an end of the works of commō Fractions for this time, not doubting, but you can applye them both vnto the rules of Progreſſion, and alſo vnto the Golden rule, without any other teaching thē you haue learned before, which might ſeeme tedious to repeate, ſaue that in ſome ſpecial diuerſities, which be peculiar to Fractions, I can not ouerpaſſe, but inſtruct you ſomewhat by the way.

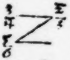
THE GOLDEN Rule.

Therefoꝛe as touching the Golden rule foꝛ the placing of the 3 numbers pꝛoponed in the queſtion, wherby to finde the thirde, and foꝛ the foꝛme of their woꝛke, with other like notes, I refer you to that which you haue alreadie learned.

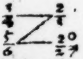
But this eaſie foꝛme of woꝛking by fractions ſhal you note, ỹ if your thꝛée numbers bée fractions, foꝛ an apt woꝛke and certain, multiply the Numeratoꝛ of the firſt nūber in the queſtion, by the Denominatoꝛ of the ſeconde: And all that againe multiplie by the denominatoꝛ of the third number, and the total therof ſhall you kéepe foꝛ to be the Diuiſoꝛ. Then multiplie the Denominatoꝛ of the firſt number by the Numeratour of the ſecond, and the whole therof by the Numeratoꝛ of the third, and the totall thereof ſhall be youre diuidend. Now diuide this diuidend by yͤ diuiſoꝛ which you founde out before, and that number ſhall be the fourth number of the queſtion whiche you ſéeke for: As in this exãple. If $\frac{3}{4}$ of a yard of Veluet coſt $\frac{2}{3}$ of a Soueraigne (eſtéemed at 20 ſhillings) what ſhal $\frac{5}{6}$ coſt?

A queſtion of Veluet.

Scholer. If it pleaſe you to let me make the anſwere, I woulde firſt place theſe thrée numbers, as I learned in whole numbers thus.



And then according to your newe rule, I muſt multiplie 3 being Numeratour in the firſt number, by thrée the Denominatour of the ſeconde, and thereof commeth 9, which I multiplie againe by 6, the Denominatour of the third number, and ſo haue I 54. which I kéepe for the Diuiſor, then multiplie I 4, the Denominator of the firſt, by 2, the Numeratour of the ſeconde, and there riſeth 8, whiche againe I multiplie by 5, the Numeratour of the thirde, and it maketh 40: then muſt I diuide 40, by 54, and it wil be $\frac{40}{54}$ that is $\frac{20}{27}$. in leſſer tearmes, and then the figure wil ſtand thus.



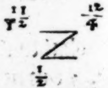
But what that is in money, I can not tell except I ſhal worke it by Reduction, as you taught me.

Ma.

Maiſter. It forceth not nowe, you may reduce it when you liſt, but it were diſorderlie done here to mingle diuerſe workes togither, where we do not ſeeke the value of the thing in common money, but in an apt nūber, whiche you haue wel done. And therefore will I yet ſhew you an other like way of eaſineſſe in worke, how you may chaunge your 3 Fractions into 3 whole numbers, by whyche you ſhall worke as if the queſtion were proponed in whole numbers. The firſt nūber you ſhall finde as I taught you: now to finde the diuiſor of the ſeconde number, take the Numerator for the ſecōd fraction: and for the third nūber take that, that riſeth of Multiplication of the Denominatour of the firſte, by the Numerator of the thirde, and then worke youre queſtion.

A queſtion of ſiluer.

Scho. For example hereof, I put this queſtion, If $\frac{11}{12}$ of 1 ℔ waighte of ſiluer, be worth $\frac{12}{4}$ of a Soueraigne what is $\frac{1}{2}$ of 1 ℔. weighte worth? For the aunſwere, firſt I place the Fractions in order thus. Then to turne theſe fractions into whole numbers, I multiplie 11 whiche is the Numerator of the firſt, by 4 (ẏ deno-



denominatoꝛ of the ſeconde) and there cõmeth 44, whiche I multiplie by 2 the Denominatoꝛ of the third, and ſo amounteth 88, whiche I ſet foꝛ the Diuiſoꝛ in the firſt place. Then in the ſeconde place I ſet 12, whiche is Numeratoꝛ in the ſeconde Fraction, and in the third place I ſet the ſumme that amounteth of 12, being the Denominatoꝛ in the firſte number, multiplyed by 1, being numeratour in the third number, & ſo the figure wil ſtand as here you ſee.

$$\begin{tikzcd} 88 \arrow[r] & 12 \arrow[dl, dash] \\ 12 \arrow[r, dash] & {} \end{tikzcd}$$

Then to woꝛke it foꝛth, I multiplie 12 by 12, and there amounteth 144, which I diuide by 88, and the quotient wil be $1\frac{56}{88}$, oꝛ in leſſer termes, $1\frac{7}{11}$ and then the figure wil ſtand thus.

$$\begin{tikzcd} \frac{11}{12} \arrow[r, dash] & \frac{11}{4} \arrow[dl, dash] \\ \frac{1}{2} \arrow[r, dash] & 1\frac{7}{11} \end{tikzcd}$$

Ma. Theſe ij. foꝛmes nowe you vnderſtand wel ynough And as foꝛ any other, at this time I will not repeate, onlie this ſhall you marke foꝛ yͤ pꝛofe of this rule, whether your woꝛke be well wꝛought oꝛ no. Multiplie the firſt number by the fourthe, and note what amounteth: then multiplie the ſeconde by the third, and marke what amounteth alſo Now if thoſe two nūbers ſo amounting be equall, then is youre woꝛk wel done, elſe you haue erred. And this

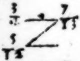
The proofe of the Golden rule.

The Backer rule.

ſhal ſuffiſe for the former rule, but in the backer rule, this ſhal you note for eaſe of worke, that you multiplie the Numerator of the firſt by the Numerator of the ſecond, and ẏ whole thereof by the Denominator of the third, and that amounteth therof, ſhal be the Diuidend. Then multiplie the Denominator of the firſt by the Denominator of the ſeconde, and that whole by the Numerator of the third, & that riſeth therof ſhall be the diuiſor. Example of this: I did lende my friend $\frac{3}{4}$ of a Porteguiſe vij. monthes, vpon promiſe that he ſhould doe as much for me againe: and when I ſhoulde borow of him he coulde lende me but $\frac{5}{12}$ of a Porteguiſe, now I demaund how long time muſt I kéepe his money in iuſte recompence of my lone, accompting 13 monethes in the yeare?

A queſtion of lone.

Scholer The firſt number muſt be the firſt money borowed, that is $\frac{3}{4}$ of the Porteguiſe: the ſecond number the 7 monethes, that is $\frac{7}{13}$ of a yeare: and the thirde number the money that was lent in recompence, that is $\frac{5}{12}$ of a Porteguiſe: then I ſet the numbers thus.



Then (as you taught me) I multiplie thrée (being Numeratour in the firſte number) by 7 the Numerator of the ſeconde number, and it maketh 21, which I multiplie by 12, the denominator of the 3 & ſo haue I 252 for the diuidēt: thē I multiply 4 yͤ denominator of the firſt, by 13 the denominator of the ſecond, and it yéeldeth 52 whiche I multiplie agayne by 5, the Numeratour of the third, and it will make 260, that is the Diuiſor. Then muſt I diuide 252, by 260, ſo it wil be in yͭ ſmalleſt fractiō, $\frac{63}{65}$ of a yeare.

Ma. And this doe you ſée ſome eaſe in working, better than to multiplie and diuide tediouſly ſo many Fractions. An other queſtion yet will I propoſe, to the intent you may ſée thereby the reaſon of the ſtatute of aſſiſe of bread and ale, which in al Statute bookes in Frenche, Latine and Engliſhe, is muche corrupted for wāt of knowledge in this art: for the right vnderſtanding whereof I propone this queſtion.

Statute of Aſſiſe of breade and ale.

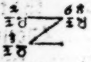
When the price of a quarter of Wheate is 2 ß the farthing white lofe ſhal wey 68 ß. then I demaunde, what ſhall ſuch a Loafe weye, when a quarter of Wheate is ſolde for 3 ß?

Queſtions

Scho. This Queſtion muſt be wroughte as it is propone in whole numbers and not in Fractions.

Mai. You ſeeme to ſay reaſonably, how be it, in that Statute of Aſſiſe, the rate is made by the proportion of parts in a pound weight Troie, elſe could it not be a Statute of anye long continuaunce, ſeeing the ſhillinges doe chaunge often, as all other monies doe: but this Statute being well vnderſtanded, is a continuall rule for euer, as I will anon declare by a new table of Aſſiſe, conuerting the ſhillinges into vnces and parts of vnces.

Therfore here by a ſhilling you muſt vnderſtand $\frac{1}{20}$ of a pound weight, and ſo by pence $\frac{1}{20}$ of an vnce, wherefore although ye mighte worke this queſtion proponed by whole nūber well ynough, for yͤ time when the ſtatute was made, yet to applie it to our time, and to make it to ſerue for all tymes generallye it is beſt to worke it by fractiōs, ſetting for 2 ſhilling $\frac{2}{20}$: and for 68 ſhillings, $\frac{68}{20}$: and ſo for three ſhillings $\frac{3}{20}$, and then will the figure of the queſtion ſtand thus.



In which queſtion becauſe all the denominators be like, you ſhal worke onely with the numerators

Sc. Then I ſhall multiplie 68 by 2, whereof commeth 136, which if I diuide by 3, the quotient will be $45\frac{1}{3}$: but how ſhal I make a Fraction of that to ſtand with the other?

Ma. Haue you ſo ſoone forgottē what was taught you ſo lately? This is his forme.

Sc. I remember it now and then it ſignifieth 45 twentie parts, and the third deale of one twentie part.

$$\frac{45\frac{1}{3}}{20}$$

Ma. So is it, and that maketh in ſhillings, 45 ß 4 d.: wherby you may note one greate error in the Statute bookes, which haue conſtantly 48 ß in that Aſſiſe. And by this rule, if you examine the Statute you ſhall finde many ſummes falſe, wherefore for the true vnderſtanding of that ſtatute and ſuche lyke as I haue made mention of it, and ſomwhat recogniſed it, ſo doe I wiſh that al gentlemē and other ſtudentes of the lawes, would not neglect this arte of Arithmetike as vnneedefull to their ſtudies. Wherfore to encourage them thereto, and to gratifie both them and all other in generall, I will exhibite a Table of that part of yͤ ſtatute in two columnes, and in a third columne I wil ad the correction of thoſe errors which haue crept into it.

Here followeth the Table.

The price of a quarter of vvheate.		The vveight of a farthing vvhite lofe by the ſtatute bookes.			The correction by iuſt Aſsiſe.		
ſ.	d.	li.	ſ.	d.	li	ſ.	d.
1	0	6	16	0	6	16	
1	6	4	10	8	4	10	8
2	0	3	8	0	3	8	0
2	6	2	14	4 $\frac{1}{7}$	2	14	4 $\frac{4}{7}$
3	0	2	8	0	2	5	4
3	6	2	2	0	1	18	10^{2}
4	0	1	16	0	1	14	0
4	6	1	10	0	1	0	2 $\frac{2}{7}$
5	0	1	8	2 $\frac{1}{2}$	1	7	2 $\frac{2}{7}$
5	6	1	4	8 $\frac{1}{2}$	1	4	8 $\frac{6}{77}$
6	0	1	2	8	1	2	8
6	6	0	19	11	1	0	11 $\frac{1}{13}$
7	0	0	19	1	0	19	5 $\frac{1}{7}$
7	6	0	18	1 $\frac{1}{2}$	0	18	1 $\frac{3}{7}$
8	0	0	17	0	0	17	0
8	6	0	16	0	0	16	0
9	0	0	15	0 $\frac{1}{2}$	0	15	1 $\frac{1}{7}$
9	6	0	14	4 $\frac{1}{2}$	0	14	3 $\frac{15}{19}$
10	0	0	13	7 $\frac{1}{2}$	0	13	7 $\frac{1}{7}$
10	6	0	12	11 $\frac{1}{2}$	0	12	11 $\frac{3}{5}$
21	0	0	12	4 $\frac{1}{2}$	0	12	4 $\frac{4}{11}$
11	6	0	11	10	0	11	9 $\frac{21}{23}$
12	0	0	11	4	0	11	4

In the common bokes there is no farther rate of aſſiſe made, than vnto 12 ſ the quarter of wheate: but in an auncīēt copie of 200 yeares olde (which I haue) there is added the rate of aſſiſe vnto 20 ſ the quarter, but yet was ỹ aſſiſe alſo eyther wrong caſt at ỹ firſt pēning, or els corrupt ſith that time, for lacke of iuſte knowledge in the rule of proportion, whiche I wil adde here alſo, to gratifye ſuche as be deſyrous to vnderſtande truth exactlye.

The price of a quarter of vvheate.		The vveight of the farthing vvhite lofe by the ſtature bokes.		The correction of the errors.		
ſ.	d.	ſ	d.	ſ.	d.	
12	6	11	0	10	10	$\frac{14}{25}$
13	0	15	0$\frac{1}{2}$	10	5	$\frac{7}{13}$
13	6	10	1$\frac{1}{2}$	10	0	$\frac{8}{9}$
14	0	9	7	9	8	$\frac{4}{7}$
14	6	9	2$\frac{1}{2}$	9	4	$\frac{16}{29}$
15	0	9	1$\frac{1}{2}$	0	0	[illegible]
15	6	0	1$\frac{1}{2}$	8	9	$\frac{9}{31}$
16	0	9	0	8	6	0
16	6	8	6	8	2	$\frac{10}{11}$
17	0	8	3	8	0	0
17	6	7	10	7	9	$\frac{9}{35}$
18	0	7	6	7	6	$\frac{2}{3}$
18	6	7	3	7	4	$\frac{8}{37}$
19	0	7	2	7	1	$\frac{17}{19}$
19	6	5	10	6	11	$\frac{9}{13}$
20	0	5	6	6	9	$\frac{1}{5}$

Theſe 2 tables I haue ſet ſeuerall, becauſe no man ſhould think that I would either ad or take away frō any law thoſe parts which might of right ſeem either ſuperfluous eyther diminute, but yet I may not be ſo curious as to neglect manifeſt errors, which is not only my part, but euery good Subiects dutie with ſobrietie to correct. And for auoiding of offēce I haue rather don it in this priuate booke rather than in any booke of yͤ ſtatutes ſelf, truſting that all men will take it in good parte.

Sc. I would wiſh ſo, but I dare not hope ſo ſith neuer good man yͭ would reforme error, could eſcape yͤ venemous tōgs of enuious detractors, which becauſe they either cannot or liſt not to do any good thēſelues, do delight to bark at yͤ doinges of other, but I beſeeke you to ſtay nothing for their peruerſe behauiour.

Ma. I cōſider many things yͭ ſome may obiect, whereunto I am not vnprouided of iuſt anſwers, but I wil not ſeem ſo haſty to make yͤ anſwers before I heare their obiectiōs, but as I truſt that men are of a bettter nature, & more grateful now thā ſom hath bin in time paſſed, as I haue don in yͤ ſtatute of Aſſiſe for bread in rate of ℔, ſo wil I ſet forth the like table in pounds & ounces, & the parts thereof.

that it may be eaſilie applyed to all tymes: but I mean not by this to alter any word of the ſtatute (being ſo good an ordinãce, & of ſo great continuance) but onely to make it as a kind of expoſitiõ & declaratiõ of yͤ ſaid ſtatut, truſting yͭ therby yͤ ſtatute may be better vnderſtãd, & conſequẽtly better put in executiõ.

A pounde vveight.

And here you ſhall note, yͭ I haue accounted the ſhillings after the rate of lx ẜ, to the poũd weight, bicauſe I eſtéem it yͤ moſt apt rate for our time. Wherfore if in yͤ firſt column you find yͤ price of wheat, directly againſt it in yͤ ſecond column, you may find yͤ weight of the farthing white lofe, in this our time: & if you double yͤ number (as I haue done in yͤ thirde columne) then haue you the weight of yͤ half pẽny white lofe & ſo in yͤ fourth column is ſet yͤ weight of yͤ pẽny white lofe. It néedeth not to tel you that, yͭ the ſight doth teſtifie, how yͭ euery colũn is parted into 3 ſmaller pillers, wherof yͤ firſt colũne hath theſe 3 titles, poũdes, ſhillings, & pennies: yͤ other 3 columnes haue ech of thẽ theſe 3 titles, pounds, vnces, & pẽnie weights. And as in the firſt colũne xij d. make a ẜ, & 20 ẜ make a pounde, ſo in the other iij columes xx pence weighte maketh an vnce. and xij ounces doe make a pounde.

The price of a quarter of wheate.

The weight of the farthing white loafe.

℔	ß	d.	℔.	unc.	d w.
0	3	0	6	9 1/2	2
0	4	6	4	6 1/4	3
0	6	0	3	4 3/4	1
0	7	6	2	8 1/2	2 4/5
0	9	0	2	3	4
0	10	6	1	11 1/4	1 2/7
0	12	0	1	8 1/8	3
0	13	6	1	6	2 2/3
0	15	0	1	4 1/8	1 2/5
0	16	6	1	2 3/4	1 9/11
0	18	0	1	1 1/2	2
0	19	6	1	0 1/2	1 1/11
1	1	0	0	11 1/2	3 1/7
1	2	6	0	10 3/4	2 2/5
1	4	0	0	10	4
1	5	6	0	9 1/2	2
1	7	0	0	9	1 1/3
1	8	6	0	8 1/3	1 15/19
1	10	0	0	8	3 1/5
1	11	6	0	7 [illegible]	0 [illegible]
1	13	0	0	7 1/4	3 [illegible]
1	14	6	0	7	1 21/23
1	16	0	0	6 3/4	1
1	17	6	0	6 1/2	0 14/15
1	19	0	0	6	0

The weight of the halfe pennie white loafe.

The weight of the halfe pennie white loafe.

li.	vnc.	d. w.
13	7	4
9	0 ¾	1
6	9 ½	2
5	5 ½	0 3/5
4	6 ¼	3
3	10 ½	2 4/7
3	4 ¾	1
3	0 ½	0 1/3
2	8 ½	2 4/5
2	5 ½	3 6/11
2	3	4
2	1	2 2/13
1	11 ¼	1 2/7
1	9 ¾	0 1/5
1	8 ¼	3
1	7	4
1	6	2 2/3
1	5	3 11/19
1	4 ¼	1 2/5
1	3 ½	0 6/7
1	2 ¼	1 6/11
1	2	3 19/29
1	1 ½	2
1	1	1 1/23
1	0	1 1/13
1	0	1

The weight of the penny white lofe.

li.	vnc.	d. w.
27	¼	3
18	1 ½	2
13	7	4
10	10 ½	1 1/3
9	0 ¾	1
7	9 ¼	0 1/7
6	9 ½	2
6	0 ½	0 2/3
5	5 ½	0 3/5
4	11 ¼	1 10/11
4	6 ¼	3
4	2	4 4/13
3	10 ½	2 4/7
3	7 ½	0 3/5
3	4 ¾	1
3	2 ¼	3
3	0 ¼	0 1/3
2	10 ¼	2 1/19
2	8 ½	2 4/5
2	7	1 1/7
2	5 ½	3 5/11
2	4 ¼	2 15/23
2	3	4
2	2	2 6/25
2	1	2 2/13
2		

Sch. Sir, I doe thanke you moſt hartily for this, not only in mine owne name and in the name of all Studentes , but alſo in the name of the whole Commons, to whom the reſtitution of this Aſſiſe (I truſt) ſhall bring reſtitution of the weight in breade, whiche long time hath bene abuſed. And if you know any like thinges more, wherein you woulde vouchſafe to declare the errors and ſet forth the truth, you cannot but obtain great thãks of all good harted men that loue the common wealth.

Mai. I haue ſundrie thinges to declare but I haue reſerued them for a priuate booke by it ſelfe, yet notwithſtanding becauſe the ſtatute of the rate of meaſuring of ground is ſo common that it toucheth all men, and yet no more common than néedefull, but ſo muche corrupt, that it is to farre out of all good rate, not only in the Engliſhe bookes of Statutes commonlye printed, but alſo in the Latine bookes, and in the Frenche alſo, for I haue read of eche ſorte, and conferred them diligẽtlye, I will giue you a Table for the reſtitution of thoſe errours, as maye ſuffiſe for this preſent time. And firſt will I propoſe one queſtion to you touching the vſe of that Statute,

tute, whereby you maye perceiue the order: how to examine the whole Statute, and euery parcell thereof, and the queſtion is this.

A queſtion of meaſure of ground.

When the Acre of grounde doth contayne foure perches in breadth, then muſt it contayne 40 perches in length: then doe I demaunde of you, howe much ſhall the length of an Acre be, when there is in the breadth of it 13 perches? but before you ſhall aunſwere to this queſtion, I will declare vnto you an other Statute, which is the grounde of the former Statute. And that Statute is this. It is ordayned that 3 Barlye Cornes, drye and rounde, ſhall make vp the meaſure of an inche: 12 ynches ſhal make a foote, and 3 foote ſhall make a yarde (the common Engliſh bookes haue an elne) fiue yardes and a halfe ſhall make a perche, and 40 perches in length, and 4 in breadth, ſhall make an Acre. This is that ſtatute: whereby you may perceiue that the intent of the Statute is, that one Acre ſhoulde containe 160 ſquare Perches. Now let me heare you aunſwere to the queſtion.

A ſtatute of meaſures.

An Acre.

Scholer. As I perceiue by the wordes of that Statute, a perche to be $\frac{1}{160}$ of an Acre, ſo could I make thoſe numbers all in Frations, and ſo worke the queſtion: but ſeeing I may doe it alſo in whole numbers I take that forme for the moſte eaſie, therefore thus I ſet the queſtion in forme. Then doe I multiplie 40 by 4, and it maketh 160, which I diuide by 13, and the quotient is 12 $\frac{4}{13}$.

4 — 40
13

Ma. Now turne that $\frac{4}{13}$ into the common partes of a perche, as they be named in the former Statute: howbe it, it ſhall be beſt to take one of the leaſt partes in Denomination for auoyding of much labour, as feete, whereof the perche contayneth 16 $\frac{1}{2}$.

Sc. Then to turne $\frac{4}{13}$ into feete, I Multiplye 16 $\frac{1}{2}$ by 4, and it maketh 66, whiche I muſte diuide by 13, and the Quotiente is 5 $\frac{1}{13}$.

Mayſt. So I finde that if the acre holde in breadth xiij perches it ſhall containe in lẽgth 12 perches, 5 foote, and $\frac{1}{13}$ of a foote, which is not fully an ynche, for the ynche is $\frac{1}{12}$ of a foote. But here all the Statute Bookes in Latine and Engliſhe (that I haue ſeene) doe note it to be 13 perches, 5 foote and 1 inche: which maketh aboue 13 perches to many in the acre, ſo that I woulde haue thoughte the erroure

Note this errour.

errour to haue crept into the printed bookes by the great negligēce that Printers in our time doe vſe, ſaue that in written Copies of great antiquitie. I doe find the ſame. Yet haue I one frenche Copie, which hath 12 perches ½, and one foote, and that miſſeth verye little of the truth.

Sc. Then I ſee it is true that I haue oftē heard ſay, that the trueſt copies of yͤ ſtatutes be the Frenche copies.

Ma. That is often true, but not generally, as I haue by conference tried diuerſlye: but in this ſtatute the frenche booke is moſt corrupt in all other places lightlye.

But now to performe my promiſe I will ſet forth the Table for meaſuring of an Acre of ground onely by ſuche partes as the Statute doth mention, becauſe at this time I doe of purpoſe write it for the better vnderſtanding of the ſtatute, and hereafter with other thinges I intende to ſet forth this ſame more at large.

In this Table following, I haue not done as in the other ſtatute before cōpared by reſtitution with the faults crept into the Statute, but onely haue written that true meaſure, which the equitie of the Statute doth pretends. For it were to vile to iudge of ſo noble Princes and worthie Councellors, as haue authoriſed & ſet forth this ſtatute, that they would make one acre in any form greater than an other, but euery one to be iuſt and equall with ech other, which is the grounde alſo of my worke, and hereby maye all men perceiue how néedefull Arithmetick is vnto the Studentes of the law. But now I think beſt to make an end of theſe matters for this preſent time, ſith the table hath in it none obſcuritie, that I ſhould néede to declare.

The breadth.	The length of the acre.		The breadth	The length of the acre.	
perche.	perche.	feete.	perche.	perche.	feete.
10	16	0	28	5	11 $\frac{11}{14}$
11	14	9	29	5	8 $\frac{31}{58}$
12	13	5 $\frac{1}{2}$	30	5	5 $\frac{1}{2}$
13	12	5 $\frac{1}{13}$	31	5	2 $\frac{41}{62}$
14	11	7 $\frac{1}{14}$	32	5	0
15	10	11	33	4	14
16	10	0	34	4	11 $\frac{11}{17}$
17	9	6 $\frac{27}{34}$	35	4	9
18	8	14 $\frac{2}{3}$	36	4	7 $\frac{1}{3}$
19	8	6 $\frac{18}{19}$	37	4	5 $\frac{13}{37}$
20	8	0	38	4	3 $\frac{9}{19}$
21	7	10 $\frac{3}{14}$	39	4	$\frac{9}{13}$
22	7	4 $\frac{1}{2}$	40	4	0
23	6	19 $\frac{18}{23}$	41	3	1 $\frac{62}{82}$
24	6	11	42	3	13 $\frac{6}{42}$
25	6	6^{5}	43	3	11 $\frac{77}{86}$
26	6	2 $\frac{7}{13}$	44	3	10 $\frac{1}{2}$
27	5	15 $\frac{5}{18}$	45	3	9^{2}

Sc. In deede Sir, I vnderſtand the Table (as I thinke) by thoſe other whiche you ſette forth before. For in the firſt Columne is ſet the perches of the breadth of any Acre, and then in the 2 columnes following appeareth how many perches and how many feete the ſame Acre muſt haue for his length

Ma. You take it well: how be it to ſpeake exactly of breadth & length, the firſt columne doth ſometime betoken the breadth: & ſometime the length, for properly the longeſt ſide of any ſquare doth limit his length, and the ſhorter ſide doth betokē the breadth, yet it is no great abuſe in ſuch tables, where a man cannot well change the title, to let the name remayne, although the proportions of the numbers doe chaunge: for ſtill by the firſt columne, is expreſſed the meaſure of the one ſide, and by the two other pillers in one Columne, is ſet forth the meaſure of the other ſide. And this ſhall be ſufficient now for the vſe of the golden Rule.

Nowe ſomewhat will I touch certaine other rules, which for theyr ſeuerall names maye ſeeme diuerſe rules & diſtinct from this, but in deede they are but braunches of it: yet bicauſe they haue not onelie ſeueral workings in appearance, but alſo pleaſant in vſe, I will giue you a taſte of eche of them. As for the rule of Felowſhippe, both ſingle and double, with time and without time, I ſhall neede to ſaye little more than I haue alreadie ſayde in teaching the workes of whole numbers, yet an example or. ij. will we haue to refreſh the remembraunce of the ſame, and to declare certaine proper vſes and applications of it, as this for one.

A queſtion of vnequal ſocietie.

Foure men get a bootie or priſe in time of warre, the priſe is in value of money 8190 li., and bicauſe ẏ men be not of like degree, therfore their ſhares may not be equall, but the chiefeſt perſon wil haue of the bootie the third part, and the tenth parte ouer: the ſecond will haue a quarter and the tenth part ouer: ẏ third will haue the firſt part: and ſo there is lefte for the fourth man a verie ſmal portion but ſuch is his lot, (whether he be pleaſed or wrothe) he muſt be content with one xx. parte of the praye. Now I demaunde of you, what ſhall euerie man haue to his ſhare?

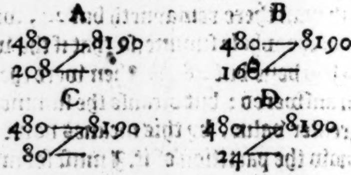
Sch. You muſt be faine to anſwere to your owne queſtion, elſe is it not like to be aunſwered at this time.

Maiſter. The forme to vnderſtande the ſolution of this queſtion, and all ſuche like, is this: Reduce al the Denominators into one number by Multiplication, except that anye of them be partes of ſome other of them, for al ſuche partes you may ouerpaſſe, and take for them all thoſe numbers, whoſe parts they be: as in this example the ſhares be theſe $\frac{1}{3}$ $\frac{1}{10}$ $\frac{1}{4}$ $\frac{1}{12}$ $\frac{1}{6}$ $\frac{1}{20}$ if I multiplie al the Denominatours togither, beginning with 3, and ſo go on vnto 20, it will make 144000: but conſidering that 3 is a part of 6, I ſhall ommitte that 3, and likewayes 10, whiche is a part of 20, I maye ouerpaſſe alſo, and then is there but 3 denominatours to multiplie, that is 4, 6, and 20, whiche make 480, whiche ſumme I take for my worke, bycauſe all the Denominatours wil be founde in it. Then I take ſuche partes of it as the queſtion imporeteth, that is for the firſte man, $\frac{1}{3}$ $\frac{1}{10}$, the $\frac{1}{3}$ is 160 the $\frac{1}{10}$ is 48 whiche I put in one ſumme for the firſte mans ſhare, and it maketh 208. Then for the ſeconde mans ſhare, I take $\frac{1}{4}$, whiche is 120, and $\frac{1}{10}$ whiche is 48, and that maketh in the whole 168. Now for the thirde man whiche muſte haue $\frac{1}{6}$ I take 80 And for the fourth man there remayneth but 24, whiche is $\frac{1}{20}$ of the whole ſumme: ſo that if the whole pray had bene but 480 ℔. then were the queſtion anſwered: but bicauſe the ſumme was of greater value, by this meanes nowe ſhall I know the partition of it. I muſt ſet my nūbers by the order of the Golden rule, putting in the firſte place the number that I founde by multiplying the Denominatours, and in the ſeconde place the ſumme of the bootie. And looke what proportion is betweene the firſte number and that ſeconde, the ſame proportion ſhall be betweene the partes of that firſt number and the partes of the ſeconde, comparing eache to his like. Therefore I muſte put in the thirde place, one of the partes or ſhares, and then worke by the former rule of proportion or Golden rule. And bycauſe I haue

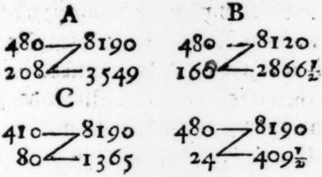
The reaſon of this rule.

haue 4 ſeueral partes of the firſt number, by which I would finde out 4 like partes of the ſeconde number, therefore muſt I make 4 ſeueral figures.

Scholer. Nowe I truſte I can aunſwere to your queſtion, as by your fauoure I wyll proue.



And to trie it, I ſet the 4 figures thus, marked with A, B, C, D, to ſhewe their order. And then in eache of them I multiplie the seconde number by the thirde, and diuide their totall by the firſt, and ſo amounteth the fourth ſum whiche I ſeeke for, for if I do multiply 8190 by 208, it maketh 1703520, whiche being diuided by 480, maketh in the quotient 3549 for the firſt mans portion: And ſo workyng with the other thrée figures, I find for the ſecond man $2866\frac{1}{2}$, and for the third mã 1365: and then for the fourth man $409\frac{1}{2}$. And ſo is euerie mans ſhare ſet forth in the figure here annexed.



And thus I thinke I haue done wel.

Ma. If you miſdoubte your working and liſte to proue it, adde all the ſhares togither: and if they make the totall, thẽ ſeemeth it wel done.

Sc. I maye ſette them thus: and then by Addition the iuſte ſumme doeth amounte, that is 8190, and therefore (as you ſaye) it ſeemeth to be wel wrought.

The proofe by Additiõ

3549
2866½
1365
409½
―――
8190

The iuſt proofe.

But I beſeeche you, is there anye doubte in this tryall, that you vſe that word, Seemeth?

Maiſter. You maye eaſilie coniecture, that if you did aſſigne the firſte mans ſhare to the laſte, and ſo chaunge all the reſt, that one had an others ſhare, yet woulde the Addition appeare al one, and therfore is not the proofe exact.

But if you wyl make a iuſte proofe for the firſte mans parte take $\frac{11}{31}$ of the whole ſum, and if it agree with the number in the figure, then it is well done. And ſo doe for the ſeconde, thirde, and fourth ſummes, and this proofe faileth not. Nowe will I propounde certaine other queſtions whiche haue bene ſette forth by certaine learned men, albeit not without ſome ouerſighte whiche queſtions I proteſt heartilie I doe not repeate to depraue thoſe good men, whoſe labours and ſtudies I muche prayſe and greatelie delighte in, but onelye accordinge to my profeſſion, to ſeeke oute trueth in all thinges, and to remoue all occaſions of errour, as muche as in me lyeth: and for that cauſe I will onelie name the queſtions wythout hurting the Authours name. The firſte queſtion is this.

A queſtion of building

Foure men did builde a houſe, which coſte them 3000 crownes, their ſhares were ſuch, that one man ſhould pay $\frac{1}{2}$ of the ſumme, and 6 crownes ouer: the ſeconde ſhould pay $\frac{1}{3}$, and 12 crownes ouer: the third man muſt lay out $\frac{2}{3}$ aba-

$\frac{2}{3}$, abating 8 crownes and the fourth mã ſhold pay $\frac{1}{4}$, and 20 crownes more, cã you anſwere to this queſtion?

Sc No in good faith ſir, and that you knowe beſt of anye man, for I knowe no more than you haue taught me.

Mayſter. Then I dare ſay you can not doe it. neither yet the beſt learned man that euer did propoſe it, for the queſtion is impoſſible: for declaration whereof I wil be bolde to vſe firſt the repreſentation of the nũbers in their apteſt form, (Although I haue not yet taught you that manner of worke) bicauſe it maye apppeare plainelie that the queſtion is not poſſible, for here I haue ſet the parts, and added them, and they make the whole ſumme and $\frac{1}{3}$ and 30 more. Now howe is it poſſible to diuide truely either gaines eyther charges ſo, that the particulars ſhall be more than the total.

An impoſſible queſtion.

$$\left\{\begin{array}{ll} \frac{1}{2}+ & 6 \\ \frac{1}{3}+ & 12 \\ \frac{2}{3}- & 8 \\ \frac{1}{4}+ & 20 \\ \hline 1\frac{1}{3}+ & 30 \end{array}\right\}$$

Sc. It is againſt the forme of proofe by Addition of partes.

Maiſter. You ſay truth And bycauſe you

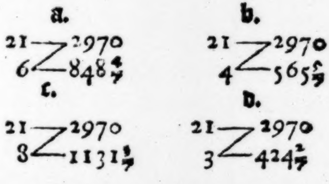
ſhal perceiue it the better, I wil trie it after the vulgare forme, as in this figure you ſee where the $\frac{1}{2}$ with 6 euer is 1506: for the totall is as you heard before 3000: the $\frac{1}{3}$ and the 12 more, is 1012: the $\frac{2}{3}$ woulde be 2000, but then abating 8, it is but 1992, and then laſte of al, the $\frac{1}{4}$ is 750, and the 20 more maketh 770: which al being added in one ſumme, doe make 5280, where the totall ſumme ſhoulde be but 3000, which ſum if you diuide by $\frac{4}{4}$, ſo ſhall you haue $\frac{3}{4}$ of it, that is 2250, and thereto adde 30 more, then wil thoſe 3 ſummes make 5280: wherby you may ſee how this forme as well as the other, doeth declare that the particulars in that queſtion woulde make more than the whole ſum by $\frac{3}{4}$, and 30 more: and therefore can that queſtion not be accepted as a poſſible thing, but yet doe certayne learned men propounde ſuche queſtions, and aunſwere to them. Therefore ſomwhat to ſay to their excuſe rather of their good meaning than for their doing, I will anone declare what may be ſaide for their defence

```
1506
1012
1992
 770
----
5280
```

```
3000
2250
  30
----
5280
```

ſence: but in the meane ſeaſon I will propounde the queſtion as it may be wrought by good poſſibilitie. As if foure men build a houſe togither, and it coſt them 3000 crownes, and then for the partition they agree thus: that as often as the firſte man doth paye 6 crownes, ſo often the ſeconde ſhall paye foure, the third man 8 and the fourth man three. Or els thus: that the firſte man ſhall pay double ſo muche as the fourth, and the ſeconde man ſhall paye $\frac{2}{3}$ of the firſte mans charge: the third manne ſhall pay double ſo much as the ſecond: (And theſe two wayes are to one ende) but further for their agreement it is appointed alſo, that the firſte man ſhal giue 6 crownes ouerplus, and the ſeconde 12, and the fourth ſhal giue 20, but the third man ſhal giue no ouerplus, but ſhal haue 8 crownes abated of his charge. Now is the queſtion poſſible to be aſſoyled, and this is the way to doe it. Marke the proportion of the ſeuerall charges, and ſet out ſmall numbers in that rate, by whiche you may reduce the worke to the Golden rule, as here in the firſt forme, the nūbers are alredie named, 6, 4, 8, 3: and in the ſeconde forme, although they be not plainly named, yet they may be the ſame nūbers: for 6 is double to 3, and 4 is $\frac{2}{3}$ of 6 : and againe 8 is double to 4. Now adde theſe togither and they make 21, whiche 21 muſt be ſet in the firſt number in the Golden rule:for if it with the ouerplus of eache mans charge woulde make the totall ſũ of the charges,thẽ were thoſe ſeueral ſums the charges of ech man, beſids his ouerplus: but now it is not ſo.

The rule. But yet this is true, that looke what proportion each of theſe ſeuerall ſums doth beare to 21, the ſame proportion doth the iuſt charges of euerie man (beſide his ouerplus) beare to the totall of the charges, the ouerplus being deducted: wherfore this maye you note, that before you do apply the total of the charges to the Golden rule: you muſt deducte the ouerplus whiche is 6, 12, and 20, that is in the whole 38: but then 8 muſt be reſtored for the abatement of the thirde man, and then remaineth to be deducted 30. Take 30 therfore out of 3000, and there will reſte 2970, whiche I muſt ſet in the Golden rule for the ſeconde ſumme: and for the third ſumme I muſt put ech of the ſmal numbers before mẽtioned, whiche although they be not the ſeuerall charges, yet they repreſent them in proportion. And ſo making for euerie mannes charge a ſeuerall queſtion, the figures will be 4, which I marke with foure letters, a.b.c.d. thus.



Where I haue ſet for brieſneſſe the ſumme of euerie mans charge in the fourthe place, preſuppoſing that you can tel how to trie out that fourth ſum by ſo many examples as yee haue had.

Sch. As I truſte that I vnderſtand thys fourme, ſo I deſire muche to knowe what maye be ſaide for thē that miſtooke this queſtion.

Maiſter. You ſeeme ſo deſirous to know this errour, that you haue forgotten to examine whether this worke bee wythoute

Scholer. Mee ſeemeth this worke to bee well done, bycauſe the Addition of the 4 ſeuerall numbers doth make the totall ſumme of 2970, whiche was to be diuided into ſuche foure parts.

Maiſter. But then haue you forgotten that the firſte man muſt pay 6 crownes more beſides this ſhare, and the ſeconde man twelue crownes more: the third mã 8 crownes leſſe: and the fourthe man 20 crownes more, for without theſe, your firſt total of 3000 crownes wil not be made.

Scho. Then muſt I adde to the firſt mans ſumme 6 more, and it wil be 854 $\frac{4}{7}$: and to the ſecond ſumme I muſt adde 12, and it wil be 577 $\frac{5}{7}$: from the thirde ſumme I muſte abate 8, and then wil the ſum be 1123 $\frac{3}{7}$: then adding vnto the 4 ſumme 20, it will be 444 $\frac{2}{7}$: and theſe 4 ſums will make 3000, whiche is the whole charge, as in this example it maye appeare, where firſte I gather the $\frac{14}{7}$ that maketh 2, and ſo procéede I in the Addition to the end.

854 $\frac{4}{7}$
577 $\frac{5}{7}$
1123 $\frac{3}{7}$
444 $\frac{2}{7}$
3000

Ma. Nowe haue you wel done, and this worke in the ſame ſums is brought of other learned men for the true ſolution of the queſtion as it was firſt proponed, which as (I ſaide) was impoſſible: and nowe examine it by theſe ſeuerall ſummes, and ſee whether it do agree with the ſummes in the queſtion proponed.

The firſt man muſt pay $\frac{1}{2}$ and 6 ouer of the total ſum: how thinke you, is $845\frac{4}{7}$ the halfe and 6 more of 3000:

Sc. No that it is not, for it would be 1506: and for the ſecond man 1012: and for ẏ thirde man 1992: and for the fourth mã 770, whereof not one ſumme agreeth to this worke. But I maruaile that ſo wiſe mẽ could be ſo much ouerſeene.

Ma. It is commonlie ſeene, that when men wil receiue things from elder writers, and will not examine the thing they ſeeme rather willing to erre with their auncients for companie, than to be bolde to examine theyr workes or writings, which ſcrupuloſity hath ingendered infinite errours in all kindes of knowledge, and in all ciuill adminiſtration, and in euerie kinde of arte: but theſe learned men did not meane anye other thing by this queſtion, than to finde ſuche numbers as ſhoulde beare the ſame proportion togither, as thoſe numbers in the queſtion proponed did beare one to an other: whiche thing you ſhal perceiue more plainelie by an other queſtion of theirs, that is this.

A queſtion of a teſtament.

A man lying vpon his death bed, bequeatheth his goods (whiche were worthe 3000 Crownes) in this ſorte. Bycauſe his wife was great with childe, and he yet vncertain whether the childe were a male or female, he made his bequeſt conditionally, that if his wife bare a daughter, then ſhould the wife haue halfe his goods, and the daughter $\frac{1}{3}$, but if ſhe were deliuered of a ſonne, then that ſon ſhould haue $\frac{1}{2}$ of the goods, and his wife but $\frac{1}{3}$. Nowe it chaunced hir to bring forth both a ſon and a daughter, the queſtion is: How ſhal they part the goods agreeable to the teſtatour his wil.

Scholer. If ſome cunning Lawyers hadde this matter in ſcanning, they woulde determine this Teſtamente to be quite voide, and ſo the man to die inteſtate, becauſe the teſtament was made inſufficient, ſith this condition was not expreſſed in it, and alſo it might haue chaunced that ſhe ſhoulde haue brought forth neither ſonne nor daughter, as often hath bene ſeene: ſo is the will vnſufficiente in that point alſo.

Maiſter. Suche ſcanners ſhoulde ſeeme to cunning, and yet not ſo cunning as cruel: for the minde of the Teſtatour is to be taken fauouranotablie, for the aid of ye legatories whē there riſeth ſuch doubts. But let vs trie this work, not by force of lawe, but by proportion Geometricall, ſéeing the teſtatour did minde to prouide for each ſort of them.

Sc. If the ſonne ſhal haue $\frac{1}{2}$ by force of the Teſtament, ſo muſt the mother haue $\frac{1}{3}$. Again becauſe ſhe hath a daughter alſo, therefore ought ſhe to haue $\frac{1}{2}$ and the daughter $\frac{1}{3}$: that is both wayes $\frac{1}{2}$ $\frac{1}{3}$, and $\frac{1}{2}$ $\frac{1}{3}$. whiche commeth to the whole goods, and $\frac{2}{3}$ more. Wherefore it ſéemeth alſo impoſſible.

Maiſter. In this matter the minde of the Teſtatour is ſo to be vnderſtande, that ſuche proportion ſhoulde be betwéene the portion of the wife and the ſonne, as is betwéene $\frac{1}{3}$ and $\frac{1}{2}$. that is, the ſonne muſte haue $\frac{1}{6}$ for $\frac{2}{6}$ to his mother, ſo ſhall he haue 3 to 2, that is as much as his mother, and half as much more: and the mother muſte haue the like rate in compariſon to hir daughter. Then muſte I finde out 3 numbers in ſuche proportion, that the firſte may be as muche as the ſecond, and halfe as muche more (that is) in proportion ſeſquialtera, and the ſeconde to the thirde in the ſame proportion, ſuche numbers be 6, 9, 4.

Sc. I pray you ſir, how ſhal I find out thoſe numbers?

M. That wil I gladly tel you.

To finde three nūbers in any proportion.

Whatſoeuer the proportion be of any thrée numbers, multiplie the Termes of that proportion togither and the number that amoūteth, ſhall be the middle number of the 3: thē multiple that middle number by the leſſer tearme, and diuide that totall by the greater, and the leaſt number of the 3 will amounte. So if you multiplie that middle number by the greater extreame, and diuide that total by the leſſer extreme, then wil the greateſt number of that progreſſion amount.

To finde the proportion betvvene 2. numbers.

Scholer Then in this example, to finde the proportion of $\frac{1}{2}$ to $\frac{1}{3}$, I muſte diuide (as you taught me in Diuiſion) $\frac{1}{2}$ by $\frac{1}{3}$, & the quotient will be $\frac{3}{2}$, that is 1 $\frac{1}{2}$. whereby I perceiue that the proportion in this queſtion is, as 3 to 2. Therefore (as you taughte me euen nowe) I multiplie 3 by 2, and the ſumme is 6, which muſt be the middle number: then I multiply the middle number 6 by 2, which is the leaſt terme, and the ſumme is 12, that doe I diuide by 3, being the greater Terme, and the quotient is 4, ſo is 4 the leaſte number of the 3. Then I multiplie 6 by 3, wherof of commeth 18, and that I diuide by 2 and ſo haue I 9, which is the greateſt number of the 3.

Ma. An other way yet maye you finde the third number in any progreſſion, if you haue 2 of them: for if the middle number be one of them which you haue, then multiplie it by it ſelf (as in this example 6 by 6 maketh 36) and that totall diuide by the other number which you haue, and the third number will be the quotient.

Sc. Then if I diuide 36 (which commeth of 6 multiplied by it ſelfe) by 4, the quotient will be 9, & if I diuide 36 by 9, the quotient will be 4. But what if I knowe the firſt nūber and the thirde, and would haue the middle number?

May. Multiplie the two numbers togither, and in their totall you muſt ſéeke the roote of that number, and it ſhall be the middle number: but becauſe as yet you haue not learned how to extract rootes, therfore vſe the firſt forme which I haue taught you, till I teache you to extract rootes. And now go forwarde with the aunſwere to the ſame queſtion.

Scholer. I perceiue then that the ſon muſte not haue $\frac{1}{2}$ of the goods, neither the mother $\frac{1}{3}$, nor yet the daughter $\frac{1}{5}$, but yet muſte the goods be diuided in ſuche proportion, that the ſonne ſhall haue 9 crownes for 6 to hys mother: and the mother ſhal haue 6 crownes for euerie 4 to the daughter. Then I applie it to the Golden rule in thrée examples thus: where the firſte number is the Addition of thoſe thrée numbers 9, 6, 4: and the thirde is one of them ſeuerallie: the ſeconde is the totall of the goods in the teſtament: & then by the worke of the Golden rule I finde out ye fourth number in euerie work that is for the ſonne 1705 $\frac{1}{19}$: for the mother 1136 $\frac{16}{19}$: and for the daughter 757, $\frac{17}{19}$, which thrée ſummes added togither do make the ſum of the whole goods, as may be ſéene by this example.

$$\begin{tikzcd}[column sep=small] 19 \arrow[r, dash] & 3600 \arrow[dl, dash] \\ 9 & \end{tikzcd}$$

$$\begin{tikzcd}[column sep=small] 19 \arrow[r, dash] & 3600 \arrow[dl, dash] \\ 6 & \end{tikzcd}$$

$$\begin{tikzcd}[column sep=small] 19 \arrow[r, dash] & 3600 \arrow[dl, dash] \\ 4 & \end{tikzcd}$$

1705	$\frac{5}{19}$
1136	$\frac{16}{19}$
757	$\frac{17}{19}$
3600	

And this (me thinketh) I doe perceiue, that bicauſe in this caſe ther is a neceſſary remedy diuiſed againſte an vrgente inconuenience, therfore thoſe lerned mē thoght they might vſe the like liberty in that other queſtion.

Maiſter. Your geſſe is good, but they had ſo good reaſon for them in the one, as they haue in the other: as in another example of theirs, it may better appeare, that is this:

Another queſtion of Teſtament.

A man left vnto his iij. ſons 7851 crownes, to be parted in this ſorte, that the firſte ſonne ſhould haue $\frac{1}{2}$. the ſecond ſonne $\frac{1}{3}$, and the third ſonne $\frac{1}{4}$, which is not poſſible, for $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ doth make $\frac{26}{24}$: or $\frac{13}{12}$, that is $1\frac{1}{12}$, ſo is it more than the whole: but reduce theſe fractions into one denominatiō, the leaſt that they wil come to, and they wil be $\frac{6}{13}$, $\frac{4}{13}$, $\frac{3}{13}$, and ſo may you part the goods in ſuch proportion as theſe 3 Numeratours beare togither: that is, the firſte to haue 6 for euery 4 to the ſeconde: and the ſeconde to haue 4 as oftē as the third hath 3: and ſo their portions wil be for the firſte, $2623\frac{7}{13}$: for the ſecond $2415\frac{9}{13}$: and for the thirde $1811\frac{10}{13}$. and thoſe 3 ſhares added togither, will make the totall ſumme of the whole goodes, as you may eaſily ſee in this example

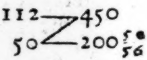
$$\begin{array}{r} 3623\frac{7}{13} \\ 2415\frac{9}{13} \\ 1811\frac{10}{13} \\ \hline 7851 \end{array}$$

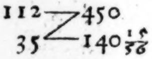
Another queſtion is there proponed thus:

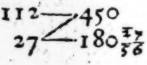
There is 450 Crownes to be diuided betwéen 3 men, ſo that the firſt man muſt haue $\frac{1}{2}$ $\frac{1}{3}$, the ſecond man $\frac{1}{3}$ $\frac{1}{4}$, the thirde man ſhall haue $\frac{1}{4}$ $\frac{1}{5}$.

Scholer. I maruaile that anye manne ſhould be ſo ouerſéene to propound that queſtion as a thing poſſible, ſith $\frac{1}{2}$ $\frac{1}{3}$, $\frac{1}{3}$ $\frac{1}{4}$, $\frac{1}{4}$ $\frac{1}{5}$, doe make 1 $\frac{1}{3}$ $\frac{1}{5}$, that is almoſt double the whole ſumme.

But I perceiue it might be thus proponed, that as often as the firſt man did receyue 50 Crownes, ſo often the ſecond man ſhould receyue 35, and the third man 27, for $\frac{1}{2}$ $\frac{1}{3}$, is equall to $\frac{50}{60}$, and ſo is $\frac{1}{3}$ $\frac{1}{4}$ equall to $\frac{15}{60}$, and $\frac{1}{4}$ $\frac{1}{5}$ is $\frac{27}{60}$, and ſo working the queſtion, the 3 figures will appeare in this forme: whereby the firſte mannes portion is founde to bée 200 $\frac{50}{56}$: the ſeconde mannes parte is 140 $\frac{15}{56}$: the third mans ſhare is 108 $\frac{24}{56}$: which in yͤ whole doth make 450 crounes yͭ was the whole ſumme to be diuided betwéene them.







Mayſter. And thus you are (I thinke) ſufficiently inſtructed in the rule of Felowſhip.

The Rule of Alligation.

The rule of mixture.

Now will I go in hād with the rule of Alligatiō, which hath his name, for that by it there are diuers parcels of ſundrie prices, and ſūdry quantities alligate boūd or mixed togither, whereby alſo it mighte be well called the rule of mixture, and it hath great vſe in compoſition of medicines, and alſo in mixtures of metals, and ſome vſe it hath in medicines of wines, but I wiſh it were leſſe vſed therein than it is now a dayes. The order of the rule is this: When any ſummes are proponed to be mixed, ſet them in order one ouer an other, and the common number whereunto you will reduce them, ſet on the left hand, then marke what ſummes be leſſer than that common number, and whiche bee grea-

The reaſō. of this rule

greater and with a draught of your penne, euermore linke 2 numbers togither, ſo that one bee leſſer than the common number, and the other greater than hee, for two greater or two ſmaller can not well be linked togither, and the reaſon is this, that one greater & one ſmaller may bee ſo mixed, that they wil make the meane or common number very wel, but 2 leſſer can neuer make ſo many as the common number, being taken orderly: no more can two ſummes greater than the meane, neuer make the meane in due order as it ſhal apeare better to you hereafter. And as it is of neceſſity to lynke euery ſmaller (once at the leaſte) with one greater, and euerye greater with one ſmaller: ſo it is at libertie to linke them oftner than once, and ſo may there bee to one queſtion many ſolutions. When you haue ſo linked them, then marke how muche eche of the leſſer numbers is ſmaller than the meane or commō number, and that differēce ſet againſt the greter nombers which be linked with thoſe ſmaller eche with his match ſtill on the right hande, & likewiſe the exceſſe of the greater numbers aboue the meane, you ☞ ſhall ſet before the leſſer numbers which bee combined with them. Then ſhall you by Addition

dition bring all theſe differences into one ſumme, which ſhal bée the firſt number in the Golden rule: and the ſeconde number ſhal be the whole maſſe that you wil haue of al thoſe particulers: the thirde ſumme ſhal be ech difference by it ſelfe and then by them ſhall bée founde the fourth number, declaring the iuſt portion of euery particular in that mixture. As nowe by theſe examples I will make it playne.

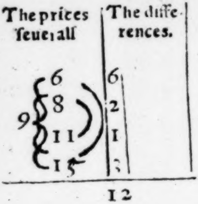
A queſtion of mixing of wine.

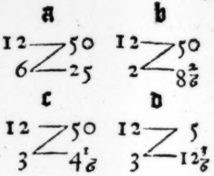
There is foure ſorts of wine of ſeueral prices, one of 6 d a gallon, an other of 8 d the thirde of 11 d, and the fourth if 15 pens the galon, of al theſe wines would I haue a mixture made to the ſumme of fiftie galons, and ſo that the price of eche galon may be 9 pens. Now demande I howe much muſt bée taken of euery ſort of wine?

Scholer. If it ſhall pleaſe you to worke the firſt example, that I may marke the applying of it to the rule, then I truſt I ſhall be able not only to doe the like, but alſo to ſée reaſon in the order of the worke.

Maiſter. Marke then this forme and the placing of euery kinde of number in it.

The common price. ☞





Here (you ſee) I haue ſette downe the ſeuerall prices which be 6, 8, 11, 15, and haue linked together 6 with 15, and 8 with 11. The common price 9, I haue ſet on the lefte ſide: And the difference betwéene it, and euery particular price, I haue ſet on the right hand not againſt the ſumme, whoſe differẽce it is, but againſt the ſumme that it is linked withall: ſo the difference of 15 aboue 9, is 6, which I haue ſet not againſt 15, but againſt 6, that is linked with 15, and the difference betweene 6 and 9 (that is 3) I haue ſet againſt 15. So likewiſe yͤ difference betwene 8 and 9 is but 1, that haue I ſet againſt 11, and yͤ difference of 11 aboue 9 (which is 2) I haue ſet againſt 8. Then adde I all thoſe 4 differences, and they make 12, which I ſette for the firſt number in the Golden rule: the ſeconde number I make 50, which is the ſumme of gallons that I woulde haue, and the thirde ſumme is euerie particular difference. Nowe if you worke by the Golden rule, you ſhall finde the number of Gallons that ſhall be taken of eache ſorte of wine: For the better diſtinction whereof, I haue ſette theſe letters a b c d both agaynſte the numbers for which the workes do ſerue, and ouer the workes alſo, whiche ſeuerallye ſerue for eache of them. And nowe if you liſte to examine the truthe of theſe workes, adde thoſe foure ſummes togither, and they wyl make fiftie, that is the totall whiche I would haue, as by this example you may eaſily perceiue. And for to proue how ye prices do agree, doe this. Multiplie this totall ſumme 50, by the common price 9, and it will make 450: then kepe that ſumme by it ſelfe, and afterward Multiplie euerie ſeuerall ſumme of Gallons, by the price belonging to the ſame Gallons, and if that ſumme doe agree with this, whiche you haue kepte firſte,

The proofe of this rule.

$$\begin{array}{l} 25 \\ 8\frac{2}{6} \\ 4\frac{1}{6} \\ 12\frac{3}{6} \\ \hline 50 \end{array}$$

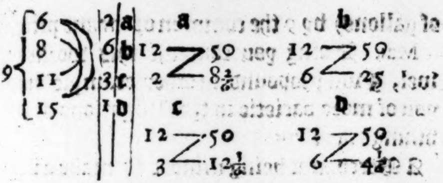
then is your worke well done. As here, 25 is the number of gallons of 6 d. price, multiplie then 25 by 6, and it maketh 150, which you ſhall ſet downe: then multiplie $8\frac{2}{6}$ by 88 which is the price for yͤ number of Gallons, and it will make $66\frac{4}{6}$: ſo againe $4\frac{1}{6}$ multiplied by 11 doth make $45\frac{5}{6}$. And laſt of al $12\frac{3}{6}$ multiplied by 15, maketh $187\frac{1}{6}$. And theſe added togither doe make 450, as in the example annexed you may ſee: wherefore ſeeing it doth agree with the former ſum of 50, multiplyed by 9, I may iuſtlie affirme this worke to bee good and wel done.

$$\begin{array}{r} 150 \\ 66\frac{4}{6} \\ 45\frac{5}{6} \\ 187\frac{1}{6} \\ \hline 450 \end{array}$$

The variation of this queſtion.

And now to proue how you can do the like, I propounde the ſame queſtion, only willing you to vſe ſome other forme of combining or linking the ſummes.

Scho. That ſhall I proue with your fauoure, and therefore I combine 8 with 15, and 6 with 11, and then the forme wyll be as foloweth:



whereby amounteth the ſame ſumme in totall of the differences, as did before: and yet nowe the differences be altered, as the combination is chaunged, whereof I vnderſtande the reaſon by your former worke. And therfore here appeareth no ſtraunge thing, but that nowe I muſte haue 8 $\frac{2}{6}$ gallons, of ſixe pence and 25 gallons of 8 d. and 12 gallons and $\frac{1}{6}$ of 11 d., and ſo conſequently 4 gallons and $\frac{1}{6}$ of 15 d., ſo ẏ multiplying 8 $\frac{2}{6}$ by 6, it maketh 50, and the 25 multiplied by 8, maketh 100: likewiſe 12 $\frac{3}{6}$ multiplyed by 11, yéelded 137 $\frac{3}{6}$, and 4 $\frac{1}{6}$ multiplied by 15, maketh 62 $\frac{3}{6}$, whiche foure ſummes added in one, will yéelde in the totall 450, whiche agréeth wyth the Multiplication of 50 (being the total ſumme

$$\begin{array}{r} 50 \\ 200 \\ 137\frac{3}{6} \\ 62\frac{3}{6} \\ \hline 450 \end{array}$$

of gallons) by 9 the common or meane price.

Mai. Seing you conceiue this worke ſo wel, I wil propound an other example vnto you of more varietie in the Alligations or cō-binings: as thus.

queſtion
ſpices.

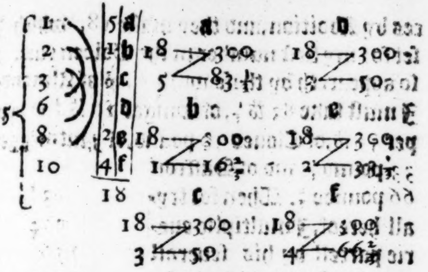
A Merchaunt being minded to make a bargaine for ſpices in a mixte maſſe, that is to ſay, of Cloues, Nutmegges, Saffron, Pepper, Ginger, and Almonds, the Cloues being at 6 ß. a pounde, the Nutmegges at 8 ß. Saffron at 10 ß. Pepper at 3 ß. Ginger at 2 ß. and Almonds at 1 ß.

Now woulde he haue of eche ſorte ſome, to the value of 300 ℔, in the whole, and ech poūd one with an other to beare in price 5 ß. howe much ſhal he haue of ech ſort?

Sch. That wil I trie thus.

Firſt I ſet down thoſe ſixe ſeuerall prices, and at the left hand I ſet the common price 5 ß. Then I linke thē thus, 1. with 10, 2, wyth 6, and 3 with 8. As in the example following.

Mai.



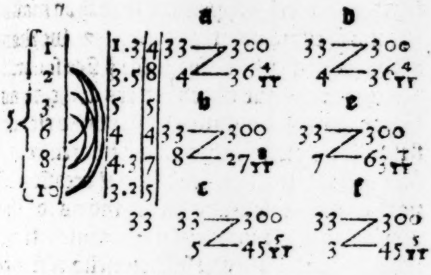
Maiſter. I hadde minded to haue combined them in more varieties, but I am content to ſee your owne worke firſte, and then more varieties in combination maye followe anone.

Scholer. Then to continue as I beganne, I ſeke the difference betwene 1 and 5 (which is 4) and that I ſet againſt 10: then againſte 1. I ſet 5, whiche is the exceſſe of 10 aboue 5: ſo I gather the difference betwene 2 and 5, whiche is 3, and that I ſet againſt 6, bicauſe it is combined with 2: and likewiſe the difference of 6 aboue 5 (which is 1) I ſet againſt 2. Then take I the difference of 3 from 5, whiche is 2, and that I ſet againſt 8, and before that 3 I ſet the difference of 8 aboue 5, whiche is 3. Then gather I al theſe differences by Addition, and they make 18, which I ſet for my firſt number in the Golden rule, & ſo appeareth by thoſe workes, yͤ of Almonds I muſt take 83 ℔ $\frac{1}{3}$, of Ginger 16 ℔ $\frac{2}{3}$, of pepper 50 ℔, of Cloues 50 pound, of Nutmegges 33 pound $\frac{1}{3}$, and of Saffron 66 pounde $\frac{2}{3}$. Then for tryall hereof, I multiplie euerie parcell by his ſeuerall price, as 83 $\frac{1}{3}$ whiche is the ſumme of Almondes, I multiplie by 1, whiche is their price.

83 $\frac{1}{3}$
33 $\frac{1}{3}$
150
300
266 $\frac{2}{3}$
1500

Alſo 16 $\frac{2}{3}$ the ſumme of Ginger I multiplie by 2, which is the price of it. And ſo each other in his kind, as this table annexed doeth repreſent: and then adding them altogither, I finde the total to be 1500, which alſo will amounte by the multiplication of the groſſe maſſe of 300 by the common price 5, wherefore it appeareth wel wrought.

Ma. Nowe will I make the Alligation to proue your cunning ſomewhat better: but bycauſe you ſhall not thinke your ſelfe preſſed ſo much, I will alſo note the differences, as in this example you may ſée.



where I haue alligate 1 with 6 and 8 : and therefore haue I ſet againſt 1, both their differences: that is 1 and 3. Likewiſe bycauſe 2 is combined with 8 and 10, I ſet before hym their differences, 3 and 5. Againſte 3 I haue ſette onlie 5, whiche is the difference of 10, with whome 3 is combined onelye : likewiſe 6 is onelie Alligate to 1, and therfore is the difference of 1 onlie ſet againſte it : 8 is lincked with 1 and 2 and therefore hathe hee againſt him both their differences, 4 and 3 : and 10 is ioyned with 2 and 3, therefore hath he their differences 3 and 2. And becauſe of eaſe for you, in an other columne I haue ſet the differences reduced into one number, for euerie ſeuerall ſort, and haue alſo added thē togither, wherby appeareth that they make 33, and ſo conſequently you ſée the workes of the golden rule ſet forth for ẏ ſixe ſeueral drugges : I haue added letters a, b, c, &c as before. But I would not wiſh you to cleaue ſtil to theſe elemētary aides, but accuſtome memorie to truſt hir ſelf, ſo ſhal occaſion of negligence be beſt auoyded. And as for the profe, trie it at more leyſure, becauſe ẏ time now is ſhort, and you ſufficientlie inſtructed in that profe. And there reſteth diuerſe things behinde yet, of whiche I woulde gladlie giue you ſome taſte before our departure.

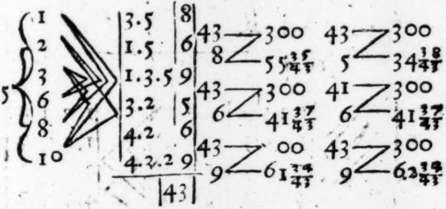
Scholer. But if it may pleaſe you to lette me ſée al the variations of this queſtion, before you go from it, for me thinketh I could varie it two or thrée wayes more yet.

Ma. I am content to ſée you make two or thrée variations, but I would be loth to ſtay to ſée al the variations, for it may be varied aboue [illegible]00 wayes although manye of them would not wel ſerue to this purpoſe.

Sc. I thought it impoſſible to make ſo many variations.

Ma. Meruaile not therat, for ſome queſtions of this rule may be varied aboue 1000 wayes, but I woulde haue you forget ſuche fantaſies, till a time of more leyſure. And now go forward with ſome variation of this queſtion.

Scholer. For the firſt variation, I linke the firſt number 1 with 8 and 10, and 2 I combine with 6 and 10, then ioine I 3 wyth 6, 8, and 10, as in this forme.



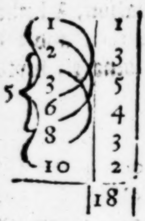
And ſo doeth there appeare the portion of weight for euery kind of drugge in this mixture. Now for the triall.

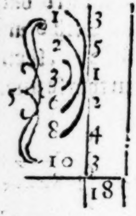
Maiſter. Nay ſtay there, you ſhal not neede to make triall in one example ſo often, or if you liſte to do it by your ſelfe, I am content. But nowe ſette forth (for declaration that you conceiue the rule) two or three examples of ſeueral combinations, and then will wee

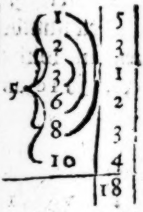
paſſe

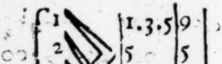
and then will we paſſe to ſome other example and ſo ende this rule.

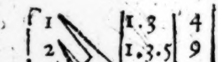
Scholer. As it pleaſeth you ſo will I doe. And theſe be the varieties in whiche as the







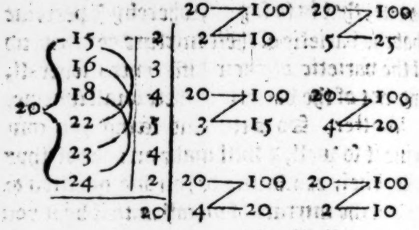




combinations are ſeuerall, ſo doth it plainely appeare, that the differences by whyche the proportion of eache ſeuerall kinde is taken, are alſo ſeuerall. And yet I ſee in the thrée firſt of theſe fiue varieties, and in one other, before, the totall ſumme of the differences to be one, that is to ſay 18, whereby I perceiue that the varietie of their mixture doth depend of the varietie of their differences ſeuerall, and not of the varietie of their totall ſumme.

Maiſter. So is it. And ſeeing you conceiue it ſo well, I will make an ende of thys rule, onely exhibiting to you one queſtion or two of the mixture of metalles, that by it you may deuiſe other like, and exerciſe your ſelf therein alſo, bicauſe the vſe of it ſerueth often in buſineſſe of charge, not ſo much for Goldſmithes, as for coynage in mintes. Firſte I demaunde of you this queſtion. If a Mintemaiſter haue golde of 22 karectes, and ſome of 23 karectes, ſome of 24: Againe, ſome of 15, ſome of 16, and ſome of 18 karectes, and would mixe them ſo, that he might haue 100 ounces of 20 karects, howe much ſhal he take of euery ſorte?

Scholer. To knowe that, I aunſweare in order thus.



Maiſter. You haue wrought the queſtion well, but howe chaunced you made no doubt of that newe name, Karecte?

Scholer. Becauſe I thoughte it out of time to demaund ſuche queſtions now, ſeeing you make ſo much haſte to ende: and againe in this caſe the proportion of the numbers is ſufficient for my purpoſe in this worke, truſting, that an other time you will inſtruct me as wel of this, as of ſundry other things, which I haue heard you talke of, ſo I haue a great deſire to knowe them.

Maiſter. Your aunſweare is reaſonable: and your requeſt and truſt with Gods helpe I intende to ſatiſfie. And to goe forwarde with this matter, let me ſee your examination of this laſt worke.

Scholer. Firſt for the one part I adde together gither all the particular ſummes as they appeare in the woꝛke, and they make 100, as here by their Addition it doth appeare.

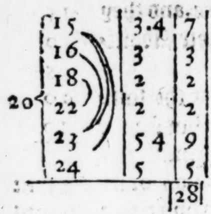
10
15
20
25
20
10
———
100

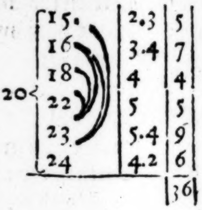
And ſo it ſeemeth, that the ſummes are well gathered, but foꝛ the farther triall of them, I multiplye firſt 20. which is the common oꝛ meane ſum of the karectes by 100, whiche is the ſumme of the whole maſſe which I woulde haue and it maketh 2000. Then I multiply euerye particular ſumme by the karects that it doeth containe, as 10 by 15, and that maketh 150. Likewiſe I multiplye 15 by 16, and it yeeldeth 240: ſo 20 by 18 maketh 360 And 25 by 22 yeeldeth 550: likewiſe 20 by 23 bꝛingeth foꝛth 460: and laſt of al 10 multiplyed by 24, yeeldeth 240: whiche ſummes all ioined togither make 2000, that doeth agree with the like ſumme befoꝛe: wherefoꝛe I may well ſaye, that the woꝛke is good. And nowe if it pleaſe you I would ſette foꝛth ſome varieties of this queſtion, to pꝛoue my witte.

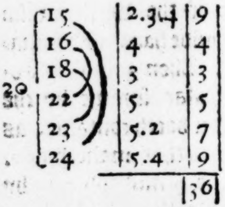
150
240
360
550
460
240
———
2000

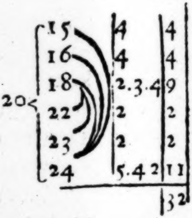
Maiſter. Goe to, let me ſee.

Scholer. Here be foure varieties.









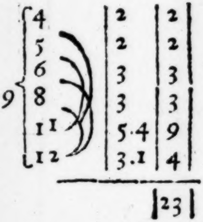
And more yet I coulde make, but not like to the number that you ſpake of in the variation of the other queſtion.

Maiſter. That will I teache you at more leiſure, ſéeing it is a thing rather of pleaſure, than of any neceſſitie.

But nowe for youre exerciſe in this rule, one other queſtion I will propoſe. A minte maiſter hathe 6 ingottes of ſiluer of ſundrie fineneſſe, ſome of 4 ounces fine, and ſome of

5 ounces, ſome of 6, and other of 8, ſome of 11, and other of 12: and his deſire is to mixe 500 pound weight, ſo that in the whole maſſe euery pound weight ſhoulde beare 9 ounces of fine ſiluer, howe muche ſhall he take (saye you) of euery ſort of ſiluer?

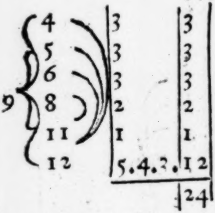
Scholer. To finde out that I ſette the numbers thus in order.

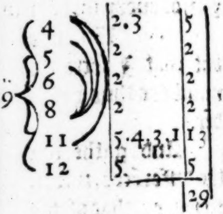


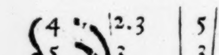
And gathering the differẽces, it will appeare, that of the firſte ſort there muſt be 43 $\frac{11}{23}$: of the ſecõde like muche: of the third ſorte, 65 $\frac{5}{23}$: and of the fourth ſorte as muche: of the fifth ſorte 195 $\frac{15}{23}$: and of the ſixth ſorte 86 $\frac{22}{23}$, whiche in the whole wyll make 300 ℔ waight: and in ounces after 9 ounces fine 4500, that is of the firſte ſorte 173 $\frac{21}{23}$: and of the ſecond ſorte 217 $\frac{9}{23}$: of the third ſorte 391 $\frac{7}{23}$: of the fourth ſorte 521 $\frac{17}{23}$: of the fifth ſorte 2152 $\frac{4}{23}$, and of the ſixth ſorte 1043 $\frac{11}{23}$, whiche al togither doe make 4500 ounces, agreable to the multiplication of 9 by 500.

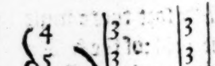
Maiſter. This is well done of you, therefore nowe make thrée or foure variaties, and ſo an ende of this rule.

Sc. Theſe 4 varieties I ſet for example.

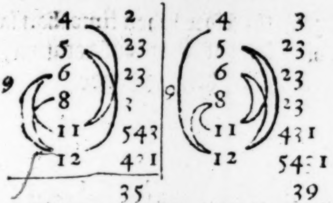








Maiſter. And by theſe it appeareth, that you can finde out more, with which I wil not nowe meddle, ſaue onely for to ſhewe you an eaſie help in drawing the lines of Combination, I wil ſet forth two varieties here.



And this ſhall ſuffiſe now for the rule of Alligation or mixture, for by theſe examples may you eaſily coniecture ſuche other as doe appertaine to it, as wel for the due working, as for variety of drawing the lines of combination.

Scholer. Sir, albeit it pleaſed you ere while, to put me from my muſing at the manifolde varieties, that may fall in theſe combinations, and termed them fãtaſies, yet my fantaſie giueth me, that the conſideration of this ſhoulde in many other examples and caſes of importance be very néedefull, and the knowledge of it moſt profitable. Therefore ye may wel thinke, that at another time conuenient I will requeſt you to aide me héerein.

Maiſter. Truth it is, that this conſideration may fall in practiſe as wel Politicke, as Philoſophicall, and ſundrie wayes in them be applyed, therefore when time ſhall fall fit for the diſcuſſing of this conſideration, you ſhal not want my helping hande.

The rule of Falſehode.

The occaſion of the name.

Nowe wil I briefly alſo teach you ſomwhat of the rule of Falſhod, whiche beareth hys name, not for that it teacheth any fraude or Falſhoode, but for that by falſe numbers taken at all aduentures, it teacheth howe to finde thoſe true numbers that you ſeeke for.

Scholer. So might any other rule be called the rule of Falſhoode, for they worke by wrong numbers, and by them finde out the right numbers, ſo doth the rule of Alligation, the rule of Felowſhip, and the Golden rule partely.

Maiſter. In the Golden rule, the rule of

Felowſhip, & the rule of Alligation, although the numbers that you worke by, be not the true numbers that you ſeeke for, yet are they numbers in iuſte proportion, and are founde by orderly worke: whereas in this rule, the numbers are not takẽ in any proportion, nor founde by orderly worke, but taken at all aduentures.

And therefore I ſometimes being merrye with my friendes, and talking of ſuche queſtions, haue cauſed them that proponed ſuche queſtions to call vnto them ſuche children or ideots, as happened to be in the place, and to take their anſweare, declaring, that I would make them ſolue thoſe queſtions, that ſemed ſo doubtfull.

And indeede I did anſweare to the queſtion and worke the triall thereof alſo, by thoſe anſweares which they happened at al aduentures to make: which numbers ſeeing they be taken as manifeſt falſe, therefore is this rule called the rule of falſe poſitions, and for briefneſſe, the rule of Falſhode, which rule for readineſſe of remembrance, I haue compriſed in theſe few Verſes following, in form of an obſcure Riddle.

Geſſe at this worke as happe doth leade,
By chaunce to truth you may proceede.
And firſte worke by the queſtion,
Although no truth therein be done.
Such falſehoode is ſo good a grounde,
That truth by it wil ſoone bee founde.
From many bate to many mo,
From to fewe take to fewe alſo.
With too much ioine to fewe againe.
To too fewe adde to many plaine.
In croſſe waies multiply contrarie kinde,
All truth by falſehoode for to finde.

The ſenſe of theſe Verſes, and the ſumme of this rule, is this:

When any queſtion is proponed appertaining to this rule, firſte imagine any number that you liſte, whyche you ſhall name the firſte poſition, and put it in ſteede of the true number, and then worke with it as the queſtion importeth: and if you haue miſſed, then is the laſte number of that worke, eyther too greate or too little: that ſhal you note as hereafter ſhall be taught you, and you ſhall call it the firſte errour.

Then beginne againe, and take an other number whiche ſhall be called the Seconde poſition, and worke by the queſtion : if you haue miſſed again, note the exceſſe or default as it is, and call that the ſecond errour. Then multiply croſſe waies the firſt poſition by the ſecond errour, and againe the ſeconde poſition by the firſte errour, and note their totalls ſeuerally by the names of Totalles. Then marke whether the two errours were bothe like, that is to ſay, both too much, or bothe too little: or whether they be vnlike, that is, the one too much, and the other too little, for if they be like, then ſhall you ſubtract the one totall from the other (I meane the leſſer from the greater) and the Remainer ſhall be your diuidend, ſo muſte you abate the leſſer error out of the greater, and the reſidue ſhal be the diuiſour. Now diuide the diuidend by that diuiſour, and the quotient will ſhew you the true number that you ſeeke for. But and if the errours be vnlike, then muſt you adde both thoſe totalles (whych you noted) togyther and take that whole number for the diuidend, ſo ſhall you adde both the errours togither, and that whole number ſhall be the diuiſour, and the quotient of that Diuiſion ſhall giue you the true number that the queſtion ſeeketh for: and thys is the whole rule.

Scholer. This rule ſeemeth ſo vnlike any other, that without ſome example I ſhal not eaſily vnderſtande it.

Maiſter. Therefore take this example: A Maſon was bouñd to build a Wall in 40 days, and it was couenaunted ſo with him, that euery daye that he wrought, he ſhoulde haue for his wages 2 ß, 1 penny, & euery daye that he wrought not, he ſhoulde be amerced 2 ß, 6 d, ſo that when the wall was made, and the reckning taken of the dayes that he wrought, and of the other that he wrought not, the Maſon had clearely but 5 ß, 5 d, for his worke. Nowe doe I demaund, how many dayes did he worke of thoſe 40, and how many did hee not worke?

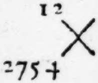
Scholer. I praye you expreſſe the order of the worke, that I may partely by imitation, and partly by comparing it with the rule, bée able againe to do the like.

Maiſter. This order ſhall you kéepe in the worke of this rule: firſte take ſome number (as you liſt) at aduenture, as for example: I ſaye he played 12 dayes, and wrought 28 dayes. Nowe caſte you the wages of euerye daye, and ſée whether it will agrée with the ſumme of 5 ß, 5 d.

Scho

Scholer. The 28 dayes that he wrought after 25 pence the daye, yeeldeth 700 d. Then the 12 dayes that hee wrought not, at 30 pence eche daye, doth amount to 360 pence, whiche if I abate out of 700 pence, there reſteth 340: but you ſay he had not ſo much.

Maiſter. He hadde but 65 pence, and by this ſuppoſition he ſhoulde haue hadde 340: therfore is this ſumme too much by 275 whiche ſumme I muſte ſet downe after this ſort, as you ſee here, where firſte I haue made a croſſe (commonlye called Sainct Andrewes croſſe) and at the ouer corner on the left hand I haue ſette the firſte poſition 12, and at the other corner vnder it, I haue ſet 275 which is the firſt erroure, with this figure +, whyche betokeneth too muche, as this line, ——— plaine without a croſſe line, betokeneth too little. On the right hande of the croſſe I haue lefte two like rowmes for the ſecond poſition and his errour. Therfore to proſecute the worke, I ſuppoſe he played 16 dayes, and wrought 24.



Scholer. I was a while in doubte why

you named the dayes of hys working, ſéyng they be not ſet in the figure: and I doubted howe you knewe them, or elſe whether that you did ſuppoſe them at all aduentures, as you didde the dayes that hée played: but now I gather, that ſéeing 40 dayes is the whole time limited, then the dayes that hée played being ſuppoſed the reſt of 40 muſte néedes be the dayes that he wrought, and therefore 28 followed 12 of neceſſitie and 24 foloweth 16 alſo of neceſſitie: but yet I ſcarce perceyue why you ſet not in the figure as well 28 as 12.

Maiſter. It forceth not whiche of them I take, ſo that in the ſeconde poſition I take the numbers of the ſame nature that is here bothe of working dayes, or bothe of ydle: but nowe examine you this ſeconde poſition.

Scholer. If he played 16 dayes, then abating 16 times 30 d. the ſumme will be 480 d. And for 24 dayes that he wrought, euery day yéelding 25 d, the totall is 600 d: ſo that abating 480 out of 600, there reſteth 120: and as you ſay it ſhoulde be but 65, therefore it is too much by 55, that muſt be ſette on the right hand of the figure at the neather parte, and ouer it on the ſame ſide 16 which is the ſeconde poſition, thus. And as I gather by your wordes, it were al one if I did ſet 28 in ſteade of 12, and 24 in ſteade of 16.

12 16
╳
275+ 55+

M. So were it. But this ſhall you marke, that of what nature ſo euer the two poſitions be, of the ſame nature is the quotient. Therefore when the poſitions in this queſtion are 12 and 16, which both being numbers of the playing dayes, the quotiente ſhall declare the true numbers of playing dayes, where as if the poſitions had bene 28 and 24, whyche are ſuppoſed to be the working dayes, then woulde the quotient declare the true number of the working dayes, & not of playing dayes as it will doe nowe. And therefore to continue the worke of this queſtion, and to finde the true number of playing dayes, I muſte multiplie croſſe wayes the firſte poſition 12 by 55, that is the ſeconde erroure, and the totall will be 660, then I multiplie 275 by 16, and it yéeldeth 4400. Now bicauſe the errours are like, that is to ſaye, both too much, I muſte ſubtracte 660, out of 4400, and ſo re-

☞ Note.

The profe of this rule.

remaineth 3740, whiche is the diuidend. Againe I muſt ſubtract the leſſer errour 55 out of 275 that is the greater errour, and there wil remaine 220, which ſhall be the diuiſor, then diuiding 3740 by 220, the quotient wil be 17. Wherefore I ſaye nowe conſtantlie, that 17 is the true number of dayes that the Maſon played: and then it followeth, that he wrought 23 dayes, and ſo is the queſtion anſwered.

Nowe for the order of triall of this worke there needeth none other proofe but only thys, to worke with this number according to the queſtion, and if it agree, then appeareth the number to be it that you would haue As here nowe ſeeing he wrought 23 daies, and muſte haue for euerie day 25 pence, the whole ſum commeth to 575. Then again ſeeing he played 17 dayes, and muſte abate 30 pence for euerie daye, the whole ſumme of the abatement will be 510: therefore I ſubtracte 510 out of 575, and there wil remaine 65, which maketh 5 ß, 5 d. the cleare wages of the Maſon for his worke, according to the queſtion.

Scholer. Nowe I truſt I vnderſtand the worke and the rule ſo well (and the better by

this

thys proofe) that I can be able to do the like. And for a proofe I take the ſame queſtion all ſaue the laſte number, where I will ſuppoſe that he hadde 10 ℔, for his wages cleere. And now to geſſe at the number of yͤ days that hee wrought, I ſuppoſe firſt that he wrought 20 dayes, then ſay I, if he wrought 20 dayes his wages muſt be 500 d, then did he play other 20 dayes, for whych muſt be abated 600 d, and then he leeſeth 100 d. And ſo am I at a ſtays, for it is not like unto your former worke.

Ma. You ſhoulde haue required of me ſome queſtion, and not haue taken a queſtion of your owne fantaſying, untill you were more expert in this arte: for ſo might you as well happen on an impoſſible queſtion as on a poſſible: but now to go forwarde, conſider that this number is too little by 220, ſeeing he ſhould gaine by your ſuppoſition 120 d, and in this poſitiõ he leeſeth 100, theſe both make 220, whiche you ſhall ſet downe for the firſt error with this ſigne —, betokening to little, as here in this forme following doeth appeare.

And nowe for the reſt go forwarde youre ſelfe once againe,

20
X
220 —

Scholer. As my errour hath vttered my follie, ſo it hath procured me better vnderſtanding. Nowe therefore conſidering this poſition not to ſolue the queſtion, I take an other, ſuppoſing that he wroughte 30 dayes, then for his wages he muſt be allowed 750 d., and for the 10 dayes whyche he wrought not, he muſte abate 300 d., and ſo remaineth cleare 450 d.: but it ſhould be onely 120 d., therfore is it to much by 330, whiche I ſette downe in the figure with the former poſition, and his erroure and the figure appeareth thus.

20 30
X
220 — 330 +

Nowe muſt I multiplie in croſſe wayes 220, by 30, and it will be 6600. Then againe I multiplie 330 by 20, and it will be alſo 6600. Wherefore if I ſhall ſubtract the one out of yͤ other, there will remaine nothing to be the Diuidend.

Ma. In this you forget your ſelfe again: for in as much as the ſignes in the errors be vnlike,

like, therefore muſt you worke by Addition, adding togither thoſe two totals to make the diuidend, and alſo adding the two errours to make the diuiſor. And bycauſe you ſhall no more forgette this part of the rule, take this briefe remembrance:

Vnlike require Addition,
And like deſire Subtraction.

Sch. You meane, that if the errours haue like ſignes, then muſte the diuidend and the diuiſor be made by Subtraction, as is taught before: And if thoſe ſignes be vnlike (as in this laſte example they be) then muſte I by Addition gather the Diuidend and the diuiſor. Therefore muſt I adde 6600 to 6600, and it will be 13200, whiche ſhall be the diuidend. Then againe I adde 220 to 330, and it will be 550, whiche muſte be the diuiſor: wherefore diuiding 13200 by 550, the quotient will be 24, whereby I knowe that the Maſon wrought 24 dayes: and then it followeth that he played 16 dayes.

M. Examine your worke whether it bée agréable to the queſtion or no.

Sc. For 24 dayes worke, the wages muſte

be 600 đ. and for 16 dayes whiche the Maſon wrought not, there muſte be abated 480, and then remaineth cleare to the Maſon 220 pence, as the queſtion importeth, wherefore it is euident, that 24 is the true number of the dayes that he wrought.

Maiſter. Althoughe you ſeeme nowe to vnderſtande this worke, yet to acquainte youre minde the better with the newe trade of this rule, I thinke it good to propone to you fiue or ſixe examples more, before I make an end of it.

Sch. Sir I thanke you, that you do ſo conſider my commoditie and profit in knowlege, for vndoubtedlie it is practiſe & exerciſe that maketh men prompt & expert in euerie kinde of knowledge.

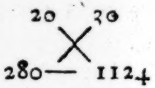
Maiſter. You ſay well, ſo that they follow ſome certaine preceptes to gouerne and rule their practiſe by, elſe maye practiſe procure cuſtome of error, and a repugnance to exactneſſe of knowledge, namelie as long as the error is not plainelie knowen to the vulgare ſorte. But to returne to our worke. There is a ſeruant that hath boughte of veluet and damaſke for his maiſter 40 yeardes, the veluet at 20 ß, a yeard, and the Damaſke at 12 ß, &

when he commeth home, his maiſter demandeth of him howe muche he hath boughte of each ſort: I cā not tel (ſaith he) exactlie, but this I knowe, that I paide for damaſk 48 ß. more than I paide for veluet, nowe muſt you geſſe how manie yeardes there is of each ſort.

A queſtion of vvares.

Scho. Although the geſſe ſeemeth difficulte, yet I wil proue what I can do: for I remember your ſaying, that it forceth not how fonde or falſe the geſſe be, ſo it be ſomewhat to the queſtion, and not an anſwere of a contrarie matter.

Therefore firſt I imagine that he boughte 20 yeardes of Damaſke, for which he ſhould paye after the former price 240 ſhillings: then muſte he needes haue of veluet other 20 yeardes (to make vp the 40 yeardes) and that woulde coſte 400 ß. So that the total of the price of the damaſke is leſſe than the ſumme paide for veluet 160 ß, and ſhould be more by 48. therefore the firſt errour is 208 too little. Then begin I again, and ſuppoſe he boughte of Damaſke 30 yeardes that coſt 360 ß, then had he but 10 yeardes of Veluet, which coſte 200 ß: and nowe the price of Damaſke is greater than the price of the Veluet by 160 ſhillings, and ſhoulde be but 48, therefore is the ſecond erroure 112 to muche, whiche I ſette in forme of a figure as here doeth appeare. Then doe I multiplie in croſſe wayes. 280 by 30 and the ſum wyll be 6240. Alſo I multiplie 112 by 20, and there wil amounte 2240. And in as muche as the ſignes of the errours be vnlike, I knowe I muſte worke by Addition, therefore adde I thoſe two totals togither, and they make 8480, whiche is the diuidend: then adde I alſo the two errors togither. 208 and 112, and they make 320, whiche is the diuiſor. Wherefore diuiding 8480, by 320, the quotient wil be. 26 $\frac{1}{2}$, whiche is the true ſumme of yeardes of Damaſke that he boughte: and in Veluet 13 yeardes $\frac{1}{2}$, and that appeared by examination thus: 26 $\frac{1}{2}$ yeardes of Damaſke at 12 ß. the yeard maketh 318 ß, then in Veluet he hadde but 13 yeards and $\frac{1}{2}$ that coſt 270 ß at 20 ß. yͤ yeard. Nowe Subtracte 270 out of 318, and there wil remain 48, which is the number of ſhillings that the Damaſke did coſte more than the Veluet.



A queſtion of debt.

M. Now ſhal you haue a queſtiō of an other kinde.

There are thrée men that do owe money to me, and I haue forgotten what the total ſum is, and what the particulars be.

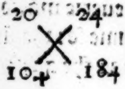
Scho. Why? then is it impoſſible to know the debt.

Maiſter. Peace ye are to haſtie: there is more helpe in it than you yet ſée: I haue thrée ſeuerall notes, whereby it appeareth that I did conferre their debts togither, and founde the debt of the firſt and the ſeconde to amount to 47 ℔, the debt of the firſt man and the third did make 71 ℔ and the ſecond man his debte with the third, did riſe to 88 ℔. Now can you tell what euerie mā did owe, and what was the whole total?

Scholer. Naye in good faith: but as I perceiue that it muſt be founde by coniecture, ſo wil I geſſe at it, ſuppoſing that the firſte man did owe 20 ℔, and the ſecond mā 30, and the third.

M. Nay ſtay there for you are to far gone alreadie, you maye not ſuppoſe a ſeuerall ſum ☞ for euerie mā, for it is ynough to ſuppoſe one ſumme for the firſte man, and let the other riſe as the queſtion importeth. Therefore ſéeing you ſet the firſt man his debt to be 20 ℔, the ſeconde man can not owe 30 ℔, for the declaration is that their debtes added togither, did make 47 ℔, ſo muſt the ſeconde man hys debte be but 27 ℔. Nowe this ſeconde debte with the thirde muſte make 88, therfore ſubtract 27 out of 88, and there wil remaine 61, as the third man his debt. Then ſaith the declaration, that the firſt and third mens debts do make 71: but by this ſuppoſitiō they make 81: that is 10 to much: whiche I muſte ſet for the firſt error. Now worke you the ſecond poſition.

I ſuppoſe the firſt mans debt to be 24 ℔, then muſt the ſecond mans debt (by your declaration) be but 23 ℔, ſeeing both they make but 47 ℔. Alſo the ſeconde man his debt wyth the third, doe make 88 ℔, and the ſecond man oweth but 23, therefore the thirde man muſt owe 65 ℔. Now the third mans debt with the firſt, ſhoulde make by the declaration 71 ℔, & they do make 89 ℔: that is 18 ℔ to much: and that is the ſeconde errour, which I ſette downe with the firſt, and their poſitiō in this forme and then do I multiply in croſſe wayes 20 by 18, &



it is 360. Alſo 10 by 24 maketh 240. And becauſe the ſignes of the errors be like, I muſt worke by ſubtraction: therfore I ſubtract 240 out of 360, and there reſteth 120, whiche is the diuidend: then doe I ſubtract 10 out of 18 by the ſame reaſon, and ſo is the diuiſor 8, which is found 15 times in 120, therefore I ſay that the firſt man did owe 15 ℔, and then the ſecond man muſte owe 32 ℔, for thoſe 2 doe make 47 ℔, and the third man his debt is 56 for ſo much remaineth if I bate 15 out of 71, or if I take 32 out of 88.

M. For the third exãple take this eaſie queſtion for yͤ variety in worke. Two mẽ hauing ſeueral ſums which I know not, do thus talk togither: yͤ firſte ſaith to the 2, If you giue me 2 ſ of your money, thẽ ſhal I haue 3 times ſo much money as you: the 2 anſwereth: It wer more reaſon, that our ſums wer made equal, and ſo will it be, if you giue me 2 ſ of youre mony. Now geſſe what ech of them had.

The third queſtion.

Scholer. I imagine that the firſte had 9 ſ.

Note.

Maiſter. Conſider euermore in your imaginations that you take a likelye ſumme, as in this queſtion take ſuche a ſum that hauing 2 added vnto it, may be diuided into 3 partes euen.

Ii.ij. Scho.

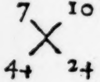
Scholer. Why? I remember you ſaid before, it forced not howe fondely ſo euer I geſſed.

M As for the poſſibilitie of the ſolution it is truth, but for eaſineſſe in worke, the apteſt numbers are moſt conuenient.

Scholer. I thoughte no leſſe, and therefore I toke 9 as an apte number to be parted into :but I perceiue I ſhoulde haue conſidered the aptneſſe of that partition after the addition of 2 vnto it, and then 7 had bene more meeter.

M That is trueth, and then ſhoulde the ſeconde man his ſum be 5: for although he haue nowe but the third part of 9 that is 3, yet you muſt remember that he lent the firſt man 2, and ſo had he 5.

Scholer Then to goe forwarde: if the ſeconde man had 3 of the firſte má, then ſhould he haue 8, and the firſt man but 4, ſo hath he double to the firſt man: yet he ſaid in the queſtion they ſhould haue equall: wherfore it appeareth that he hath 4 to much. Therefore I note that error with his ſuppoſition, and geſſe againe that he hath 10 ß: whereunto I adde 2 ſhillings borrowed of the ſeconde man and then hath he 12 ſhillings, ſo the ſeconde man hath remayning but foure, whereunto if I adde the 2 that he lent to the firſte man, ſo had he but 6 ß at the beginning. Thē take 3 ſhillings from the firſt man, and giue to the ſeconde, then hath the firſt man but 7, and the ſeconde hath 9, whiche are not equall, but there are 2 to manye, wherfore I ſet down bothe the poſitions with their errors as here you ſée, and multiplie a croſſe, ſo commeth there 40 and 14: and bycauſe the ſignes be like. I take 14 out of 40, and ſo reſteth 26 to be the diuidend, thē likewiſe I take 2 out of 4, and there reſteth 2 by which I diuide 26, and the quotient wyll be 13, which is the ſumme that the firſte man had. And ſo appeareth that two being added thereto, the ſumme will be 15, ſo hath the ſecond man nowe but 5, and before he had 7: then take thrée from the firſt, and put to hys ſeuen ſo haue eache of them 10, and that is equal, as the queſtion would.



The fourth example.

Maiſter. For the fourth example take this queſtion. One man ſaide to an other: I think you had this yeare two thouſand lambes: ſo had I ſaide the other: but what with paying the tith of them , and then thꝛee ſeueral loſſes they are muche abated: for at one time I loſt halfe as many as I haue now left: and at an other time the thirde time of ſo many: and the third time $\frac{1}{4}$ ſo many. Now geſſe you how manie are left.

Sch. Bicauſe here is mention made of certaine parts, I muſt take a number that may haue all thoſe parts: that is to ſay, $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$, whiche wil be 24, howbeit 12 hath the ſame partes. Therefore firſte I take 12 to be the number that doth remaine, ſo hath he loſte 6, 4, and 3, that is 13: and in the whole 25, but it ſhoulde be 2000.

Maiſter Ye are deceiued yet ſtil: you haue forgotten the 10 part, whiche muſt be defalked, that is 200, ſo there remaineth but 1800 and now go on againe.

Scho. Then to finde the error, I take 25 out of 1800, and there remayneth 1775 to fewe, whiche I ſet for the firſte errour. Then for the ſecond poſition I take 24, whoſe halfe is 12, the third part 8, & the quarter 6, whereby riſeth 50, which is to little by 1750, therfore I ſet downe both the poſitions with their errors thus.

12 24
X
1775- 1750-

And multiplie in croſſeways

1775 by 24, whereof commeth 42600. Alſo I multiplie 1750 by 12, and there ariſeth 21000. And bicauſe the ſignes are like, I doe ſubtract the one from the other, and ſo remaineth the diuidend 21600: then do I ſubtract 1750 out of 1775, and there reſteth 25, by whiche I diuide 21600, and the quotient is 864, whereof the halfe is 432, and the thirde part is 288, the quarter is 216, which all being added togither, will make 1800. And if you adde thereto the tenth which was abated before, then wil the whole ſum be 2000. And nowe doeth there come a queſtion to my memorie which was demaunded of me, but I was not able to anſwere to it, and now me thinketh I could ſolue it

200
864
432
288
216

1800

M. Propone your queſtion.

Sch. There is ſuppoſed a Lawe made, that (for further tillage) euerie man that doth keepe ſheepe, ſhall for euerie 10 ſheepe eare and ſowe one acre of grounde: and for his allowance in ſheepe paſture, ther is appointed for euerie foure ſheepe one acre of paſture. Nowe is there a riche ſheepemaiſter which hathe 7000 acres of grounde, and woulde gladlie keepe as manye ſheepe as he might by

A queſtion Of ſheepe and tillage.

that

that Statute, I demaunde how many ſhéepe ſhal he kepe.

M. Anſwere to the queſtion your ſelfe.

Scho. Firſt I ſuppoſe he maye kepe 500 ſhéepe, and for them he ſhall haue in paſture after the rate of 4 ſhéepe to an acre, 125 acres, and in arable grounde 50 acres, that is 175 in al: but this error is to litle by 6825. Therfore I geſſe againe, that he may kéepe 1000 ſhéepe, that is in paſture 250 acres: and in tillage 100 acres, which maketh 350: that is to little by 6650.

500 1000

6825— 6650—

Theſe both errours wyth their poſitions I ſette downe as you ſée, and multiplie in croſſe 6825 by 1000, & it maketh 6825000. Then I multiplie 6650 by 500, and it doeth amounte to 3325000, whiche ſumme I doe ſubtract out of the firſte and there remaineth 3500000 as the diuidende. Alſo I doe ſubtracte the leſſer error out of the greater, and ſo remaineth 175, by which I diuide the ſaid diuidend, and the quotient wil be 20000: ſo that I ſée, that by this rate he yͭ hathe 7000 acres of grounde, may kéepe 20000 ſhéepe:

and thereby I coniecture, that many mennes may kéepe ſo many ſhéepe, for many men (as the common talke) haue ſo manye acres of ground.

Maiſter. That talke is not likely, for ſo much ground is in cõpaſſe aboue $48\frac{3}{4}$ miles, leaue this talke and returne to your queſtions, leaſte your pointing be ſcarce well taken.

Scholer. Indéede I doe remember, that the Egiptians did grudge ſo muche againſte ſhepeheardes, till at length they ſmarted for it, and yet they were but ſmal ſhéepemaiſters to ſome men that be now, and the ſhéepe are waxen ſo fierce nowe & ſo mightie, that none can withſtand them but the Lion.

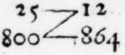
An other way of working.

Maiſt. I perceiue you talke as you heare ſome other: but to the worke of your queſtion: both this laſte queſtion, and the nexte before might be wrought without the ſeconde poſition, by the rule of proportion, as this. When in this queſtion ye found in the firſte errour, that for 500 ſhéepe, there muſte be 175 acres, then might you reduce it to the Golden rule, thus.

If 175 acres wil admit in allowance 500

175 — 500
70000 — 20000

ſheepe than 7000 will haue 200000. And ſo by one poſition with the helpe of the Golden rule may you anſweare that queſtion. Likewiſe for the queſtion of Lambes, when you had founde that 12 came of 25, you mighte haue ſet the figure thus as you ſee I haue ſaid: If 25 do leaue but 12, what ſhall 180 leaue? and it woulde appeare to be 864.

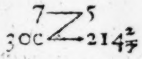


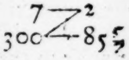
Scholer. Sir, I thanke you for this ayde, for it doeth muche ſhorten the worke of thys rule.

An other way yet.

Maiſter. Yet againe I will ſhewe you an other way, to anſweare to this laſte queſtion without this rule of Falſe poſition, and that by the rule of Felowſhippe, for it appeareth in the proponing of the queſtion, that 10 ſheepe muſt haue in paſture 2 acres and $\frac{1}{2}$, and for them muſt there be eared but one acre: ſo it followeth, that for two acres eared, there muſt be fiue ſet to paſture. And if you putte them both into one ſumme, they wil make 7. Therfore looke what proportion 7 being this totall, doth beare to 5 and to 2, ſuch proportion ſhall any totall in this queſtion beare to the paſture ground, and the eared ground.

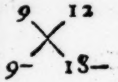
Scholer. This ſerueth wonderous aptly. Therfore to proue it, I demaund this by the former ſuppoſition: If a man haue 300 acres, how much ſhall he leaue in paſture, and howe much ſhal he turne to tillage? You ſay that as 7 is to 5, ſo ſhall 300 be to the acres of paſture: and as 7 is to 2, ſo is 300 to the acres of tillage, whereof for both I haue ſette examples here following, whereby appeereth that of paſture there ſhall be 214 $\frac{2}{7}$ acres, and of tillage 85 $\frac{5}{7}$ which both ſummes added togyther, doe make 300.





Maiſter. Now take an other example: A man hath three ſiluer cuppes with one couer, the couer wayeth 18 ounces, the ſeconde cup weyeth euen halfe the waight of the firſt and the third. Now if the couer be put to the firſt cup, they wey iuſte as muche as all the three cups doe wey: and if the couer be ioined with the ſeconde cuppe, they wey as muche as the ſecond twice, and the third: and if the couer be put to the thirde cup, they wil make twice as much as the firſt and the ſecond cup. Now trie you what was the iuſt waight of euery cup.

Scholer. I doe ſet the waight of the firſte cup to be 9 ounces: then in as much as theſe two (that is to ſay, the couer and the firſt cup) do wey the waight of the thrée cups, I ſée that the thrée cuppes muſt wey 27 ounces, for ſo much is 18 and 9. Alſo becauſe the firſte and the third do wey double ſo much as the ſecōd, therefore is it the third parte of that waight, that is 9, and then woulde it followe, that the third cup alſo ſhould wey 9 ounces, but then the queſtion ſaith, that the couer being ioined to the ſecond cup, they wey as muche as the ſecond twice, and the third once, that ſhould be 27, and ſo it doth: then being ioined with the third cup, they ſhould wey twice as much as the firſt and the ſecond, that ſhould be 36, and they wey but 27, ſo is that errour 9 too little. Then beginne I againe, and ſaye, that the firſte cup doth wey 12 ounces, whiche I ioine with the couer, and they make 30 ounces: then ſéeing the ſecond is $\frac{1}{3}$ of that waight, it muſte néedes wey 10 ounces, and the third muſte wey 8 ounces, ſéeing the firſt and the third muſt wey 20 ounces. Now putte I the couer to the ſeconde cuppe, and they wey 28 ounces, which ſhould be euen ſo: then ioine I the couer with the thirde Cuppe, and ſo ſhould it wey twice the firſte, and the ſecond, that is 44 ounces, and they doe wey but 26, that is 18 too little: thoſe errours with their poſitions I ſet downe, and multiply in croſſe ways 9 by 12, whereof commeth 108. Alſo 9 by 18, and that yeeldeth 162: and in as much as the ſignes be like, I abate the leſſer out of the greater, and there doth remaine 54. Then doe I alſo abate the leſſer errour from the greater, and ſo remaineth 9, by which I diuide 54, and the quotient is 6: which I take for the true waight of the firſt cuppe: which being ioined with the couer muſte wey as much as the three cups, ſo do they wey but 24 ounces. Then ſeeing the ſecōd cup is the third part of that weight, for the other two cuppes (you ſay) muſte wey double his weight, the weight of the ſecond cuppe is 8 ounces, and ſo the waight of the the thirde muſt be 10 ounces. Nowe put the couer to the ſeconde cuppe, and it will make 26 ounces: that muſte be the waight of the ſeconde twice, and the thirde once, that is twice 8, and once 10, and ſo is it. Againe, putte the couer to the third cuppe of 10 ounces,



ces, and they muſt wey twice as much as the firſte and the ſecond, that is 28 : and ſo is all agreable.

Maiſter. Then anſweare to this queſtion.

A queſtion of water.

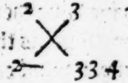
There is a Ceſterne with foure cockes, containing 72 barrels of water : and if the greateſt cocke be opened, the water wil auoid cleane in ſixe houres : at the ſeconde cocke it wil aſke eight houres : at the thirde cocke it will auoide in no leſſe than nine houres : and at the ſmalleſt it wil require twelue houres. Nowe I demaunde, in what ſpace will it auoide, all the cockes beeing ſet open?

Scholer. Firſte I imagine that it will auoide in two houres.

Maiſter. Then muſte there auoide by the firſt cocke $\frac{1}{3}$ of the water, that is 24 barrels, and by the ſecond cocke $\frac{1}{4}$, that is 18, and by the third cocke $\frac{2}{9}$ that is 16 barrelles, and by the ſmalleſt cocke $\frac{1}{6}$, that is 12 barrelles, all which ſummes put togither do make 70, as by their addition it doth appeere, but it ſhould be 72, therefore the errour is 2 to fewe.

24
18
16
12
70

Scholer. Then I begin againe by youre fauour, bycauſe I think I vnderſtand the worke, and putte thrée houres for the due time: ſo ſhall there runne out at the greateſt cocke $\frac{1}{2}$ that is 36 barrels and at the ſeconde hole $\frac{3}{8}$, that is 27, and at the thirde cocke $\frac{1}{3}$ that is 24, and at the ſmalleſt hole $\frac{1}{4}$, that is 18 barrels, which al togither do make 105, and ſhould be but 72, ſo is it too muche by 33, therefore do I ſet the errors in order of the figure with their poſitions, and worke by multiplication, in croſſe, ſaying: 2 times 3 is 6 and 2 times 33 maketh 66: and becauſe the ſignes are vnlike, I muſt adde thoſe two totalles togither, which make 72: alſo I adde the two errours, and they make 35, by which I diuide 72, and the quotient riſeth $2\frac{2}{35}$, whereby I ſee that all the cockes beyng ſet open, the water wil auoide in 2 houres, and $\frac{2}{35}$ of an houre.



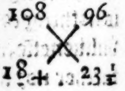
Maiſter. This exerciſe maketh you to grow expert in the rule. Therefore I wil inure you ſomewhat more wͭ a queſtiõ or two.

There were two men that had bene parteners,and had in accompt betwene them 300 duckets: whereof the one ſhoulde haue for his parte 180, and the other 120 : but in the parting of them they fel at variannce, ſo that eche of them catched as many as he coulde: yet afterwarde being reconciled, they agréed that he which had gotten moſt parte of them, ſhoulde lay downe $\frac{3}{4}$ of them againe, and hée that had gotten leaſt,ſhoulde laye downe $\frac{1}{3}$ of thoſe which he had taken , and then parting them vnto two equall partes, eache man to haue halfe thereof,and ſo had they their iuſte portions,as they ought: nowe I demaunde of you what eache of them had gotten by the ſcambling?

Scholer. I ſuppoſe he that had leaſte,gotte 108 duckats,then the other had 192 : wherefore in laying downe againe of the 192,there was put downe $\frac{3}{4}$ that is, 144, and ſo had he left but 48. Alſo of the 108 : there was layde downe 36,that is $\frac{1}{3}$, and ſo he had lefte 72. Then I put togither 144,and 36,and it maketh 180 which I parte into two partes euen,and ſo commeth 90 to be giuen to eache of them: which ſumme put to 72,maketh 162, and ioined to 48,it maketh 138:and now I doubt how I ſhal go forwarde.

Maiſter. You neede not to take but one of them which you liſt, the greater or the ſmaller, for all commeth to one purpoſe : and ſo may you compare it that you take to anye of the other ſummes, remembring that you make compariſon to the ſame in the ſeconde worke: as for example of the firſte parte, If you compare 138 with the leſſer ſumme due, that is 120, ſo is it 18 to muche: and if you compare it with the greater ſumme, then is it 42 to little. Againe, if you compare 162 to the greater ſumme, the errour will be 18 as it was in the other: but it wil haue a contrarie ſigne: and if you compare it with the leſſer ſumme it will be 42 to much: ſo that the errour both wayes is eyther 18 or 42: & as for the ſignes it little forceth, for in them is nothing conſidered here, but likeneſſe and vnlikeneſſe, which in this caſe, doth neyther further nor hinder. But now go on with the worke.

Scholer. If it be ſo, then am I out of my greateſt doubt. Then I ioine that 90 (which I founde as the halfe of the latter partition) vnto 48, which is left with the one man, and ſo hath he 138, whiche (I may ſay) is 18 to many for the leaſt ſhould be but 120: that error doe I note, and then make a newe poſition, ſuppoſing the one man to haue 204, and the other to haue 96, wherefore of the 204 there muſt be laide downe 152, and ſo remaineth with him 51. Alſo of the 96 there muſte be laid downe $\frac{1}{3}$, that is 32, and ſo reſteth with that man. 64. Nowe of the 153 and 32 I make one ſumme as 185, whiche I muſte diuide into 2 equall partes. and ſo eche man ſhall haue 92 $\frac{1}{2}$, whereunto if I adde their former portions reſerued, then the one ſhall haue 156 $\frac{1}{2}$ and the other hath 145 $\frac{1}{2}$. Wherfore I take the leſſer ſumme nowe agayne, as I did before, that is 143 $\frac{1}{2}$, and finde that he hath to many by 23 $\frac{1}{2}$, for he ſhoulde haue but 120, ſo haue I for my two poſitions two errors, which I ſet down, as here may be ſeene, eche errour vnder his poſition, and then by the rule I doe multiplye in croſſe wayes 108 by 22 $\frac{1}{2}$ and there riſeth 2538 whiche I note then againe I multiplye 96 by 18, and thereof amounteth 1728. Now becauſe the ſignes are both like, that is bothe to many, I muſt work be Subtraction, and ſo aba-



ting 1728 out of 2538, there wil reſte for the diuidend 810: then for the diuiſor I ſubtract 18 out of 23 $\frac{1}{2}$ and there remaineth 5 $\frac{1}{2}$, by whiche I diuide 810, and the quotient will be 147 $\frac{2}{11}$, which is the laſt portion of him that had the leaſt ſumme. And if I doe ſubtracte it out of 300 being the totall ſumme, then wil there remaine 152 $\frac{9}{11}$, as the portion that the other did get.

Maſter. For the proofe of this worke, you may chooſe whether you will examine thoſe numbers according to the forme of the queſtion, or elſe worke by other two poſitions for to finde the ſecond number: and if thoſe poſitions bring the ſame numbers that didde amount by the firſt two poſitions, then dothe eche worke confirme other.

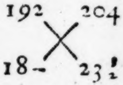
Scholer. By your patience, I will proue both wayes, not only to ſee their agreement, but alſo to accuſtome my mind to thoſe workes: for I perceiue it is exerciſe that muſt bee the chiefe engrauer of theſe rules in my memorie.

Maſter. You conſider it wel: then goe to.

Scholer. Firſte I will by two other poſitions trie to finde the portion of him which had moſte.

Maiſter. Although you may doe it with any poſitions, yet to ſee the agreement of your worke the better, take the ſame poſitions that you did before, comparing them nowe to the greater, as you did before vnto the leſſer.

Scholer. Then I ſuppoſe, that he that had moſte, had 192, ſo had the other 108. Nowe if I take ¾ of 192, that wil be 144, and there will reſte to that man but 48. And from the ſecond which had 108, if I take ⅓, that is 36, there wil remaine to him 72: then ioyning 144 with 36, it will make 180, the halfe whereof being 90. If I adde to eche of thoſe two mens portions remaining with them, the one ſhal haue 138, and the other 162, of which two I take the greater (that is 162) and ſee it to be 18 to fewe, for it ſhoulde bee 180, that errour I note vnder his poſition. Then for the ſeconde poſition I take (as I did before) 204 for the one, and ſo reſteth 96 for the other: then take I ¾ of 204 and it wil be 153, and there reſteth to him 51. Alſo of the 96 I take ⅓ that is 32, and there remayneth to him 64. Nowe put I that 32 to 153, and it yeeldeth 105: whiche being parted in equall halewes, maketh 92 ½. to be added to eche mans remainder, and ſo the one hathe $143\frac{1}{2}$, and the other $156\frac{1}{2}$: wherefore I take the greateſt ſumme, and it is $23\frac{1}{2}$ too little, that doe I note alſo, and ſette both theſe errours vnder their poſitions, as in this example folowing doth appeare.



And then multiplying 192 by $23\frac{1}{2}$, there doth ariſe 4512.

Again, I multiply 204 by 18, & it maketh 3672, whiche I doe ſubtracte out of 4512, becauſe the ſignes be like, & there reſteth 840 for the diuidend: then ſubtracting 18 out of $23\frac{1}{2}$, there will remaine $5\frac{1}{2}$, which I muſt take for the Diuiſor. And ſo diuiding 840 by $5\frac{1}{2}$, the quotient wil be $152\frac{8}{11}$: whereby I haue found an agreable ſumme to that whiche I found by the former poſitions, for hym that had moſte, whiche if I doe ſubtracte out of 300, that is the totall, there wil reſt $147\frac{3}{11}$, which was the portion of him that hadde the leaſt parte.

Maiſter. So by diuers poſitions you ſee, that one doeth confirme the worke of the other. Nowe examine thoſe two numbers by the forme of the queſtion, and ſo ſhall you proue your worke good alſo.

Scholer. If that he whiche gate moſt, had $152\frac{2}{11}$, then muſte he laye downe $\frac{3}{4}$ of hys ſumme, that is $114\frac{6}{11}$ and ſo ſhall remaine with him, but onely $38\frac{2}{11}$. The other which had leaſte, that is $147\frac{3}{11}$. muſte put downe of his ſum $\frac{1}{3}$. that is $49\frac{1}{11}$. and ſo doth there remaine with him yet $98\frac{2}{11}$. Then doe I adde togither $114\frac{6}{11}$ and $49\frac{1}{11}$, and it wil make $163\frac{7}{11}$. whiche I muſte part into two equall partes, and that will be $81\frac{9}{11}$ to bee giuen to eche of them: ſo putting $81\frac{9}{11}$ vnto $38\frac{2}{11}$, there doth amount 120 iuſte, whyche is the true portion of him that ſhoulde haue the leſſer ſumme: and adding $81\frac{9}{11}$ to $98\frac{2}{11}$ the totall will be 180. the true portion of the other. And ſo is the worke by this proofe alſo tried to be good. And this I marke by the way, that in their ſcambling, hee gate moſte (as it chanceth often) that ought to haue had leaſt by iuſt partition.

Maiſter. Let your ſtudy be to learne truth and iuſte arte of Proportion, and to diſtribute and parte according thereunto, as often as occaſiō ſhal be miniſtred. And here would I make an ende of this rule, ſaue that I remember one pleaſant queſtion whiche I can not ouerpaſſe, which I wil declare ſomwhat largely, becauſe you ſhal as well vnderſtand ſome reaſon in the pleaſaunt inuention, as apte proceeding in the wittie working thereof.

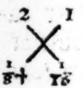
An example of mixture of Gold and Siluer.

Hiero King of the *Syracuſanes* in *Sicilia*, hadde cauſed to bee made a Crowne of Golde of a wonderfull waight, to be offered for hys good ſucceſſe in warres: in making whereof, the Goldſmith fraudulently tooke out a certaine portion of Golde, and putte in ſiluer for it, ſo that there was nothing abated of the full weight, although there was much of the valewe diminiſhed. Which thing at length being vttered, (as no euill can alwayes lye hidde) the King was ſore moued, and beyng deſirous to trie the truth without breaking of the Crowne, propoſed the doubt to Archimides; vnto whoſe wit nothing ſeemed vnpoſſible, whiche althoughe preſentlye hee coulde not aunſweare vnto, yet hee had good hope to deuiſe ſome pollicie for that inuention. And ſo muſing thereon, as he chanced to enter into a baine full of water to waſhe him, he obſerued that as his body entred into the baine, the water did runne ouer the tub:

[illegible]

whereby his ready wit of ſuche ſmall effectes coniecturing greater workes, conceyued by and by a reaſon of ſolution to the Kings queſtion, & therefore reioicing exceedingly more than if he had gotten the Crowne it ſelfe, forgate that he was naked, and ſo ranne home, crying as he ran, εὕρηκα, εὕρηκα. I haue founde, I haue founde. And thereuppon cauſed two maſſie peeces, one of golde, and an other of ſiluer to be prepared of the ſame weight that the ſaide Crowne was of: and conſidering that golde is heauier of nature than ſiluer, and therfore golde of like waight with ſiluer, muſt needes occupie leſſe roume, by reaſon it is more compact & ſound in ſubſtance, he was aſſured, that putting the maſſe of golde into a veſſel brimme full of water, there would not ſo much water run ouer, as when he ſhoulde putte in the ſiluer maſſe of the like weight. Wherefore he tried both, & noted not onelye the quantities of the water at eche time, but alſo the difference or exceſſe of the one aboue the other, wherby he learned what proportiō in quantitie is betweene gold and ſiluer of equall waight. And then putting the crowne it ſelf into the veſſell of water brimme full (as before) marked how much water did run out then, & comparing it with the water that rã out when the gold was put in, noted howe much it did exceede that : & likewiſe comparing it to the water that rã out of the ſiluer, marked how much it was leſſe than that : & by thoſe proportions found out the iuſt quãtitie of gold ỹ was taken out of the crown, & how much ſiluer was put in ſteede of it. But ſeeing Vitruuius whiche writeth this hiſtorie, doth not declare the particular worke of this trial, it ſhal be no inconuenience to ſuppoſe an example for declarations ſake, wherin although the true and iuſt proportions be not expreſſed, yet the forme of triall ſhall be truely ſet forth And for an example, I ſuppoſe the weighte of the Crowne to be 8 ℔, and ſo of each of the other two Maſſes. And when the maſſe of Golde was put into the water, I imagine that there ran out 2 poũd of water: and when the maſſe of ſiluer was put in, I ſuppoſe there ran out 3. pound $\frac{1}{2}$. Againe when the crowne was put in, there rã out 2 pound $\frac{1}{4}$. Now to know what quantitie of ſiluer was in the Crowne, worke by the rule of falſe poſition, and imagine that there was 2 pound of ſiluer: then muſt there be 6 pounde of Golde Then ſay thus by the rule of Proportion: If 8 pound of gold doe expell 2 ℔ of water, what ſhal 6 ℔. expelle, and it wil be 1 pound $\frac{1}{2}$. Againe for the ſiluer: If 8 ℔ of ſiluer expell 3 ℔ $\frac{1}{2}$ of water, what ſhall 2 ℔ of ſiluer put out? it will be $\frac{7}{8}$. Nowe adde thoſe two weightes of water togither, and they will make 2 ℔ $\frac{3}{8}$ and it ſhoulde be by the ſuppoſition 2 ℔ $\frac{1}{4}$, ſo is it too muche by $\frac{1}{8}$.

Sch. Nowe doe I underſtande the worke as I thinke, therfore I pray you let me work the reſt of the queſtion. And bycauſe this firſt ſuppoſition did erre, I note that poſitiõ, and his errour, and take a newe poſition, eſteming the ſiluer to be but one pound, ſo muſte there be in Golde 7 pounde, Then ſay I: If 8 ℔ of Gold yeeld 2 ℔ of water, what ſhall 7 ℔ yeelde? and it will be 1 ℔. $\frac{3}{4}$. Againe if 8 ℔, of ſiluer expel 3 ℔. $\frac{1}{2}$ of water what ſhall 1 ℔ expell? and it wil be $\frac{7}{16}$. Nowe muſte I adde thoſe two ſums togither, and they make 2 ℔, $\frac{1}{16}$ and they ſhoulde make 2 ℔ $\frac{1}{4}$ ſo is it too little by $\frac{2}{16}$ Therfore I ſet yͤ poſitiõs with their errours in order, as here foloweth. And thẽ I multiplie in croſſewaies 2 by $\frac{1}{16}$ and it maketh $\frac{1}{8}$: likewiſe 1 multiplied by $\frac{1}{8}$:



maketh $\frac{1}{8}$. And becauſe the ſignes be unlike, I muſt adde thoſe two ſumme, whiche make $\frac{1}{4}$ and that is the diuidend. Again I muſt adde $\frac{1}{8}$ to $\frac{1}{16}$, and it wil be $\frac{3}{16}$ that is the diuiſour. Now I ſhall diuide $\frac{1}{4}$ by $\frac{3}{16}$ and the quotient wil be $\frac{16}{12}$ that is, 1 $\frac{1}{3}$, whereby I knowe that there was put 1 ℔ and $\frac{1}{3}$ of ſiluer into the Crowne, and ſo muche Golde taken out for it.

M Proue it now by examinatiō according to the queſtion.

Scho. If there were 1 pounde $\frac{1}{3}$ of Siluer then was there of Golde 6 pounde $\frac{2}{3}$. Nowe ſay I by the rule of proportion: if eighte pound of Golde expell two pounde of water, what ſhal 6 pound $\frac{2}{3}$ expel?

8 — 2
6 $\frac{2}{3}$ — 1 $\frac{2}{3}$

It wil be one pounde $\frac{2}{3}$.

8 — 2 $\frac{1}{2}$
1 $\frac{1}{3}$ — $\frac{7}{12}$

Againe, if 8 ℔ of Syluer expell 2 ℔ $\frac{1}{2}$ of water, what ſhal 1 $\frac{1}{2}$ expel? It wil be $\frac{7}{12}$.

Now muſt I ad togither 1 ℔ $\frac{2}{3}$. and $\frac{7}{12}$ and they wil make 2 ℔, $\frac{9}{36}$, that is 2 ℔ $\frac{1}{4}$, according to the ſuppoſition of the queſtion, whereby I perceiue the worke to be wel done. And as I can not but muche reioyce of thys excellent inuentiō,ſo my deſire is kindled vehemently to be perfectlye inſtructed in euerie part therof,and namelie in this point,whether the portion betwéene water and golde be ſuch,that for 8 ℔ of gold into a veſſel full of water, there ſhall run out 2 ℔ of water:& for as much ſiluer, whether 3 ℔ $\frac{1}{2}$ of water would auoide?

Ma. I perceiue your meaning, and coniecture your imagination to be thus: that if you knewe the exacte proportion betwéene Gold and Siluer and Water both in theyr waight and quantities, then coulde you eaſilie finde out the mixtures of them, whiche thing I haue reſerued for an other worke that intreateth ſuch matters ſpeciallie. And at this time you muſte conſider, that you learne Arithmetike, which intreateth of the maner to ſolue doubtful queſtions touching number, without regarde what matter is ſignifyed by that number, elſe were it neceſſarie in Arithmetike to teach all arts, ſéeing in it may be moued queſtions of al arts. But ſéeing you are ſo deſirous to know this thing, I wil tell it you in ſuchs a ſorte, that you ſhall practiſe your arte in finding it, and propounde it in forme of a queſtion. Gold beareth greater proportion to water than ſiluer doth, and their two proportiõs be in proportion togither as $\frac{48}{25}$. But to help you ſomewhat in this riddle, you ſhall note that the proportion of quicke ſiluer vnto water, is the iuſt middle number proportionall in Progreſſion Geometricall, betwæne the proportions of Gold and ſiluer vnto water. And his proportion is as $\frac{296}{21}$. Now if you will know the iuſt numbers of theſe 3 proportions, then muſt you finde out 3 numbers in Progreſſion geometricall, whereof the middlemoſt muſt be $\frac{296}{21}$, and the firſt muſt be vnto the laſt, as 25 to 48. And thus I will leave you to finde thoſe numbers when you be at leaſure.

Scholer. Yet ſir I thanke you heartilie for this muche, for nowe I ſæ the poſſibilitie to finde them out. Howbeit, bycauſe this queſtion ſæmeth ſtraunge, if it might pleaſe you to inſtructe me ſomewhat in the order of working for it, I ſhould the more eaſilie finde the true working.

M. You deſire to much eaſe if you wil ſtudie for nothing: therefore to occaſion you to ſtudie yͤ better, I will leaue this doubt wholy to your own ſearch. But as touching the generaltie of the rule, Archimedes næded not to take two maſſes of golde and ſiluer equall in waight with the crowne, for the proportion might as wel be foũd in any other waight yea althoughe the maſſe of golde were of one waight and the maſſe of ſiluer of another. As for example. If the crowne were of 8 pounde waighte, as I did ſuppoſe, and I haue not ſo muche other fine golde, but onely 1 ℔, and trying that by water, & finding that it doth expell but ¾ of an ounce of water, yet then by it may I inferre, that 8 pound of gold would expell 6 ounces of water. And likewiſe of the ſiluer: wherof if I had but 2 pound, and finde that it doth expel thrée ounces of water, then might I affirme that 8 pound woulde expell 12 ounces, that is 1 ℔ waight. And ſo is it, as good as if the 2 maſſes were al of one weight. And thus for this time I wil make an end of this other part of Arithmetike.

Sc. Although I can not ſufficientlie thanke you for this, yet your promiſe made me to loke for the art of extraction of rootes, wherof hitherto I haue learned nothing.

Maiſter. I wil not breake my promiſe, but intend (God willing) to performe it within theſe thrée or foure monethes, if I perceyue this my paines to be well taken in the mean ſeaſon. And you ſhall not repent the tarrying for it: for it ſhal be increaſed by the tarrying. And in the meane time, you ſhall take thys Addition, not for the ſecond part of Arithmetike which I promiſed, but for an augmentation of the firſte parte, vnto which I woulde haue annexed the extraction of Rootes ſquare and cubike, namely for examples of the Statute of Aſſiſe of wood, but that in the ſeconde parte I muſte write of diuers other rootes, and thought it beſte to reſerue thoſe rules alſo with their examples vnto the ſame ſeconde parte.

Scholer. Sir, althoughe I can not recompence your goodneſſe, yet I ſhall alwayes do mine endeuour to occaſion you not to repent your benefite on me thus employed.

Maiſter. That recompence is ſufficient for your parte.

FINIS.

The thirde parte or Addition to this Booke entreateth of briefe rules, called rules of Practize, of Rare, Pleaſant and commodious effect, abridged into a brieffer Method than hitherto hath bene publiſhed:

VVith diuers other neceſſarie Rules, Tables, and Queſtions not only profitable for Merchants, *but alſo for Gentlemen, and all other occupiers whatſoeuer, as by the Contents of this Booke may appeare.*

Set foorth by *Iohn Mellis*
Scholemaiſter.

The firſte Chapter of this Addition entreateth of briefe Rules, called Rules of *Practize*, with diuerſe neceſſarie queſtions profitable, not onlie for *Merchaunts*, but alſo for all other occupiers whatſoeuer.

THe working of Multiplicatiõ in Practize, is no other thing, than a certaine manner of multiplying of one kind by another: whervpon is brougt forth the product of the proponed number whych is accompliſhed by the meanes of Diuiſion in taking the halfe, the third, the fourth, the fifte, or ſuch other parts of the ſumme whichs is to be multiplyed.

And for the better vnderſtanding of ſuche conuerſions: you ſhall vnderſtand that in the manner and vſe of theſe Rules of Practize, you oughte firſt to knowe the euen or aliquot parts of a ſhilling, whiche in this Table following doth appeare.

$$\text{Item d} \left\{\begin{matrix}6\\4\\3\\2\\1\end{matrix}\right\} \text{is the} \left\{\begin{matrix}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\\ \frac{1}{6}\\ \frac{1}{12}\end{matrix}\right\} \text{of a ß.}$$

Wherin as you ſee according to the order of theſe rules of Practiſe at 6 d ye yeard of any thing, you muſt take the $\frac{1}{2}$ of your number whiche is to be multiplied, and the product, that commeth thereof ſhal be ſhillings, if any vnitie do remaine it is 6 d.

For 4 d take the $\frac{1}{3}$ of the number that is to be multiplied and the product alſo produceth ſhillings if anye vnities doe remaine, ech one ſhal be worth in valew 4 pence. The like is to be vnderſtoode of the other 3 &c.

Example I.

At 6 d the yearde what 379 yeardes

II. 189 ß—6 d

At 4 d the yearde:
what are 104 yeardes worth

---

34 ß—8 d

III.

At 3 pence the yeards
What ——— 5014 yeardes
maketh ——1253 ß ——— 6 d.

IIII.

At 2 pence the yeards
What ——— 532 yeardes
maketh ——— 88 ß ——— 8 d

V.

At 1 pennie the yearde
What ——— 409 yeardes
maketh ——— 34 ——— ß ——— 1 d

Here you may ſée in the firſt example yͤ 379 yeards at 6 d the yeard, are worth 189 ß 6 d in taking the $\frac{1}{2}$ of 379. And in the ſeconde example the 104 yeardes at 4 d the yeard: are worth 34 ſhillings 8 d: in taking yͤ $\frac{1}{3}$ of 104. Likewiſe in the third example 5014 yeards at thrée pence the yearde bringeth forth 1253 ß 6 d in taking the $\frac{1}{4}$ of 5014. Alſo in yͤ fourth example at 2 pence the yeards, maketh 88 ſhillings 8 d.

And laſtly in the fifth example: 409 yeards at 1 d the yeard, amounteth to 34 ß and 1 d, in taking the $\frac{1}{12}$ of 409: And ſo is to be done of all other queſtions the like, when the number of the pence is anye of the euen oꝛ aliquot parts of 12 d.

Item to bꝛing the pꝛoductes of theſe ſhillings and all other the like in poundes is verie eaſie in diuiding of it into your minde by 20, foꝛ it is to be vnderſtoode that as often as 20 is found in that pꝛoduct: So many pounds doth it containe, whiche with facilitie to perfoꝛme, allwaies ſtrike of the figure towarde your right hande, with a right downe daſhe of your penne foꝛ the o that appertayto 20: And then beginne at the lefte hand, in taking the $\frac{1}{2}$ of the reſt. And if at the laſte any vnitie do remaine, the ſame ſhall be ioyned with the figure that is cut of, which ſhall repꝛeſent the odde ſhillings contained in that woꝛke.

As foꝛ example in your third queſtion at 3 d the yearde whiche amounteth to 1253 ß. 6 d: the pꝛoducts whereof maketh 62 ℔ 13 ß 6 d: as here you maye ſee is eaſily perfoꝛmed in the mergent.

```
  1
125|3
-------
62-13-6
```

Item alſo for the working of 1 pennie the yearde, it is ſomething harſhe and harde to take the $\frac{1}{12}$ part of ſome products: Therefore to eaſe that hard worke you ſhall firſt bryng your deliuered ſumme into groats, by taking the $\frac{1}{4}$ part of the product. And if any vnites remaine of that $\frac{1}{4}$ part, as ſomtimes there may they are pence: and muſt be ſignifyed wyth a line from the groates with theyr title of pence: And becauſe that 60 groates maketh a pounde or twentie ſhillings, ſtrike of the firſte figure towarde your righte hande for the 0 that apperteyneth to 60 (as you did euen nowe for the 0 that belongeth to 20:) then in taking the $\frac{1}{6}$ of that product, if there do remaine any vnities the ſame ſhal you ioyne with the figure that you cut of, eſteeming thē as groates: whiche keepe in your mind. And by taking the $\frac{1}{3}$ part of them, you ſhall turne into ſhillings: And ſo haue you done as for example by a queſtion or 2 hereafter proponed ſhal more plainly by the worke appeare.

At 1 d the yearde

what 54368 yeardes

1359|2 groates

$\frac{1}{6}$ ℔ — 226 — 10 ß — 8 d

Here in taking the $\frac{1}{6}$ part of 1359: in comming to the laſt worke the $\frac{1}{6}$ part of 39 being taken, the remainer is 3 whiche ioined with the 2 that was cutte off, maketh 32 groates: which conuerted into ſhillings by taking the $\frac{1}{3}$ part: maketh as appeareth 10 ß 8 d: Many other wayes there are, but none more apter for a yong learner to vnderſtande than this: wherfore this one way wel impreſſed in memorie is better thã 20 waies doubtfully vnderſtoode.

At 1 d the yeard, what 4533 yeards

113|3 groats —1 d

$\frac{1}{6}$ ℔ 18—17—9

At 1 d the yeard what 64768 yeardes

1619|2 groats

℔ —209—17—4 d

Nowe followeth alſo to be vnderſtood ẏ if the number of pence be not an aliquot part of 12, you muſt reduce them into ſome aliquot part of 12 And after the aforeſaid maner, you ſhall make of them 2 or 3 products as neede ſhal require: And adde them togither into one ſum: And here for thy furtherance appeareth a note of the order of their partes, as they are to be taken. 2. Rule.

For

For pence	5. take.	3 & 2	or	4 and 1
	7. take.	4 & 3		6 and 1
	8. take.	4 & 4		6 and 2
	9. take.	6 & 3		4.4 & 1
	10 take.	6 & 4		4.4 & 2
	11 take.	6.4 & 1		4.4 & 3

Here in the firſte note of this table at 5 d, you ſhall firſt take for 3 d the $\frac{1}{4}$ of the number that is to be multiplied: And likewiſe for 2 d: the $\frac{1}{6}$ of the ſame number, adding togither both the products. But if you wil worke by 4 and 1 you muſt for 4 d firſte take the $\frac{1}{3}$ of the number that is to be multiplied: And for 1 d take the $\frac{1}{12}$ of the whole ſumme or rather, which is more better for 1 d. you maye take the $\frac{1}{4}$ of the producte whiche did come of yͭ 4 d: Bicauſe yͭ 1 d is yͤ $\frac{1}{4}$ of 4 d: The total ſums of theſe two nūbers ſhall be the ſolution to the queſtion. And in like maner is to be done of all others: As by theſe examples following ſhal appeare.

I.

At 5 pence the yearde
What will ——758 yeardes amount to

3 d 189——6 d
2 d 126——4

ſ ——315——10 d

Otherwiſe.

At 5 d yͤ yard what are 758 yeardes worth

4 d 252——8 d
1 d 63——2

ſ——315——10 d

II.

At 7 d the ell what 562 elles

4 d 187——8
3 d 140——9

ſ——328——5 d

III.

At 8 d the lb what 112 pounds

4 d 37——4
4 d 37——4

ſ——74——8 d

Other-

Otherwiſe.

What coms —112 pound at 8 d the pound

6 d — 56—0
2 d — 18—8
ß—74—8 d

IIII.

At 9 d the Ell
What coms —356 elles to

6 d — 178—0
3 d — 89—0
ß—267—0 d

V.

At 10 d the péece
What coms —795 péeces to

6 d — 397—6
4 d — 265—0
ß—662—6 d

VI.

At 11 d the pound

What — 7578 pound

6 d 3789 — 6

4 d 2526 — 4

1 d 631 — 7

ß — 694|7 — 5 d

maketh ℔ — 347-7 — 5 d

Here in this firſt exãple where it is demãmaunded (at 5 d the yeard) what will 758 coſt: Firſt for 3 d I take the $\frac{1}{4}$ of 758: And thereof commeth 189 ß — 6 d: Then for 2 d I take the $\frac{1}{8}$ of the ſame product whiche amounteth to 126 ß 4 d: theſe two ſummes added togither do make 315 ß 10 d: And ſo much are the 758 yards worth at 5 d the yard.

Item alſo for the ſame again: Firſt for 4 d I take the $\frac{1}{3}$ of 758: and thereof commeth 252 ß — 8 d: then for 1 penny I take the $\frac{1}{4}$ of the ſame product, that is to ſay of 252 ß — 8 d, and it yéeldeth me 63 ß 2 d: whiche both added togither make 315 ß — 10 d, as before.

Item, for 7 d there is take then $\frac{1}{2}$ and the $\frac{1}{3}$ of the whole ſumme: which is to be multiplied, plyed, and adde them togither, that is to ſay, firſt, for 4 d there is taken the $\frac{1}{3}$ of 563: whiche coms to 187 ß - 8 d as appeareth by the worke: and for 3 d there is taken the $\frac{1}{4}$ of the whole ſum which amounteth to 140 ß -9 d. Both which products added togither maketh 328 ß—5 d: And ſo much coms 563 elles to at 7 d the Ell.

Item, for the firſt 8 d there is taken for 4 d the $\frac{1}{3}$ of the whole ſumme: and an other $\frac{1}{3}$ for the other 4 d, which added togither as in the example doth euidently apppeare, amounteth to 74 ß—8 d.

Againe, for the ſecond work of 112 ℔, there is taken firſt the $\frac{1}{2}$ of the whole ſumme for 6 d, whiche coms to 56 ß: then for the 2 d you haue to take $\frac{1}{6}$ of the whole ſumme, or if you will the $\frac{1}{3}$ part of the product that came of 6 d either which maketh 18 ß 8 d. Theſe two ſommes being added togither dos make 74 ß 8 d: as in the third example appeareth.

Item, for 9 d there is taken for 6 d, the $\frac{1}{2}$ of the whole ſumme: and the $\frac{1}{4}$ of the whole ſumme for 3 d, or otherwiſe for the 3 d you may take the $\frac{1}{2}$ of ẏ product that came of 6 d, bicauſe 3 d is the $\frac{1}{2}$ of 6 d: which added togither as plainly appeereth in the fourth example, amounteth to 267 ß—0 d.

Item, for 10 d, firſt there is takẽ for 6 d the ½ of the whole ſumme, which amounteth to 397 ß—6 d. Then for 4 d there is foũd 265 ß: bothe whiche added togither maketh 662 ß—6 d as appeareth in the fift example: it may alſo be wrought, as appeareth by the ſecond note in the table by 4 d twice taken, and the ½ of the product of 4 d: or els by the ⅙ of the whole ſumme, &c.

Item, for 11 d, there is firſt taken the ½ for 6 d: then the ⅓ of the whole ſumme for 4 d: laſtly, the ¼ of the laſt producte for 1 d: All which 3 ſummes added togither maketh in ß 6947—5 d, & in pounds 347—7 ß—5 d.

Item, likewiſe by the ſame reaſon, when you will multiply (by ſhillings) any number that is vnder 20 ß you ſhal haue in the product pounds, if you know the euen or aliquot partes of 20, which are here in this little table ſet downe to ſight. 3. Rule.

Item ß	10	is the	½	of one ℔
	5		¼	
	4		⅕	
	2		1/10	
	1		1/20	

So that for 10 ß which is the $\frac{1}{2}$ of a poũd you may take the $\frac{1}{2}$ of the number whiche is to be multiplyed: and you ſhal haue in your product pounds: if a vnitie do remaine, it ſhal be worth 10 ß.

Likewiſe for 5 ß you muſt take the $\frac{1}{4}$ of the number whiche is to be multiplyed: And if there doe remaine any Vnities, they ſhall be fourth partes of a pound, euery Vnitie being in valewe 5 ß.

For 4 ß take the $\frac{1}{5}$ of the number which is to be multiplied: And if there doe remaine anye Vnities, they ſhall be fifte partes of a pound, eche vnitie being worth 4 ß.

For 2 ß you muſt take the $\frac{1}{10}$ of the number to be multiplied: wherefore to take the $\frac{1}{10}$ of any number: you muſt cut off the laſte figure of the ſame number (whiche is neareſt your right hand) from all the other figures with a ſmall right downe line or daſh with a pen, and ſo haue you done: for all the other figures which do remaine toward your lefte hand from the ſame figure that you doe ſeparate ſhal be the ſaide $\frac{1}{10}$ of a pound: And that figure ſo ſeparated towards your right hand ſhall be ſo manye peeces of 2 ß the peece: the whiche figure you muſte double to make

thereof

therof the true number of ß, as by the example ſhall appeare.

Finally, for 1 ß, néedeth ſmal worke, for it is ſo many ſhillings as be proponed in the ſumme, whiche to bring into poundes hathe bene already taught in the firſte Rule.

Example.

At 10 ß the péece
What are — 6543 —— péeces worth
1/2 ℔ — 3276 —— 10 ß

At 5 ß the Ell
What coms — 4373 —— Elles to
1/4 ℔ — 1093 —— 5 ß

At 4 ß the yards
What — 7839 —— yeards
1/5 ℔ — 1567 —— 16 ß

At 2 ß the pound waight
What are — 752|7 —— pounds waight
1/10 ℔ — 752 —— 14 ß

At 1 ß the péece.
What coms — 757|3 péeces to
1/20 ℔ — 378 —— 13 ß

Nextly

4. Rule. Nextly, nowe followeth in order to bée vnderſtoode, that if the number of ſhillings be not ſome euen, or aliquot parte of 20, you muſt then conuerte the ſame number of ſhillings into the aliquot parts of 20: And thereof make two or thrée products, as néede ſhall require: which done, adde them togither, and bring them into poundes. And here for thy furtherance I haue ſet down a note of the order of their parts, as they are to be taken.

ſ					ſ		
3		2 & 1			13		10.2 & 1
6		4 & 2		5.&.1	14		10. & 4
7		5 & 2			15		10. & 5
8	of	4 & 4	or	5.2.1	16	of	10.5. 1
9		5 & 4		4.4.1	17		10.5. 2
11		10 & 1			18		10.4. 4
12		10 & 2			19		10.5. 4

For 3 ſ according to the tenor that you ſée is expreſſed in the Table, you muſte firſte take for 2 ſ the $\frac{1}{10}$ of the number that is to be multiplied: Then for 1 ſ you muſte take the $\frac{1}{2}$ of the product which didde come of the ſame $\frac{1}{10}$ parte, and adde thoſe two ſums ad-

ther, produceth the effecte deſired.

Item, for 6 ß according to the note ſet forth in the table, firſt for 4 ß I take the $\frac{1}{5}$ of the number that is to be multiplied: Then for 2 ß the $\frac{1}{2}$ of the product that came of 4 ß, and adde them togither.

Or elſe, as appeareth alſo in the table, for 5 ß you may take the $\frac{1}{4}$ and the $\frac{1}{5}$ parte of the product that came of 5 ß, and adde them togither.

Item, for 7 ß, firſte take for 5 ß the $\frac{1}{4}$ of the producte, that is to be multiplied, then for 2 ß, take the $\frac{1}{10}$, of the number that is to be multiplied, and adde them togither, &c.

Item, for 8 ß, according to reaſon, and the intent of the Table, for the firſte 4 ß take the $\frac{1}{5}$ of the product, and the ſame number againe for the other 4 ß: and adde them togither.

Item, for 9 ß: firſte for 5 ß take the $\frac{1}{4}$: then for 4 ß take the $\frac{1}{5}$: and adde them togither.

Otherwiſe as you ſée by the intente of the table, work twice for 4 ß, as was taught euē now for 8: and then take the $\frac{1}{4}$ of the laſt product for the 1 ß: But 5 and 4 is the ſhorter.

Itē, for 11 ß: firſt diſpatch 10 ß: for which you muſt take the $\frac{1}{2}$ of the product: then laſtly for 1 ß take the $\frac{1}{10}$ parte of the ſumme produced of the $\frac{1}{2}$ of the product and adde them togither.

Item, for 12 ß where I will end wyth the firſte part of my Table: Firſt take the $\frac{1}{2}$ for 10 ß: And then for 2 ß take the $\frac{1}{5}$ of the ſum that came of 10 ß, and adde them togither: or elſe, if you pleaſe for 2 ß you may take the $\frac{1}{2}$ of the whole giuen number.

To write more of the maner of taking the true parts, I thinke ſuperfluous. The deſirous practitioner will (no doubt) conceiue it. Alſo the Table is ſome aide to helpe the vnperfect: whereupon by & by I will ſet downe three or foure of theſe notes in examples: and the reſt I wil leaue to thine own induſtrie & practiſe to labour vpon.

This is the order moſt commonly vſed in Practiſe when the number of the ß is not an aliquot part of a pound. But louing Reader) after I haue touched the euē or aliquot parts of a ℔ that falleth out in đ and ß, I will deliuer 2 new Rules that ſhal drowne this common order quite and cleane: wherein ſhal be

com

comprehended in one line, or working bothe euen and odde part of ß vnder 20: without regard whether it be an aliquot or not an aliquot parte: which 2 Rules, when they come in place, I committe to thy friendly iudgement in working: Nowe followeth the examples vpon the notes before ſaide.

At 6 ß the yard

What — 3215 — yards

4 ß 643 —

2 ß 321 — 10

℔ — 964 — 10 ß

Otherwiſe by multiplication of 6:

3215 —

6 ß 1929|0

℔ — 964 — 10 ß

At 7 ß the Ell

What — 4563 Elles

5 ß 1140 . . 15

2 ß 456 . . 6

℔ — 1597 . . 1 ß

Otherwiſe by multiplication of 7:

4563

7 ß 3194|1

1597 . . 1 ß

At 8 ß yͤ péece what 7563 péeces

4 ß 1512 · · 12

4 ß 1512 · · 12

℔ ——3025 · · 4 ß

Otherwiſe by Multiplication.

7563

8 ß 6050|4

℔ — 3025 — 4 ß

At 13 ß yͤ péece what 401 péeces

10 ß 200 · · 10

2 ß 40 · · 2

1 ß 20 · · 1

℔ ——260 · · 13 ß

Otherwiſe by Multiplication.

401

13 ß 1203

401

5213|3

℔ · · 260 — 13 ß

Theſe & ſuch like queſtions of Compound numbers, which I haue here in this fourth rule for orders ſake ſet down, I count but as ſuperfluous. For, in the ſeconde parte of my new promiſed Rules ſhall appeare, that the giuen price of any odde nūber of Shillings, either vnder or aboue 20: ſhall bee wrought at two workings at the moſte howe difficult ſo euer the queſtion be.

Item, there reſteth yet a kind of Practize, howe to bring pence into poundes at the firſt working: wherevpon you muſt vnderſtand, that 240 pence maketh one pound, or 20 ß, I cutte off the laſte figure or 0: and there remaineth but 24 (of whiche 24) 8 d is the $\frac{1}{3}$ parte thereof: 6 d is the $\frac{1}{4}$ parte, 4 d the $\frac{1}{6}$ parte: and 2 d is the $\frac{1}{12}$ parte thereof.

Wherevpon if it were demaunded what 1486 yeardes or poundes of any thing commeth to: at 8 d the yeard, in pricking or cutting off the firſte figure towardes your right hande: for the 0 that appertaineth to 240: There is remaining of the ſaide ſumme 148: whereout I take the $\frac{1}{3}$ parte: and it cōmeth to 49 ℔: and there reſteth one: which I putte to the 6: that I pricke or cutte off, and it maketh 16 pieces of 8 pence, whiche I double to make into groates and they make 32. whereof the $\frac{1}{3}$ part maketh 10 ſ and ther remaineth $\frac{2}{3}$ ſ: which is 8 pence, whereby it followeth, that the 1486 yeardes at 8 pence the yeard, maketh 49 ℔ 10 ſ 8 d: as by the example ſhall appeare.

Item, for 6 pence, take the $\frac{1}{4}$ parte of the number from the prickt figure: And if any vnities do remaine, they are ſo many ſixepences, whereof taking the $\frac{1}{2}$, they are ſhillings, if there do remaine yet one, it is in valewe 6 pence.

Item, for 4 pence, take the $\frac{1}{6}$ parte of the number from the prickte figure: If any vnities remaine, they are ſo many groates, which to conuert into ſhillings, take the $\frac{1}{3}$ part: And if any thing yet remaine, they are thirds of ſhillings, echeone in valewe being worth 4 pence.

Item, for 3 pence, take the $\frac{1}{8}$ parte from the prickt figure, if any vnities remayne, they are ſo many peeces of 3 pence whereof in taking the $\frac{1}{4}$ part, maketh ſhillings: If any thing yet remaine, they are fourth partes of ſhillings, echone being in valewe 3 pence.

Item, for 2 pence, as appeareth alſo by the table, take the $\frac{1}{12}$ parte of the number from the prickt figure: If any thing remaine, they are ſo many peeces of 2 pence: whiche by taking the $\frac{1}{6}$ parte, you ſhall turne into ſhillings: and if any vnities remaine, they are ſo many ſixte parte of ſhillings, or peeces of 2 pence, whether you will.

If one pound coſt — 8 d
What —1486 ℔
1 ——— 8—1486
maketh — ℔ —49 · —10 — 8 d

If one coſt 6 d
What 7865
1 ——— 6 ——— 7865
maketh — ℔ ——— 196 · —12 —— 6 d

If one yeard coſt 4 d
What —8736
1 ——— 4 ——— 8736
maketh — ℔ ——— 145 · —12—0

At 3 d the yeard
What 9874
1 ——— 3 ——— 9874
maketh — ℔ ——— 123 · —8—6 d

At 2 d the ell what 7894
1 —— 2 —— 7894

maketh poundes —— 65 · — 15 — 8 d

6. Rule. But if your number of pence be not an aliquot or euen part of 24: then muſt you bring them into the aliquot partes of 24, and make thereof diuers products, which muſt be added togither, as by the queſtions hereafter following ſhall appeare.

Item, for 5 d, firſt take for 3 d, then for 2 d: and adde them togither according to the inſtruction of the ſecond Rule: Or elſe firſte take for 4 d, then for 1 d.

Item for 7 d, firſt take for 4 d: then for 3 d and adde them togither.

Item, for 9, firſt take for 6 d: then for 3 d, and adde them togither.

Item, for 10 d, firſte take for 6 d: then for 4 d, and adde them togither.

Item for 11 d firſte take for 8 d then for 3 d and adde them togither: as by theſe examples folowing doth appeare.

Examples.

If one yearde coſt 5 d what 7596

1——5——7596

4 d 126· ——12

1 d 31——13

maketh poundes—158——5

Otherwiſe.

1——5——7596

3 d 94·——19

2 d 63——6

maketh poundes————158——5

If one coſt 7 pence what 987

1——7——987

4 d 16·—9—

3 d 12—6—9

maketh pounds——28—15—9

Otherwiſe.

1——7——987

6 d 24·—13—6

1 d 4——2—3

maketh pounds————28——15—9

I

If one coſt 9 pence what 987

1——9———987

6 d 24 ·—13—6

3 d 12 6 9

maketh poundes——— 37——·0—3

Otherwiſe.

1——9———987

3 d 12·———6—9

maketh pounds ——— 37———0—3

If one yearde coſt 10 d what 987

1——9———987

6 d 24 ·—13—6

4 d 16——9—0

maketh poundes———41———2—6

If one coſt 11 pence what 987

1——11———987

8 d 32 ·———18

3 d 12 12—6

maketh pounds———45———11—6

But if you haue any ſhillings, & pence to be multiplied togither: Then are you to take for the ſhillings according to the enſtruction of the third Rule: And for the pēce according to the firſt Rule before mentioned : vnleſſe you can ſpie the aduauntage therof: and thereby helpe your ſelfe : as appeareth in this ſeconde example, where firſt I worke for 6 d : which is to be rebated out of the giuen number, and I haue 719 ℔ 11 ß my deſire.

At 19 ß 6 d: the yearde
What 738 yeardes

	738	
10 ß	369.	
5 ß	184.	10
4 ß	147.	12
6 d	18.	9
℔.	719.	11 ß

	738	
6 d	18.	9
℔.	719.	11 ß

The like againe is done by rebating as by theſe 2 examples appeareth:

Item, 418 elles at 18 ß

2 ß 41—16——

℔——376—4——ß

Item 517. at 16 ß

4 ß 103-- -8

℔———413—12 ß

And

7.Rule. And now I wil touch a little the euen parts of a pound that falleth out in pence and ſhillings, wherof foꝛ thoſe partes you ſhall take ſuch like part of the giuen number that is to be multiplied, as the pꝛice of that giuen nũber beareth in pꝛopoꝛtion to a pound whiche alſo foꝛ thy better aide is here ſet down.

1 ß. 8 d	is the	$\frac{1}{12}$	of a ℔.
2. 6		$\frac{1}{8}$	
3. 4		$\frac{1}{6}$	
6. 8		$\frac{1}{3}$	

Item firſt foꝛ 1 ß 8 d take the $\frac{1}{12}$ parte of the giuen number & if any thing do remaine, they are twelue parts of a pounde, eache one being in value 1 ß 8 d.

Item foꝛ 2 ß 6 d take the $\frac{1}{8}$ part of the nũber that is to be multiplied. And if any thing do remaine they are eight parts of a pounde each one being in value 2 ß 6 d.

Item foꝛ 3 ß 4 d as appeareth by the table, you muſt take the $\frac{1}{6}$ part of the giuen nũber. And if anye thing do remaine they are 6 parts of a ℔: each one being in value 3 ß 4 d.

Item foꝛ 6 ß 8 pence take the $\frac{1}{3}$ part of the number that is to be multiplied: And if anye vnities do remaine, they are thirds of a pound euerie one being woꝛth 6 ß 8 pence.

Other infinite numbers there are, that may be reduced by abbꝛeuiation into the pꝛopoꝛtionate parts of a pounde: as 16 ß 8 pence maketh $\frac{5}{6}$: whiche 16 ß 8 d. is eaſilie reduced into groates by multiplying 16 by 3: & therto adde 2: which maketh 50 groates: Then ſet 60 the groates of a pounde vnder 50, cutting of the 2 Ciphers as is here perfoꝛmed in the margent. And then haue you bꝛoughte 16 ß 8 pence into the knowen partes of a ℔ which maketh $\frac{5}{6}$.

16—8
3
5|0
6|0

But yet gentle Reader, foꝛ thy further enſtruction, I haue herevnto annexed in a table, howe pence and ſhillings beareth pꝛopoꝛtion to a ℔: which I cõmitte to thy friendlie beneuolence, it will be ſome aide vnto the vngrounded pꝛactitioner: but I counte him the beſt woꝛkeman that can pꝛeſentlie reduce his giuen pꝛice vnto the knowen and pꝛopoꝛtionate parts of a ℔.

ß	ꝺ	℔	ß	ꝺ	℔
0	2	1/120	8	4	5/12
0	3	1/80	8	9	7/16
0	4	1/60	9		9/20
0	6	1/40	10		1/2
0	8	1/30	11		11/20
1	0	1/20	11	3	7/16
1	3	1/16	11	8	7/12
1	8	1/12	12		3/5
2	0	1/10	13		13/20
2	6	1/8	13	4	2/3
3	0	3/20	13	9	11/16
3	4	1/6	14		7/10
3	9	3/16	15		3/4
4	0	1/5	16		4/5
5		1/4	16	8	5/6
6		3/10	17		17/20
6	3	5/16	17	6	7/8
6	8	1/3	18		9/10
7		7/20	18	4	11/12
7	6	3/8	18	9	15/16
8		2/5	19	19	19/20

Here followeth 4 examples vpon the 4 notes deliuered.

At 1 ß 8 pence the yeards
What 3884 yeardes

℔ — 323 — 13 — 4 d

At 2 ß 6 pence the yearde
What 4563 yeardes

℔ — 570 — 7 — 6 d

At 3 ß 4 pence the yeards
What 5833 yeardes

℔ — 972 — 3 — 4 d

At 6 ß 8 pence the yeards
What 7562 yeardes
℔ — 2520 — 13 — 4 d

Nowe by cuſtome you are able to woꝛke by all ſoꝛtes of ſummes, being deliuered in ſhillings & pence, as 1 ß 1 pennie: 1 ß 2 pence 1 ß 3 pence, and ſo of all other: wiſhing you to haue ſome conſiderations of your queſtions, when they are ſet downe, foꝛ there are many ſubtill abbꝛeuiations, and great aduantages to be gotten, and eaſilie to be perceiued

As 2 ß. 8 d of 2 ß & 1 ℔ 8 d.

4 ß 2 d: of 3 ß 4 d: and 10 d whiche 10: is $\frac{1}{4}$ of 3 ß 4 d

5 ß 8 d. of 4 ß: and 1 ß. 8 d.

5 ß 10 d, of 5 ß and 10 d: whiche 10 d is $\frac{1}{6}$ of 5 ß.

And by this meane when you haue taken one product, you maye oftentimes vppon the ſame take an other more brieflie then vpon the ſum which is to be multiplied &c.

8.Rule. Nowe gentle Reader that you haue ſeene the vertue of the euen or aliquot partes of a ℔:in ſhillings alone, And alſo in the aliquot parts of ſhillings and pence: according to my promiſe hereafter followeth a briefer and eaſier method for any euen number of ſhillings either vnder or aboue 20, then euer yet hath bene publiſhed: Notwithſtanding Maiſter Humfrey Baker, whoſe trauel is worthie cõmendation, And whom for knowledge ſake I reuerence hath in ſome part touched thys firſt parte: though not in this method: The worke of the Rule is both pleaſante, readie and briefe. As by the varietie of the examples deliuered thereupon ſhall appeare. And firſt I wil ſet forth a queſtion: Thereby the better to expreſſe or teach you the order thereof: which is this.

If one yarde coſt 6 ß what 8574

1 —— 6 —— 8574

---

maketh pounds —— 2572 : —— 4

To the vnderſtanding of this exāple, after you haue ſet down your giuen nūber in form of the rule of 3, with a line drawen vnder it; you ſhal preſētly ſet a prick vnder your firſt figure 4, towards your right hād, drawing frō the pricke as heretofore hath bene practiſed, a little ſhort line, thereto ſette downe the ſhillings anone, which done, multiplie the firſt figure 4 by 6. the value of your price, (whyche here you ſee ſtandeth in ſight aboue the line) it maketh 24: which is 1 ℔ 4 ſ. The 1 ℔ keep to carrie to the next place, & the 4 ſ ſet downe at the end of the preſcribed line towards your right hande: Thus haue you done nowe with 6 aboue the line, and alſo with 4 in the firſte place (for the pricke vnder the 4. doeth repreſent that 4 hath done his office.) Then ſecōdarily for a general rule take but the ½ of the giuen price whiche here is 3, which 3, is the number that ſhal now continue the reſte of ẏ multiplication and end the worke, wherupon I multiply 3 into 7 ſtanding in the ſeconde place it maketh 21, and with the 1 ℔ I kept in minde 22, ſet downe 2 & keepe 2 in minde working according to the rule of multiplication, deliuering the tens in mind in their due place, which done, the product from the pricke

Note. A general rule.

to your left hand repreſenteth the pounds and the other at the ende of the line the ſhillings: as appeareth by the examples.

If one yearde coſt 2 ß what 7536
1 ——— 2 ——— 7536

maketh pounds ——— 753 ——— 12 ß

If one yearde coſt 4 ß what 8792
1 ——— 4 ——— 8792

maketh pounds ——— 1758 ——— 8 ß

If one piece coſt 6 ß what 9537
1 ——— 6 ——— 9537

maketh pounds ——— 2861 ——— 2 ß

If one coſt 8 ß what 7509
1 ——— 8 ——— 7509

maketh pounds ——— 3003 ——— 12 ß

If one coſt 12 what 5794
1 ——— 12 ——— 5794

maketh pounds ——— 3476 ——— 8 ß

If one coſt 14 ß what 3705
1 ——— 14 ——— 3705

maketh pounds ——— 2593 ——— 10 ß

If one coſt 18 ß what 5703
1 ——— 18 ——— 5703

maketh poundes ——— 5132 ——— 14 ß

If

Iſ one coſt 22 ß what 953

1 ———— 22 ———— 953

maketh pounds ———— 1048. ———— 6 ß

Let theſe ſuffiſe gentle reader for an entrance into euen numbers: And now I wyll ſhew the like rule for any od or vneuen parts of a pound.

To help you to the vnderſtanding of theſe other queſtions ye hereafter followeth: where in my firſt example the giuen nūber is 6487. 2 Rule.
At 3 ß the yeard: I multiply 3 aboue the line into 7. it maketh 21 : The 1 ſhilling I ſet down & the 1 lb I keepe: Nowe am I to take the $\frac{1}{2}$ of 3: which becauſe it is an odde number I cannot. Therefore I ſhal keepe and continue my multiplication by 3 ſtil: And worke by the $\frac{1}{2}$ of the reſt of the giuen figures or nūbers: To wit 648: And firſt the $\frac{1}{2}$ of 8 whiche is 4 multiplied into 3 maketh 12 therto ioine ye 1 lb in minde, it maketh 13: ſet down 3 keepe one. Then againe multiply by 2 the $\frac{1}{2}$ of 4 it maketh 6, and with 1 in minde it maketh 7. Then laſtly take the $\frac{1}{2}$ of 6 which is 3. ſaying 3 times 3, is 9: whiche 9 ſet downe and ſo is the queſtion aunſwered as appeareth by the practiſe, and the examples following.

At 3 ß the yearde what 6487

1——3——6487

maketh pounds. 973.——1 ß

If one yearde coſt 5 ß what 4769

1——5——4269

maketh pounds 1067.——5 ß

At 7 ß the elle what 6489

1——7——6489

maketh poundes 2271.——3 ß

If one elle coſt 9 ß what 2807

1——9——2807

maketh pounds 1203.——3 ß

At 11 ß the piſtolet what 8263

1——11——8263

maketh pounds 4544.——13 ß

If one piece coſt 13 ß what 4629

1——13——4629

maketh pounds 3008.——17 ß

But nowe note gentle Reader, when the giuen price falleth vppon anye odde number. As 3.5.7.9.11.13.&c: Then it is to be preſuppoſed, that the giuen ſumme to be multiplied muſte be a ſumme made of euen numbers, as 2.4.6.8.0 &c. elſe can not that queſtion be wrought at one line or working.

Prouiding alwayes that it maye beare an odde figure in the firſte place towardes your righte hand : as appeareth in theſe 6 Examples which laſt were wrought, and ſuch like ꝛc. which may beare an odde number for the price, and be done at one line or working very wel.

But if the giuen price be an odde number, and the ſumme to be multiplyed odde numbers alſo : Thē can it not be done at one working, but requireth the aide of 2 workings: for odde with odde will not agrée, which notwithſtanding to bring to paſſe. Take this for a general rule: Firſt work for the euen number, contained in that queſtion, or giuen price, according as you haue learned , And thē afterwards for the one odde ſhilling, take the $\frac{1}{2}$ of the ſumme giuen to be multiplied, omitting the firſt prickt place, As was taught for yͤ working of one ſhilling in my firſt rule of practiſe, And adde thoſe two togither. And you ſhal haue your deſire.

A general rule.

Example.

At 3 ß the yearde what 7539

1 —— 3 —— 7539

---

2 ß 753·—18

1 ß 376·—19

---

maketh pounds 1130·—17

At 7 ß the ell what 7539

1——7——7539

6 ß	2261 ·	14
1 ß	376	19
maketh pounds	2638	13

At 13 ß the yearde what 7534

1——13——7534

12 ß	4520 ·	8
1 ß	376 ·	14
maketh pounds	4897	2

And thus haue I abbridged into theſe two Rules how to bring any number of ß: whatſoeuer they be into pounds, wᵗ a briefer method, then euer yet hath bene publiſhed, whiche I commende vnto thy friendlye cenſure and iudgements in the vſe of practiſe thereof.

If one coſt 6 ß 5 d what 1231

1——6——5——1231

6 ß	369 ·	6	
4 d	20 ·	10	4
1 d	5 ·	2	7
maketh poundes	394	18	11

At 14 ß 2 d what 2825.

1 —— 14 — 2 —— 2825

14 ß 1977 — 10

2 d 23 — 10 — 10

maketh pounds 2001 — 00 — 10 d

At 16 ß 4 d what 2531

1 —— 16 —— 4 —— 2531

10 ß 2024 — 10

4 d 42 — 3 — 8

maketh pounds 2060 — 19 — 8

At 3 ß the Piſtolet what 8325

1 —— 3 —— 8325

maketh poundes 1248 —— 15 ß

At 7 ß the crowne what 6529

1 —— 7 —— 6529

maketh pounds 2285 —— 3 ß

At 9 ß the péece what 6567

1 —— 9 —— 6567

maketh pounds 2955 — 3 ß

Theſe thrée laſt queſtions may ſéeme ſome thing harde, yet are they eaſie ynough, if you marke them well, if I ſhould explaine them, then are they tóo eaſie: therfore I leaue them to whet the minds of the deſirous.

10. Rule. Item, when any one of the ſummes whiche is to be multiplied, is compoſed of manye Denominations: and the giuen number but of one figure alone: Then ſhall you multiplye all the denominations of the other ſum, by the ſame one figure, beginning firſt with that ſumme which is leaſt in valew towards your right hande, and bring the produce of thoſe d into ß, and the produce of the ß into ℔, as by this example doth appeare.

At 3 ℔ — 7 ß — 4 d what 9

Maketh —— ℔ — 30 — 6 — 0 d

11. Rule. But if in any of the ſummes that are to be multiplied ther be a broken number: Firſt worke for the whole according to the inſtructions that you haue learned: and then take ſuche part of the giuen price: as that broken number beareth in proportion to the price, as in the example: after you haue wrought, for 3 ß and for 6 d: then are you to take the $\frac{1}{2}$ of 3 6 d for the $\frac{1}{2}$ yeard: and adde that to the ſumme: So adding all 3 productes togither which maketh 42 ℔ — 2 ß — 11 d the iuſte price of 246 $\frac{1}{2}$ Els: and thus muſte you do of all other.

At 3 ß 6 d the Ell what 246 $\frac{1}{2}$
1———3—6—246 $\frac{1}{2}$
3 ß 36·—18
6 d 6— 3
·1—9
maketh ———43 —2—9

At 16 ß 4 d the péece
What 14 $\frac{3}{4}$
1 ———16—4—14 $\frac{3}{4}$
10 ß 11·—4
4 d —48
$\frac{3}{4}$ 12—3
maketh ——— ℔ ——— 12 -00—11

If one péece coſte 4 ℔ —3 ß —6 $\frac{1}{2}$
What ———12

4 ℔ 48
3 ß 1—16
6 d 0 6
$\frac{1}{2}$ · ·—6
maketh ——— ℔ ———50——2—6 d

The Proofe.

If 12 peces coſt 50 ℔· 2 ß· 6 d. what one pece

50 —— 2 —— 6

---

maketh pounds 4 —— 3 —— 6½

12.Rule. Item, touching the manner howe to underſtande the order of this proofe, and others the like: firſt ſeeke howe many times 12 is contained in 50: maketh 4: reſteth 2 ℔ whiche conuerted into ſhillings, and ioined with the other 2 ß, maketh 42 ß: wherein is foūd 12 three times: reſteth 6 ß which turned into pence, putting thereto the 6 d in the firſte place, it maketh 76: wherein 12 is founde 6 times, reſteth 6 d, which containeth 12 but ½ a time, put that ½ to the 6 d: And then the ſolution is 4 ℔ — 3 ß — 6½ as appeareth by the practiſe thereof.

13.Rule. Item, the like is to be done of any thing that is bought or ſolde after 5 ſcore to the hundreth, or the Quintall: As for example.

If 100 ℔ coſt 27 ℔ — 13 ß — 4 d
What one pound.

```
        27—13—4
          20
ß      5|53
        |12
       1|10
       5|3
d      6|4 |0      2
       1 0|0       5
```

maketh 5 ß — 6 2/5 d

I haue wrought this at lēgth for yͤ aide of yͤ yōg learner, bicauſe he ſhould vnderſtād how al yͤ Multiplication is ſet downe.

```
        27—13—4
          20
ß      5|53
        |12
d      6|40     2
        |100    5
```

makes 5 ß 6 2/5 d

But to worke it more neatly, it is by a little vnderſtanding ended thus.

Item to the vnderſtanding of this and ſuche like queſtions, the right downe line is all the guide, which is pulled down cloſe by 20, as you ſee in the example, where 27 ℔ — 13 ß is reduced all into ß: maketh 553.

The 5 towards your lefte hād being ſeparated with the hanging or right downe line, is the iuſt number of ſhillings: that aunſweareth to the queſtion: Nextly, 53 ß is multiplied by 12 to reduce them to pence, putting to the 4 đ: it yéeldeth for the multiplication of the firſt figure two: 1 10: the one beyond the line towards the left hande: is 1 penny towardes the reſte of the price: then 53 alſo multiplied by 1 yéeldeth 53: but the 5 behinde the line towards the left hande, is alſo 5 đ more, towards the price, which 1 and 5: I adde togither vnder the line: it maketh 6 đ: So is there found nowe as appeareth by the Titles of ß and đ: 5 ß 6 đ.

Finally, I come nowe on this ſide the line, towards the right hand: and vnder 12: I find firſt 10: and then 3: whiche added togither maketh 40: vnder whiche 40, you muſte putte the 100: and it maketh —— $\frac{40}{140}$ which abbreuiated commeth to $\frac{2}{7}$: So the iuſte price of one pounde after 5 ſcore to the hundreth, maketh — 5 ß — 6 $\frac{2}{7}$ đ.

One example more, and ſo will I leaue this rule.

At

Iſ 100 coſt 10$\frac{3}{4}$ d
What — 9874
100 .. 10$\frac{3}{4}$ .. 9874

6 d	246.	— 17 —
4 d	164.	11 — 4
$\overset{2}{\underset{1}{4}}$	20	11 — 5
	10	5 — 8$\frac{1}{2}$

maketh	℔	4	42 . . 5 . 5$\frac{1}{2}$ 20	
	ß	8	45 12	
	d	5	$\frac{45\frac{1}{2}}{100}$	$\frac{91}{200}$ parts of a d.

Alſo the like maye be done of our vſuall waightes here in England (whiche is 112 ℔ for euery hūdreth waight) in caſe you know the Aliquot parts of a hundreth waight, whiche are theſe, 56 ℔, 28 ℔, 14 ℔, and 7 ℔: For 56 ℔ is the $\frac{1}{2}$ of 112 ℔, 28 ℔ is the $\frac{1}{4}$ of 112 ℔, 14 ℔ is the $\frac{1}{8}$, and 7 ℔ is the $\frac{1}{16}$.

Therfore for 56 ℔, take the $\frac{1}{2}$ of the ſumme of mony that 112 ℔ waight is worth.

For 28 ℔ take the $\frac{1}{4}$ of the ſumme of money that 112 ℔ waight is worth.

For 14 ℔, take the $\frac{1}{8}$ of the ſumme that 112 ℔ is worth.

And for 7 ℔, the $\frac{1}{16}$ of the ſumme of money that C. is worth.

As for example: at 17 ℔ — 19 ß the hundreth poundes waight, that is to ſaye, the 112 ℔, what ſhall three quarterns and 7 ℔ coſte?

	1	17	19	3—7
2 quarternes	8	19	6	
1 quarterne	4	9	9	
7 pounds	1	2	5 $\frac{1}{4}$	
maketh pounds	14	11	8 $\frac{1}{4}$	

The ſecond Chapiter entreateth of the Reduction of diuers meaſures to others valewe by Rules of Practiſe.

v. Rule. NOW will I ſhewe a fewe examples of Practiſe in reducing of meaſures: as Elles Yeardes, Braces, pawnes of Jeanes &c. Muche more I woulde haue touched but that I feare the booke wil riſe to too great a volume.

In 864 elles of Antwerpe, howe many yeardes of London?

864	864
432	210
216	648

maketh: 648. —— yeards of London.

Item, in theſe and ſuch like queſtions of Flemmiſh meaſure to be brought into yards Engliſhe: firſte take the ½ of the giuen number, as appeareth in the firſt example towards your lefte hand: Then take the ½ of that product: or the ¼ of the giuen number: and adde thoſe 2 products togither, they ſhal be yeards Engliſhe, as by the example you maye perceiue.

The ſecond example towards your right hand is yet briefer than the firſt, whoſe work is this: take the ¼ of the deliuered number, and that product, ſubtray out of the gyuen number: and the reſt ſheweth your deſire: of theſe two wayes vſe whiche you thinke beſt.

The Proofe.

In 648 yeards of London,
how many Elles of Antwerpe
648

216

maketh —804 Elles of Antwerpe.

Item for the underſtanding of this work: firſt take the $\frac{1}{3}$ part of the yeards of London, which found, adde that $\frac{1}{3}$ part, and the yeards togither, as appeareth by the practiſe: and the product ſheweth the Elles of Antwerpe.

In 320 yeards of London,
how many Elles of Antwerpe
maketh 426 $\frac{2}{3}$ Elles.

320 yeardes	Proofe.
$\frac{1}{3}$ 106 $\frac{2}{3}$	426 $\frac{2}{3}$ elles
426 $\frac{2}{3}$ elles	$\frac{1}{3}$ 106 $\frac{2}{3}$
	320 yeards.

Other Reductions.

Item you ſhall underſtand, that for as much as 6 Braces of Millane make 5 Elles of Antwerp, whereuppon, according to the Rules of Practiſe, you maye reduce the one into the other by the like Reaſons aforeſaide in taking the $\frac{1}{6}$ parte, and then ſubtraye the ſame to make Elles of Antwerp: And againe by the contrary in taking the $\frac{1}{5}$ part, with adding the giuen number, to turne the Elles to Braces, as for example.

In 876 Braces how many elles of Antwerp

	The contrarie
876 Braces	730 Elles Fle.
146	146
elles 730 Antwerp	876 Braces.

Elles 730 Antwerp.
182 $\frac{1}{2}$
Yeards 547 $\frac{1}{2}$ Engliſh.

Thus appeareth, that 876 braces by pratiſe, make 730 Elles Flemmiſhe: whiche Elles Flemmiſh reduced into Engliſh yeardes by the Rules aforeſaide, make 547 $\frac{1}{2}$ yeardes.

7.Role. So againe vpon the ſame firſte Queſtion of Braces: I woulde knowe howe manye yeardes Engliſh they make.

After the rate that 100 Braces are worth 62 $\frac{1}{2}$ yeardes.

876 Braces

438

109 $\frac{1}{2}$

I anſwere 547 $\frac{1}{2}$ Yeardes

Item, to the vnderſtanding of this worke and ſuche like, firſte take the $\frac{1}{2}$ of the giuen Braces: And after take the $\frac{1}{4}$ of that halfe: or the $\frac{1}{8}$ of the giuen number, and adde them togither: And the product are alſo yeards Engliſhe.

18.Rule. Item 3 Elles of Rochell make 5 Elles at Liſbone: So likewiſe 3 Elles at Lions make 5 Elles at Antwerpe.

To worke theſe and ſuch like, double the Elles of Lions, and the Elles of Rochel: and from their products Subtray the $\frac{1}{8}$: And the reſt ſhall be Elles of Antwerpe, or Elles of Liſbone.

Example.

In 63. Elles of Lyons how many Els of Antw.	In 100. Elles of Rochel how many Elles of Liſb.
03	
63	100
126	200
$\frac{1}{6}$) 21	$\frac{1}{6}$) $33\frac{2}{3}$
Anſwer 105 Elles Ant.	Anſw. $166\frac{2}{3}$ els of Liſb.

Touching the proofe or returne of theſe & ſuch like queſtions for a generall Rule, you ſhal firſt take the $\frac{1}{5}$ of the giuen number: And adde that $\frac{1}{5}$ and the giuen number togither. And the $\frac{1}{2}$ of that producte ſhall be your deſire.

Example.

In 105. Elles of Antw. how many Elles of Liſbõ	In 166.$\frac{2}{3}$ elles of Liſbon how many elles of Roch.
$\frac{1}{5}$) 105	$\frac{1}{5}$) $166\frac{2}{3}$
21	$33\frac{1}{3}$
$\frac{1}{2}$) 126	$\frac{1}{2}$) 200
Anſwere.63.els of Liſbõ.	Anſwer.100.els of Roch.

The thirde Chapiter treateth of the order and worke of the Rule of three in broken Numbers, after the trade of Merchants, digreſsing ſomething from Maſter *Records*, which is comprehended in 3 Rules.

NOw that I haue ſomwhat intreated of the Rules of Practiſe, I will giue a fewe inſtructiõs after my ſimple order, for the working of the Rule of thrée in broken numbers, wherein I ſhall néede to ſaye the leſſe, becauſe I hope the ſtudious Learner, that hath trauelled any thing in the Grounde of Artes, is not vnfurniſhed of knowledge capable to vnderſtande me. But before I deliuer any inſtructions for broken numbers, I will prepone a queſtion, whiche ſhall bée wrought 3 ſundrie wayes, thereby to ſhew as it were 3 degrées of compariſon: how far the Rule of thrée, in broken, for more ſpéede of worke, differeth from whole, which I rather ſet downe for a view, that the ſtudious herein may be more deſirous to attaine broken: leauing any more to diſcourſe in Dialogue forme: but only to giue inſtructions, where néede is: and in the reſt to putte forth the queſtions with their aunſweares.

My firſt queſtion is this.

If one yeard coſt 6ß —8 d.
What are ——789 worth at that rate

$$\begin{array}{ccccccc} 1 & — & 6 & — & 8 & — & 789 \\ & & 12 & & & & 80 \\ \hline & & 80 & & & & 63120 \text{ d.} \end{array}$$

Here the product of the ſumme are pence according to the nature of the midle nūber.

$$\begin{array}{lll} \cancel{1}\,\cancel{1} & & \\ \cancel{1}\,3\,\cancel{7} & \vert\,\cancel{1} & \\ \cancel{6}\,\cancel{3}\,\cancel{1}\,\cancel{2}\,0 & \vert\,\cancel{5}\,\cancel{2}\,\cancel{6}\,0 & (263 \\ \cancel{1}\,\cancel{2}\,\cancel{2}\,\cancel{2}\,\cancel{2} & \vert\,\cancel{2}\,\cancel{2}\,20 & \\ \cancel{1}\,\cancel{1}\,\cancel{1} & & \end{array}$$

I aunſweare 263 li.

$$\frac{1}{3} \text{———} \frac{6\frac{2}{3}}{20} \text{———} \begin{array}{r} 789 \\ 20 \\ \hline 15780 \end{array}$$

Here the producte of the ſumme is ß, according to the nature of the middle number.

$$\begin{array}{l|l|l} \cancel{1} & & \\ \cancel{1}\cancel{5}\cancel{7}\cancel{8}0 & \cancel{5}\cancel{2}60 & 263 \text{ pounds.} \\ \cancel{3}\cancel{3}\cancel{3}3 & \cancel{2}\cancel{2}\cancel{2}0 & \end{array}$$

$$\frac{1}{3} \text{———} \frac{\frac{1}{3}}{1} \text{———} \begin{array}{r} 789 \\ 1 \\ \hline 789 \end{array}$$

Here the producte is pounds according to the title of the ſecond number.

$$\begin{array}{l} \cancel{1} \\ \cancel{7}\cancel{8}9 \\ \cancel{3}\cancel{3}3 \end{array}(263$$

I anſwere 263 ℔

Now that you haue ſeene the 3 former vertues of the rule of three, whoſe products hath firſte broughte forth d, nexte ß, and laſtlye, poundes: I will deliuer 3 notes in order following: And with them a doſen queſtions:

that ſhall ſhewe the worke of the Rule of thrée in broken numbers or Fractions.

1 The firſte foure ſhall bée ſundrie Queſtions of a Fraction comming in the ſecond place.

2 The ſecond foure ſhall be of 2 Fractions comming in the ſeconde or thirde place.

3 The thirde foure of Fractions in all thrée places.

My firſt queſtion is this.

If one yeard coſts me 3 ß — 4 d what are 756 worth at that price. 1. Rule.

In ſetting downe the Queſtion to performe the worke I turne 4 d into the part of a ß.

TO the ready working of this queſtion, & all ſuch other like, my firſt note is this: which take for a generall rule, that when any one Fraction ſhall come either in the ſecond or third place: that the Denominator of that Fraction or Fractions, muſt alwayes be brought vnto the number or Numerator of the firſt place: and thereby multiplyed the one into the other.

And this benefit is allwayes gotten by the vertue of bringing the Denominator of the ſecond numbers fraction vnto the firſt place. For the fraction in the midle number is now releaſed: and the producte that commeth of the Multiplication, is of the nature and lyke Denomination of the whole number in the ſecond place which here are ſhillings.

Whereuppon now to worke the queſtion I bring 3 the Denominator of the Fraction in the ſecond place, vnto my firſte number 1 with a line ſet vnder $\underline{1}$ thus: and the 3 vnder it $\frac{1}{3}$ thus: ſaying, once 3 is my Diuiſor: That done, reduce 3 $\frac{1}{3}$ ſaying, 3 times 3 is 9, and the 1 ouer 3 make 10: my ſecond number in the Rule of 3: by whych 10 I do multiply my laſt number 756 as appeareth by yͤ worke thereof: And it yéeldeth 7560 ß my diuidend.

Then diuiding 7560 by 3 my diuiſor, it yéeldeth in Quotient 2520 ß: whiche, as appeareth by the worke maketh 126 poundes.

At 3 ß 4 d. the yeard what 756 yeards

$\frac{1}{3}$ ——— 3 $\frac{1}{3}$ . . 756 .

10

10 7560 ß

```
1    | 1    |
7560 | 2520 | 126: I anſwere 126 ℔
3333 | 2220 |
```

If one yearde of cotten coſt $8\frac{1}{4}$ d what 859.

```
1 —— 8 1/4 ——— 859
—    ——        ———
4    33        33
               2577
              2577
          ————————
              28347
```

```
      | 1       |      |
 2(3  | 2       |      |
28347 | 7088(6  | 11   | ℔          ß         d
 4444 | 1222    | 590  | 29 —— 10 ——— 6 3/4 d
      |  11     | 220  |
```

This queſtion was alſo wrought like the firſt and bringeth forth 29 ℔ — 10 ß — $6\frac{3}{4}$ d the price of 859 yeardes.

If 7 pounde of anye thing coſt 3 ℔ 10 ß
What comes 987 ℔ to.

$$\begin{array}{lcr} 7\text{———} & \underline{3}\text{———} & 987 \\ \underline{2} & 7 & \underline{7} \\ 14 & & 6909 \end{array}$$

$\cancel{1}\cancel{4}\cancel{1}$

$\cancel{2}\cancel{3}\cancel{4}\,(7$

$\cancel{6}\cancel{9}\cancel{0}9\;|493\frac{7}{14}|\frac{1}{2}$ I aunſwere 493 ℔ 10 ß.

$\cancel{1}\cancel{4}\cancel{4}4$

$\cancel{1}1$

Item I will worke vpon my firſt queſtion now againe: but altered into the proportion it beareth to a ℔: for that 3 ß 4 d is $\frac{1}{6}$ of a pound.

$$\begin{array}{lcr} \dfrac{1}{6}\text{———} & \underline{1}\text{———} & 756 \\ & \underline{6} & \underline{1} \\ & 1 & 756 \end{array}$$ poundes.

As ſone as I haue carried 6 the denominator of my middle number vnto my firſte place, as afore hath bene taught: I pul down one, the numerator of 6, with a line vnder $\frac{6}{1}$ thus: And that one of cuſtome I pul downe in ſight being the figure ẏ I ſhal multiply my laſt number by: According to the tenor of the

Rule of 3: And becauſe one can neither multiplie noꝛ yet diuide (thoughe here it is ſette downe in foꝛme of multiplication,) the pꝛoducte of the multiplication, accoꝛding vnto the declaration of my firſt note, is conuerted into the title of my ſeconde number whiche here are poundes: nowe followeth the diuiſion perfoꝛmed by my diuiſoꝛ 6, to make an end of the queſtion.

$$\left.\begin{array}{r}\cancel{2}\,\cancel{3}\\ \cancel{7}\,\cancel{5}.\cancel{6}\\ \cancel{6}.\cancel{6}.\cancel{6}\end{array}\right|\ 126.\text{maketh } 126\ ℔.$$

Notes vpon my ſecond Rule for two Fractions cōming in the ſecond and third place.

2.Rule.

My firſt queſtion is this.

If one ell coſt 13 ß 4 d: what halfe a quarter oꝛ $\frac{1}{8}$ of an ell?

Anſwer. Firſte bꝛing 13 ß 4 d into the parts of a ℔: which is $\frac{2}{3}$ and then wil the queſtion ſtand thus. 1 — $\frac{2}{3}$ — $\frac{1}{8}$.

Item for the performance of this worke do as before was taught in the firſt Rule, firſte bring 3 the denominator of the ſecond fraction vnto your firſt number 1 : ſetting a line vnder $\underline{1}$ thus: Saying once 3 is 3 : that done bring 8 the denominator of the third fraction ſetting it vnder 3 : and multiplye them togither, ſaying 3 times 8 makes 24: which 24 is your diuiſor : (Now haue you done with the denominator 3. and alſo with the denominator 8.) Therefore you ſhal put a line vnder $\underline{3}$ thus. And the like line alſo vnder $\underline{8}$: ſetting or pulling downe vnder them their own numerators, that is 2 vnder 3, and alſo 1 vnder 8, as appeareth in the example, which Numerators for a general rule are euermore to be pulled down, of cuſtome in ſight, to multiplie the one by the other according to the tenor of the Rule of thrée. Then I multiplie the one by the other ſaying once 2 is 2 : which ſignifyeth 2 ℔: being of the Nature and like denomination of the middle number, which 2 ℔ is to be reduced into ſhillings, otherwiſe it can not be diuided by my firſte number 24 : Then diuiding 40 by 24 : the quotient bringeth forth 1 $\frac{2}{3}$ ß : So muche is $\frac{1}{8}$ of an ell worthe after that rate. Otherwiſe although 2 pound coulde not be diuided by 24: yet it might haue bene abbreuied to $\frac{1}{12}$ of a pounde: whiche is worth 1 ß 8 pence, as before.

$$\begin{array}{ccccc} & & & & (1 \\ \underline{1}\text{———} & \underline{\frac{2}{3}}\text{———} & \underline{\frac{1}{8}} & & \cancel{2}6 \\ 3 & 2 & 1 & & \cancel{4}\cancel{0}(1\frac{2}{3}ß \\ 8 & 20 & & & \cancel{2}\cancel{4} \\ \overline{24} & \overline{40} & & & \end{array}$$

Second queſtion.

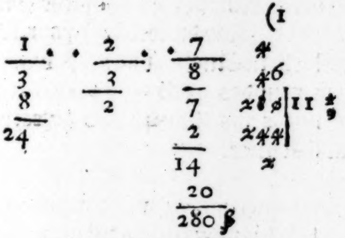
If one pound of any waighte coſte 13 ſhillings 4 pence: what are $\frac{7}{8}$ of the pound worth after that rate: Anſwere. Reduce the 13 ſhillings 4 pence into the partes of a pounde: which is $\frac{2}{3}$: and then will the queſtion ſtand thus.

$$\begin{array}{c} \text{lb} \\ 1\text{———}\frac{2}{3}\text{———}\frac{7}{8} \end{array}$$

Item for the vnderſtanding of this, if you marke wel the laſt example, this and the reſt lyeth open, and néedeth ſmall inſtruction. For as you did laſte. So nowe againe bring the Denominator of the ſeconde and third fraction vnto the firſt figure, 1, multiplying the one into the other which maketh alſo 24, as before your Diuiſor.

Then making a line vnder 3 thus, and a line vnder 8 thus, And pulling downe their Numerators vnder each figure, that is, 2 vnder 3, and 7 vnder 8, whiche as I ſaid before for a generall Rule I pull downe of cuſtome in ſight: to be the two numbers that of dutie ought to be multiplied togither, whiche done I bring 2 being yͤ leſſer figure vnder 7: multiplying them togither, it maketh 14: whiche are of the nature of the middle number. That is to wit pounds: which 14 cannot aptlye bée diuided among 24: Therfore are reduced into ß, as is plainly to be ſéene in the example. Then 280 ß parted among 24: yéeldeth for his quotient 11 ß 8 d: your deſire: and yͤ iuſt price of $\frac{7}{8}$ of an ell. Otherwiſe, 14, thoughe it could not be diuided by 24: might by mediation or diuiſion in broken numbers haue bin diuided or abbreuiated to $\frac{7}{12}$ whiche in effect being reduced to his knowen partes maketh 11 ß 8 d as before. But my good wil & meaning is to aide yong beginners. Therefore haue I reduced the 14 ℔ into ß: which is the eaſier way.

Now followeth the Example.



The third example.

If one yarde coſte me 2 ß 6 d. what 345 ¼ yardes.

Anſwer. Firſt put 6 d. into the parts of a ß: and then the queſtion ſtandeth thus.

1 — 2 ½ — 345 ¼:

Itē to the readie vnderſtanding of this & all ſuch like, according as before hath bene declared: Bring the denominators of the ſecond & third fractions vnto the firſt place, multiplying them the one into the other, al which maketh 8. your diuiſor cōmon. Thē next reduce your ſecond number, ſaying 2 times 2 is 4, and 1, is 5 : as appeareth in the example. Laſtly, reduce your third number 345 $\frac{1}{4}$ all into fourths, and they make 1381 : whiche 1381 is to be multiplied by 5, accordyng to the tenor of the Rule of thrée : whiche done, maketh 6905 ſ : And diuided by 8 your Diuiſor : yéldeth in Quotient 863 $\frac{1}{8}$ ſ whyche maketh in poundes 43 ℔ — 3 ſ — 1 d $\frac{1}{2}$: And ſo muche are the 345 and $\frac{1}{4}$ yeardes worth at that price.

The ſame queſtion wrought again but
2 ſ 6 d : is now conuerted into
the parts of a ℔, and
ſtandeth thus:

1 ——— $\frac{1}{8}$ ——— 345 $\frac{1}{4}$

Item, after I haue brought here my ſecond and third fraction unto my firſt place, & found 32 to be my deuiſor : hauing thus furniſhed my firſte place with all thinges unto him belonging (whiche is meant of bringing and multiplying the Denominators of the ſecond and third Fractions into him) I then goe in hand to ſée what is to doe in my ſecond place, where preſently of cuſtome I pull downe my Numerator 1 under 8 : béeing the figure in ſight that ſhal multiply my third number.

Then laſtlye I reduce 345 $\frac{1}{4}$ all into fourthes, as afore was practiſed, whych maketh 1381, the whyche 1381 I am to multiplie by one my ſeconde number : they are nothing encreaſed, but by the Methamorph of my worke, they are nowe 1381 pounde : being of the nature of the middle number, as I haue often ſhewed you, whiche diuided by 32 my Diuiſour, yéeldeth 43 pounde and $\frac{5}{32}$: whiche $\frac{5}{32}$ of a ℔ reduced into knowen numbers ,˙ make 3 ſhillings 1 d $\frac{1}{2}$ as before.

Example.

$$
\begin{array}{ccc}
1 \text{———} & \frac{1}{8} & \text{———} 345\frac{1}{4} \\
32 & 1 & 1381
\end{array}
\qquad
\begin{array}{l}
\quad\;\cancel{1} \\
\;\cancel{1}\cancel{0}(5 \\
\cancel{1}\cancel{3}\cancel{8}\cancel{1} \\
\;\cancel{3}\cancel{2}\cancel{2}(43\frac{5}{32} \\
\quad\;\cancel{3}
\end{array}
$$

Rule.

Now followeth 4 other queſtions which are in al 3 places, bꝛoken numbers oꝛ whole and bꝛoken togither.

Item firſte foꝛ the finding out of your diuiſoꝛ: you ſhal take this foꝛ a moſt certaine and generall rule. That you muſt multiply the numeratoꝛ of the firſt number in the queſtion by the denominatoꝛ of the ſecond: And alſo al that againe by the denominatoꝛ of the third: and the total therof ſhal be your diuiſoꝛ.

Secondarily foꝛ a generall rule to find out your diuidend, multiplie the denominatoꝛ of the firſt number by the numeratoꝛ of the ſeconde, and the whole thereof by the denominatoꝛ of the third. And the total thereof ſhal euermoꝛe be your diuidend.

Nowe foꝛ an example I pꝛopone this queſtion thereby to make my meaning the moꝛe plaine, and to ſhew you as I haue done in the reſt the manner and oꝛder of the woꝛke.

If $\frac{3}{7}$ of any waight oꝛ meaſure coſt $\frac{5}{8}$ of a ℔ oꝛ 20 ß what are $\frac{2}{8}$ of the like waight oꝛ meaſure woꝛth after that rate.

Example.

$\frac{2}{3}$ — $\frac{5}{6}$ — $\frac{3}{8}$

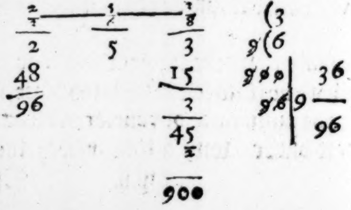
Item for the more plainer vnderſtanding hereof, and all other the like, in broken numbers: Firſt you ſhal pul downe 2 the Numerator of the firſte number or fraction, with a line vnder $\underline{2}$ thus: that done according as you haue learned before, bring 6 the denominator of the ſecond fraction, and ſet it vnder 2 multiplying the one into the other whiche maketh 12. Then laſtly bring 8 the denominator of the third fractiō: and ſet it vnder 12, multiplying that 12 by 8: which amounteth to 96: (or elſe for more briefe multiply 6 by 8: ſaying 6 times 8 makes 48: which 48 ſette vnder 2, and multiplie the one into the other, maketh 96 as before) And this 96 is the firſt nūber in the rule of thrée. That ſhal alwaies for a moſt general rule be your diuiſor.

Secondly to worke for your diuidend you ſhal as hath bin ſufficiētlie declared afore, pul down 5 the numerator of your ſecond fractiō. And ſet it vnder 6 with a line vnder $\underline{6}$ thus.

That done as you knowe, you are to pull downe 3 the numerator of the third fraction and ſet it vnder 8. with a line vnder 8 thus multiplying the one into the other according to the tenor of yͤ rule of 3: which maketh 15: thẽ according to my note, forget not to bring the Denominator of the firſt fraction which is 3 vnder 15: And multiplie them togither, which maketh 45: which 45, is your diuided.

Which 45 are of the nature or denominatiõ of the middle number, as I haue ofte taught you before And therefore are 45 ℔, whiche aptly cannot be diuided by 96: Therfore you ſhal reduce that 45 ℔ into ß as you ſée is performed in the example, which amounteth to 900 ß whiche diuided by 96, your diuiſor it yéeldeth 9 ß & $\frac{36}{96}$ of a ß which in leſſer termes is $\frac{3}{8}$ which $\frac{3}{8}$ in money maketh 4 $\frac{1}{2}$ d: And ſo much wil the aforeſaid $\frac{3}{8}$ coſt, as by the work following ſhal appeare.

Example.



Otherwiſe though 45 coulde not be diuided by 96: yet by diuiſion in brokē numbers it might haue bene abbreuiated to $\frac{15}{32}$ of a ℔, which reduſed into knowen parts will make 9 ß 4 $\frac{1}{4}$ d. as before.

Now my ſecond example ſhal be the proofe of this queſtion.

If $\frac{3}{8}$ yeards coſt $\frac{15}{32}$ of a ℔: or 20 ß what ſhal $\frac{2}{3}$ coſt?

Anſwer. Worke as was taughte you before, and you ſhal haue your deſire.

$$\begin{array}{ccc} \frac{3}{8} & \frac{15}{32} & \frac{2}{3} \\ \hline 3 & 15 & 2 \\ 32 & 2 & \\ \hline 90 & 30 & \\ 3 & 8 & \\ \hline 288 & 240 & \end{array}$$

Here as appeareth by the worke, the multiplications being ended 240 is to be diuided by 288: which to ſome perchance may ſeeme harde, yet notwithſtanding is the work good: Therefore abbreuiate 240 by 288: as you ſee here is practiſed: and the end of your abbreuiation ſhal come to $\frac{5}{6}$ your deſire. $\frac{240}{288} \Big| \frac{30}{36} \Big| \frac{5}{6}$

Otherwiſe. $\frac{240|120|60|30|5}{288|144|72|36|6}$

Otherwiſe. $\frac{240|40|5}{288|48|6}$

The third queſtion.

If $\frac{3}{4}$ elles coſt 13 ß 4 d what 156 $\frac{1}{2}$ elles? Anſwer. To worke this queſtion the ſhorteſt way: reduce 13 ß 4 d into the partes of a ℔ which is $\frac{2}{3}$.

Then as you did afore after, you haue ſette downe the queſtion, the numerator of ye firſte Fraction 3, is pulled down vnder 4. and the denominators of ye other 2 fractiõs multiplyed into him which maketh 18 your diuiſor.

Then the Numerator of the ſecond fraction 2: is pulled down vnder 3, of cuſtome now in ſight readie to multiply my third number by: which is performed as ſoone as the laſte number 156 $\frac{1}{2}$ is reduced into halfes.

Then laſtly I multiply that product by 4: the denominator of my firſte fraction: it yeeldeth 2504: which I diuide by 18: And my quotient is 139 ℔ & $\frac{1}{9}$ of a ℔ remayning which is worth 2 ß 2 $\frac{2}{3}$ d: And ſo much will 156 $\frac{1}{2}$ elles coſt as by the worke following doth appeare.

The

```
1/3 . . 2/3 . . 156 1/4          1
3       2       313             47
6                 2             78(2
18              626           2504|139 1/9
                  4           1888
               2504             11
```

The fourth Example.

If 2 ½ Elles coſte 1 ⅔ lb what commeth 29 ¼ elles to?

Item to the workmanſhip of this queſtion: firſte reduce your firſte number to one direct Numerator: in ſaying, 2 times 2 is 4, and 1 is 5: Then bring the Multiplication of the Denominators of the ſeconde and thirde Fractions, which maketh 12: and multiplie that 12 by 5 your firſt Numerator, it maketh 60, which is your diuiſor.

Then the reduction of the ſecond number, which is 5 multiplied by 117 the product of yͤ laſte numbers reduction, make 585, whiche 585 yet reſteth to bée multiplied by 2, the denominator of the fraction in the firſte place yéeldeth 1170: whiche diuided by your diuiſor 60: yéeldeth 19 ℔ — 10 ß as appeareth by the worke therof.

Thus hauing now touched the 12 queſtions wherof I firſt pretended, which with diligence and oft practiſe I truſt are ſufficient to aide the deſirous, unto the working of any broken numbers, therefore I will propone a queſtion or two more, and ſo ende this treatiſe of the Rule of thrée in broken numbers.

Example.

$2\frac{1}{2}$ —— $1\frac{2}{3}$ —— $29\frac{1}{4}$

5   5   117 $\cancel{5}$(3

12   5 $\cancel{1}\cancel{1}\cancel{7}$0

60   585 $\cancel{6}\cancel{6}$0(19½ ℔

2

1170

If $\frac{2}{3}$ of an ell coſt 3 ß 4 d. what one ell.

$\frac{2}{3}$ —— $3\frac{1}{3}$ —— 1

2   10   30(5 maketh 5 ß

6   3   6

30

Proofe.

If one Ell coſt 5 ß what $\frac{2}{3}$

$$\begin{array}{rr|l} \frac{1}{3}\ \cdot\ \cdot\ 5\ \cdot\ \cdot\ \frac{2}{3} & \cancel{1} & \\ \hline \frac{2}{3} & \cancel{1}\cancel{0} & 3\frac{1}{3} \\ \hline 10 & \cancel{3} & \end{array}$$

maketh 3 ß—4 d

If one péece of Kerſy coſt 2 ℔—5 ß, and one yeard thereof ſold for 2 ß—4 d, how many yeards long was the whole péece.

$$\begin{array}{lr|l} 2\frac{1}{3}\ \cdot\ 1\ \cdot\ 2—5 & \cancel{6}\cancel{2} & \\ 20 & \cancel{1}\cancel{3}\cancel{5} & 19\frac{2}{7} \\ \hline 7 \qquad 45 & \cancel{7}\cancel{7} & \\ 3 & & \\ \hline 135 & & \end{array}$$

maketh 19 $\frac{2}{7}$ yeards

A Grocer hathe bought a bagge of Almonds weyghing 385 ℔: tare 4 $\frac{1}{4}$ ℔ at 27 ß—6 d the C: The queſtion is what they amoũt to in money.

$$\begin{array}{r} 385 \\ 4\frac{1}{4} \\ \hline 380\frac{3}{4} \end{array}$$

```
100—27 5/7 · ·   380 4/7 ·
  8      55      5523          5
800                55     83765|10|4
                 7615     88800| 5—4 ß
      ·          7615  · 565
                83765     12       3
                        1130   6780|8 19/40
                         565    800|
                        0780
```

maketh 5 ℔—4 ß—8 19/40 d

The fourth Chapiter treateth of Loſſe and Gaine, in the trade of Merchandiſe.

IF one yard coſt 6 ß—8 d: and the ſame is ſold againe for 8 ß—6 d: the Queſtion is, what is gained in 100 ℔ laying out on ſuch commoditie.

Anſwere, the Rule of thrée direct, applieth two manner of wayes to doe the ſame: the one is to ſay: If $6\frac{2}{3}$ giue $8\frac{1}{2}$ what giueth 100? Multiply and diuide, and looke what your quotient bringeth forth aboue your laying out, is the neat gaines, and the ſolution to your Queſtion: If you follow the worke, your Quotient wil bring forth 127 ℔—10 ß.

Item, to worke it the other way, which I take the neareſt, ſeeke the difference betwixt the iuſt price, and the ouer-price, whiche is 1 ß — 10 d: Then ſay by the Rule of three: If 6 $\frac{2}{7}$ ß gaine 1 $\frac{5}{6}$ ß what ſhall 100 ℔ gaine: Multiply and diuide, and you ſhall finde — 27 ℔ · 10 ß: and ſo much is gained in 100 ℔ laying out.

Uſe which of theſe two wayes you thinke good.

The Proofe.

If a yeard of cloth be deliuered for 8 ß — 6 d, whereupon was gained after the rate of 27 ℔ — 10 ß in 100 ℔ laying out: The Queſtion is what the yard coſt at the firſt hand?

Anſwer. Put your gains to 100 ℔, al maketh 127 ℔ — 10 ß: then ſay, if 127 ℔ — 10 ß giue but 100 ℔, what giueth 8 $\frac{1}{2}$ ß: worke, and you ſhall finde 6 ß — 8 d, the true ſolution to your queſtion.

Yet an other Branch or proofe vpon the ſame firſt Queſtion.

If one yeard coſt 6 ß — 8 d: the queſtion is, what price the ſame is to be ſolde againe for, to gaine 27 ℔ — 10 ß in 100 ℔ laying out.

Anſwere, ſay by the Rule of three, if 100 ℔ giue 127 ℔ — 10 ß : what giueth 6 $\frac{1}{2}$ ß: multiply and diuide, and you ſhal finde 8 ß — 6 d, your true ſolution.

If one Ell coſt 7 ß — 8 d, and ſolde againe for 8 ß — 6 d Queſtion : what is gained in 20 ℔ laying out, in ſuch commodities.

Anſwere, Seeke the difference betwixte the iuſt price & the ouer-price, which is 10 d, and then apply the Rule of three, as before is taught : ſaying, if 8 $\frac{1}{2}$ giue $\frac{5}{6}$ ß, what giueth 20 ℔ : multiplie and diuide, and you ſhal find 2 ℔ — 3 $\frac{1}{2}\frac{1}{3}$ ß: And ſo much is gained in 20 ℔ laying out.

The Proofe.

A Merchant hath bought Holland cloth at 8 ß — 6 d the Ell, whiche proueth not to his expectation : whereuppon he is content to loſe 2 ℔ — 3 ß — $\frac{1}{2}\frac{1}{3}$ ß in 20 ℔ laying out: The queſtion is what price ought to be made of the Ell abating this loſſe.

Anſwere, doe as before in gaines hathe bin taught: putting 2 ℔ — 3 $\frac{1}{2}\frac{1}{5}$ ß to your 20 ℔, all togither maketh 22 — 3 $\frac{1}{2}\frac{1}{5}$ ß: Then ſay by the Rule of thrée: If 22 — 3 $\frac{1}{2}\frac{1}{5}$ ß giue but 20 ℔, what ſhall come of 8 $\frac{1}{2}$ ß: worke and you ſhall finde 7 ß — 6 d, the iuſt price that the Ell ought to be ſold for after the rate of this loſſe.

Thus you ſée as in companye the Rule is appliable as wel to gaines and loſſe.

If 20 $\frac{1}{4}$ yeards coſt 36 ℔ — 10 ß howe ſhall I ſel the ſame againe to gaine $\frac{1}{3}$ of the principall: or to make of 3. 4: whyche is all one.

Anſwere, by the Rule of thrée: if 3 doe do giue 4: what will 36 $\frac{1}{2}$ giue? Multiply & diuide, and you ſhall finde 48 ℔ — $\frac{2}{3}$: Then ſay againe, if 20 $\frac{1}{4}$ yeards do giue 48 $\frac{2}{3}$ poũds, as well principall as gaine: what will one yeard be worth at that price? Multiplie and diuide, and you ſhal finde 2 ℔ — 8 $\frac{2}{4}\frac{1}{33}$ ß.

If one Ell of Cloth coſt me 8 ß — 8 d: And afterwardes I ſell, 10 $\frac{1}{2}$ Elles thereof for 5 ℔ — 12 ß — 4 d: I would know whether I winne or loſe: and how much vpon the 100 ℔ of money.

Anſwere. See firſte, at 8 ß — 8 d the Ell, what $10\frac{1}{2}$ elles commes to, and you ſhall finde 4 ℔ — 11 ß: and I ſolde the ſame for 5 ℔ — 13 ß — 4 d: So that I did gaine vpon the $10\frac{1}{2}$ yeardes 1 — 2 — 4 d: Then if you would know how much is gained in the 100 ℔, ſay by the rule of three: If 4 ℔ — 11 ß did gaine $22\frac{1}{3}$ ß what will 100 ℔ gaine: Multiply and diuide and you ſhall finde — 24 ℔ — 10 ß — $10\frac{12}{13}$ d: And ſo much is gained in the 100 ℔ of money.

If $12\frac{1}{2}$ yeards coſt me 11 ℔ — 5 ß: And I ſel the yeard againe for 16 ß: the queſtion is, whether I do win or loſe, and howe much in or vpon the pound of money.

Anſweare. Looke what the $12\frac{1}{2}$ yeardes come to at 16 ß the yeard, and you ſhall find 10 ℔: But they coſt 11 ℔ — 5 ß: So there is loſte vpon the whole 1 ℔ — 5 ß: Then to know how much is loſt in the pound: Saye by the rule of 3: if $11\frac{1}{4}$ ℔ do loſe $1\frac{1}{4}$ ℔, what will 1 ℔ loſe: Multiplie and diuide, and you ſhall finde $6\frac{2}{5}$ d: and ſo much is loſt in the ℔ of money.

If I ſel the C. waight of any cōmoditie, for 4 ℔: whereupon I do loſe after 10 ℔ in the 100 ℔: I demaund howe muche I ſhall loſe or gaine in the 100 ℔, if in caſe I had ſolde the ſame for 4 ℔ — 10 ß.

Anſwer. Say, if 90 ℔ yéeld 100, how much wil 4 give? Multiplie and diuide, & you ſhall finde $4\frac{4}{9}$: Then ſay againe, if $4\frac{4}{9}$ giue me $4\frac{1}{2}$ what wil 100 come to? Multiplie & diuide, and you ſhal find 101 ℔ $\frac{1}{4}$ which is more thā 100 ℔ by 1 ℔ - 5 ß: And ſo muche is gained in the 100 ℔.

A Merchant hath ſolde Currans for the ſum of 430 ℔: & he hath gained therein after 10 ℔ in the 100 ℔: The queſtion is, to know how much he gained in all?

Anſwere, Say by the Rule of thrée: If 110 ℔ do gaine 10 ℔, what will 430 ℔ gain: Multiplie & diuide, and you ſhall find 39 ℔ - 1 - 9 $\frac{9}{11}$ d, And ſo much hath he gained in al.

If one yeard be worth 28 $\frac{1}{2}$ ß, for howe much ſhall 10 yeards be ſolde to gaine after 8 ℔ - 6 ß — 8 d in the 100 ℔.

Anſwere. Firſt adde 8 ℔ — 6 ß — 8 d to 100: thē ſay, if 100 ℔ do giue 108 $\frac{1}{3}$ for principal and gaine, what will 28 $\frac{1}{2}$ ß principall yéelde? Multiplie and diuide, and you ſhall finde 30 $\frac{7}{8}$ ß: Then ſaye againe by the Rule of thrée: If 1 yearde doe giue 30 $\frac{7}{8}$ ß (whyche is as well the principall as the gaine) what ſhall 10 yeardes giue? Multiplie and diuide, and you ſhal find 15 ℔ —8 ß —9 d. : And for the ſame price ſhall the 10 yeards be ſolde, for to gaine after the rate of 8 ℔ —6 ß —8 d. vpon the 100 ℔.

A Branch or Proofe out of this Queſtion.

A Merchant hathe ſolde Clothes for 15 ℔ —8 ß —9 d. , and he hath gained in the whole, the ſum of 1 ℔ —3 ß —9 d. : The queſtion is to know howe muche hee hath gained in the 100 ℔.

Anſwere, to knowe this, firſte rebate the gaines from the price, and there will remaine 14 ℔ —5 ß —0 d. : Then ſaye by the Rule of three direct, if 14 ℔ ¼ giue me —— 1 ℔ — 3 ß — ¾ d.: what will 100 ℔ giue? Multiplie and diuide, and you ſhall finde — 8 ℔ —6 ß —8 d. the effect deſired: the proofe is apparant in the queſtion before.

Yet an other Branch or Proofe of the firſt Queſtion.

If —10 yeardes be deliuered for —

15 ℔

15 ℔ 8—9 whereupon was gained after yͤ rate of 8 ℔—6 ß — 8 d upon the 100 ℔: the queſtion is, what the yeard did coſte at the firſt hande.

Anſwere, firſt ſay by the Rule of thrée, if 10 with principall and gaine yéelde 15 ℔ - 8 $\frac{3}{4}$, what ſhal 1 yéelde? Multiplie and diuide, and you ſhall finde 30 $\frac{7}{8}$ ß: Then ſaye againe by the Rule of thrée: If 108 $\frac{1}{3}$ principall and gaine giue but 100: what ſhall 30 $\frac{7}{8}$ ß of principall and gaine yéelde: work, and you ſhall finde 28 $\frac{1}{2}$ ß: And ſo muche did the yeard coſt at the firſte penny.

If one yeard coſt 36 ß how much ſhall 12 yeardes be ſolde for to gaine after the rate of 10 ℔ in the 100 ℔.

Anſwere, Firſte ſay, if 100 giue 110 ℔ principall and gaine, what will 26 ß giue? Multiply and diuide, and you ſhal find 39 $\frac{3}{5}$ ß Then ſaye againe by the rule of thrée: If 1 yeard of principall and gaine yéelde 39 $\frac{3}{5}$ ß, what ſhall 12 yeards gaine? Multiplie, and diuide, and you ſhall finde 23 ℔—15 $\frac{1}{5}$ ß, which $\frac{1}{5}$ ß is 2 $\frac{2}{5}$ d: And for the ſame price ſhall the 12 yeards be ſold, to gaine after the rate of 10 in the 100.

The Proofe.

If 12 yeards be ſolde for 23 ℔ - 15 ß - 2 $\frac{2}{5}$ d whereupon is gained after 10 ℔ in the 100 ℔: The queſtion is, what the yeard coſt at the firſt peny?

Anſwere. Firſt ſay, if 12 giue 23 ℔ — 15 $\frac{1}{5}$ ß, what 1 yeard? Multiplie and diuide, and you ſhall finde 39 $\frac{3}{5}$ ß: Then ſaye againe by the Rule of thrée: if 110 ℔ giue but 100, what ſhall 39 $\frac{3}{5}$ giue: worke and you ſhall finde 36 ß, the iuſte price of the yeard at the firſte hand.

Itẽ, when one Merchant ſelleth wares to another, and he giueth to the buyer 1 ℔ — 6 ß — 8 d vpon the ſcore, or 20 ℔: The queſtion is, howe muche ſhall the buyer gaine vppon the 100 ℔ after that rate.

Anſwere. Firſte adde 1 ℔ — 6 ß — 8 d vnto 20 ℔, and they are 21 $\frac{1}{3}$: Then ſay, if 20 ℔ giue 21 $\frac{1}{3}$, what ſhall 100 giue? multiplie and diuide, and you ſhall finde 106 $\frac{2}{3}$: So the buyer getteth after the rate of $6\frac{2}{3}$ lb vpon the 100 lb.

Gentle Reader other neceſſarie queſtions appertaining to Loſſe and Gaine, you ſhall haue in the ~~ſeauenth~~ 8 Chapiter of this Treatiſe.

The fifth Chapiter entreateth of Loſſe and Gaine vppon Time, wrought by the double Rule of three: or by the Rule of three compoſed, whiche is contained in foure ſpeciall ſelected braunches or queſtions of diuers formes, eche one of them ſpringing from the firſte queſtion, and eache one of them alſo beeing a proofe to other, &c.

If one yeard coſt me 2 ß 8 d readie mony: and after I ſell the ſame againe for 2 ß 10 d to be paide for it at the ende of 3 moneths: the queſtion is, what I gaine vpon the 100 lb in 12 moneths?

Anſwere. Firſt ſaye, if $2\frac{2}{3}$ gaine $\frac{1}{6}$ what ſhal 100 lb gaine: Multiplie, and diuide, and you ſhal finde — $6\frac{1}{4}$ lb: Then ſaye againe by the Rule of three: if 3 moneths gaine 6 $\frac{1}{4}$ lb what ſhall 12 moneths gaine: Worke, and you ſhall finde 25 lb: and ſo muche ſhall I gaine in 12 moneths after that rate.

Itẽ you may alſo work it at one working by yͤ firſt part of the Rule of 3 compoſed: ſaying, if 2 $\frac{2}{3}$ ß in 3 moneths do gaine $\frac{1}{6}$ of a ß, which is 2 d, what wil 100 lb gaine in 12 moneths: whiche for thy further encouragement, the worke of this one Example I wil here putte downe, to verifie that I affyrmed when I delivered in the firſte parte of this Ground of Artes, that this Rule, and ſo all others more reioiceth in Broken than in Whole.

$$\begin{array}{lcr}
\underline{2\frac{2}{3}} \cdot 3 \cdot \frac{1}{6} \cdot 100 \cdot 12 & & \\
8 & & \underline{100} \\
3 & \cancel{1} & 1200 \\
24 & \cancel{7} & \underline{3} \\
\underline{6} & \cancel{1}\cancel{0}\cancel{1} & 3600 \\
144 & \cancel{3}\cancel{6}\cancel{0}\cancel{0} \,\Big|\, 25 & \\
 & \cancel{1}\cancel{4}\cancel{4}\cancel{4} & \\
 & \cancel{1}\cancel{4} &
\end{array}$$

maketh 25 lb your deſire

If a yeard be deliuered for 2 ß—10 d: whereupon was gained after the rate of 25 ℔ in the 100 for 12 moneths: The queſtion is nowe, what the yeard coſte at the firſt hande.

Anſwere. firſte ſaye, if 12 moneths gaine 25 ℔, what ſhal 3 moneths gaine: and you ſhall finde $6\frac{1}{4}$ ℔: Then ſay againe the ſecōd time, if $106\frac{1}{4}$ ℔ giue but 100, what ſhall $2\frac{5}{6}$ ß giue? worke, and you ſhall finde $2\frac{2}{3}$ ß, which is the iuſt price that the yeard coſte at the firſte hand.

If one yeard of cloth coſt me 2 ß—8 d ready mony, for what terme ſhal I ſel the ſame again for 2 ß 10 d: So that I might gaine after the rate of 25 ℔ vpō the 100 ℔ in 12 moneths.

Anſwere. Firſt ſay, if $2\frac{2}{3}$ gaine $\frac{1}{6}$ what ſhall 100 ℔ gaine: Multiply and diuide, and you ſhall finde $6\frac{1}{4}$ ℔. Then ſaye againe for the ſecond worke, if 25 ℔ become of 12 moneths, what ſhal come of $6\frac{1}{4}$ ℔: Work, and you ſhall finde 3 moneths: the iuſte terme of time that the cloth ought to bée deliuered at

2 ß

2 ß 10 d, to gaine 25 ℔, vppon the 100 ℔ in 12 moneths.

If one yeard coſte me 2 ß 8 d readye mony for what price ſhal I ſel the ſame againe to be paid at the end of 3 moneths: So that I may gaine after the rate of 25 ℔ in the 100 ℔ for 12 moneths?

Anſwere. Firſte ſaye, if 12 gaine 25 ℔ what ſhal 3 moneths gaine? Multiplie, and diuide, and you ſhal finde $6\frac{1}{4}$ ℔: then ſaye for the ſecond worke: if 100 ℔ gaine $6\frac{1}{4}$ ℔, what ſhall $2\frac{2}{3}$ ß gaine? worke, and you ſhall find 2 d, whiche muſte be added to 2 ß—8 d: and then it maketh 2 ß—10 d: and for that price muſte the yeard be ſolde to gaine after the rate of 25 in the 100 for 12 moneths, you may alſo worke it by the firſt part of the rule of three compoſed, ſaying if 100 ℔ in 12 monethes doe gaine me 25 ℔? what ſhall $2\frac{2}{3}$ ß gaine in 3 moneths: work and you ſhall find $\frac{600}{72000}$ of a ℔, which in leſſer termes is $\frac{1}{120}$, and is in knowen mony worth 2 d, as before.

Many other of theſe queſtions I might heere haue deliuered, but for feare the booke woulde riſe to too thicke a volume, and ſo to make the price ſo much the dearer, whereby it mighte not bee ſo partable to my Countrymen as I wiſh it. But theſe foure, I haue of purpoſe framed in this order, hauing relation one to an other: Aſſuring you, that what queſtions ſoeuer maye bee proponed within the compaſſe of this rule, you ſhall finde by one of theſe foure to make a ſolution: And moreouer, diuers other are yet to be deliuered: where the Creditour giueth diuers dayes of payment, which can neuer bee well wrought, nor yet vnderſtoode, vnleſſe you can firſte finde by Arte, the iuſt time that al thoſe payments, how different ſoeuer they be, ought to be paide at once: whereupon firſt I thinke good here to giue ſome inſtructions into ſuch a Rule, for it is the onelie aide for the finiſhing of ſuch queſtions as hereafter ſhall follow.

The ſixth Chapiter entreateth of Rules of Payment, which is a right neceſſary Rule, & one of the chiefeſt handmaids that attendeth vpon buying and ſelling, &c.

Example.

A Merchant doth owe a ſumme of money, whereof the $\frac{1}{3}$ is to be paid at 6 moneths, the $\frac{1}{2}$ at 8 moneths, and the reſt at a yeare, If he woulde paye at one payment, the queſtion is, what time ought to be gyuen hym.

Anſwere I haue omitted the quantitie of the ſumme, for you ſhal vnderſtande, the Rule is appliable, and yéeldeth a true ſolution to what ſum ſoeuer ſhal be proponed. But now for orders ſake in teaching, I doe imagine the ſumme to be 60 ℔: wherevppon the maner of this worke is to multiplie the proportionate parte of the mony by the time, as in company: Then 20 being the firſte payment, and the $\frac{1}{3}$ of 60, whiche $\frac{1}{3}$ multiplied in broken numbers by 6: his time of payment, maketh $\frac{6}{3}$ whiche in whole numbers, as appeareth by the example in the margēt, maketh 2 monethes: next 30, whiche is the $\frac{1}{2}$ multiplyed by his tearme 8, yéeldeth 4 monthes, then ỹ reſt which is 10 ℔ muſt néedes be abbreuiated into the proportion it beareth to 60, which is $\frac{1}{6}$: which $\frac{1}{6}$ multiplied by his time 12 monthes produceth $\frac{12}{6}$: maketh 2 monthes. All whiche added togither as appeareth in the margente. maketh 8 monthes: whiche is the iuſt time that all thoſe payments ought to be paide at once.

$\frac{1}{3}$ by $\frac{6}{1}$	
$\frac{6}{3}$	2
$\frac{1}{2}$ by $\frac{8}{1}$	
$\frac{8}{2}$	4
$\frac{1}{6}$ by $\frac{12}{1}$	
$\frac{12}{6}$	2
	8

A Merchāt hath 800 ℔ to pay, the $\frac{1}{6}$ therof readie money the $\frac{1}{3}$ at two monethes, the $\frac{1}{2}$ at foure monthes, and the reſt at a yeare. The queſtion is, if he woulde paye all at one payment, what time ought to be giuen him.

Anſwer. The readie money is neuer multiplyed : then $\frac{1}{4}$ multiplyed by 2 monthes as you did before, maketh $\frac{1}{2}$ then $\frac{1}{2}$ by 4 produceth 2 monthes, as appeareth here in the margent: But nowe for the reſt of the money you can not multiplye it vntill you haue ſought what proportion it beareth to 800 lb. Therefore you muſt ſubtract the readie money: the $\frac{1}{4}$ and the $\frac{1}{2}$ out of the principall: The reſt wil be 66 $\frac{2}{3}$ lb: which you muſt looke what parte it beareth to the principal, which you ſhal finde to be $\frac{1}{12}$: The ſame you muſt alſo multiply by his time 12 monthes. And it yeeldeth 1 month : So all make 3 $\frac{1}{2}$ monthes as appeareth in the margent.

$\frac{1}{4}$ $\frac{2}{1}$
$\frac{2}{4}$ | $\frac{1}{2}$
$\frac{1}{2}$ $\frac{4}{1}$
$\frac{4}{2}$ | 2
$\frac{1}{12}$ $\frac{12}{1}$
$\frac{12}{12}$ | 1

3 $\frac{1}{2}$

A Merchant is to paye 1200 lb : in 3 termes, That is to wit, 400 lb. at 2. weekes And 600 lb at 4 monthes, laſtly 200 lb: at 5 monthes. The queſtion is in what time, they ought to be paide at once.

Anſwer. Proportionate the partes, and you ſhall hal finde for $400\frac{1}{3}$ and for $600\frac{1}{2}$. And 200 s the $\frac{1}{6}$ part: which multiplied by their times s before, and you ſhal haue $\frac{2}{3}$ weekes: more weekes, and laſtly $3\frac{1}{2}$ weekes, which togiher maketh 12 weekes, or 3 monthes youre eſire.

A Merchaunt is to paye 600 ℔ in 6 termes, whereof 100 ℔ is paid preſent, more 300 ℔ at 20 dayes: And the reſt at 5 monthes, accompting 30 dayes to a month. The queſtion is, what time oughte theſe payments to be payde at once.

I anſwer two monthes.

The ſeuenth Chapter entreateth of buying and ſelling in the trade of Marchandiſe, wherin is taken part readie money, and diuerſe dayes of payments giuen for the reſt: & what is won or loſt in the 100 ℔. forbearance for 12 monthes more or leſſe according to the quantitie of money, or proportion of time &c.

A Merchant hath bought Sattins whiche coſte 8 ß the yearde readie money: And he ſelleth the ſame again to another mā for 10 ß the yearde: But he giueth two dayes for the payment: That is to ſay thrée monthes for the one half, and fiue monthes for the other half: The queſtiō is to know how much the ſeller doth gain vpon 100 ℔ in 12 monthes after that rate.

Aunſwer. Séeke firſt by the Rules of payment, at what time thoſe two paymentes ought to be paide at once, and you ſhall finde 4 monthes, at whiche time the ſeconde Merchant oughte to haue paide the whole entire payment: And therefore ſay by the firſt parte of the rule of thrée cōpoſed: If 8 ß in 4 monthes doe gaine two ſhillings, what wil 100 pounde gaine in twelue monthes multiply and diuide and you ſhall finde 75 ℔ as appeareth in the example, and ſo much doth the firſte Merchaunte gaine vppon 100 ℔ in 12 monthes.

```
8—4—2—100—12
 4        12
---     -----
32       1200     2400|75
            2       32
        -----
         2400
```

A Merchaunt hath ſold 50 clothes at 9½ ℔ the péece, to be paide the one ½ at 4 monthes:

the $\frac{1}{3}$ at 5 monthes, & the $\frac{1}{6}$ at 7 monthes: And yͤ ſellers minde is to take no more but after 8 ℔ in the 100 for 12 monthes. The queſtion is now what the firſt Merchaunt gaineth in the ſale of theſe clothes after that rate.

Anſwer. Firſt looke what the 50 clothes come to at that price: and you ſhall finde 475 ℔. Then ſecondly, according to your direction in the Rules of paymente, ſeeke at what time all the payments are to be performed at once. And you ſhal finde 4 $\frac{5}{6}$ monthes. Then thirdly ſay by the firſte part of the Rule of 3 composed: If 100 ℔ in 12 monthes gaine 8 ℔ what wil 475 ℔ gaine in 4 $\frac{1}{6}$ monthes: work and you ſhal finde 13 ℔ and $\frac{7}{36}$ of a ℔: whiche is the neate gaines that the firſte Merchaunt hath after the rate aforeſaid:

A Merchant hath bought Holland at 7 ß 3 đ the ell readie money: And he ſelleth the ſame againe, for 8 ß 4 đ the ell, to be paide $\frac{1}{4}$ part in readie mony, more $\frac{1}{3}$ parte at three monthes, and the reſte at foure monthes. The queſtion is nowe to knowe

how

hoѡ muche the firſt Merchant doth gai vppon the 100 ℔ in 12 monthes after t ſame rate.

Anſwer. According to the direction de uered you in the rule of payment, the read money is not to be multiplyed: Then wo king for the other 2 paymentes, to finde ou the true proportion at what time they oug to be paide at once, you ſhall finde for $\frac{1}{3}$ at monthes, $\frac{2}{3}$ of a mōth: And the reſt of the mo ney which is $\frac{5}{12}$ multiplyed by his tearme monthes, yeeldeth 1 $\frac{2}{3}$ monthes: both which added togither make 2 and $\frac{1}{3}$ monthes: Th iuſt time, that both the payments ought to b performed at once. And therefore ſaye by th firſt part of the Rule of three compoſed, if 7 in 2 $\frac{1}{3}$ monthes do gaine $\frac{13}{240}$ of a ℔: what ſha 100 ℔ gaine in 12 monthes after that rate, worke and you ſhall finde 76 $\frac{172}{201}$ poundes. And ſo much doth he gaine vpon 10 pounde in 12 monthes.

A Merchant hath bought 30 clothes at 6 ℔ the peece for readie money: Afterwarde he ſelleth 10 of them for 7 ℔ the peece, for three monthes tearme. And the other 20 he

ſel

ſelleth for 8 ℔ the péece for foure monethes terme: The queſtion is nowe, what hée gaineth vpon 100 ℔ in 12 monethes?

Anſwere. Firſte finde the value of the 30 Cloths, which amount to 180 ℔: Secondarily, ſéeke what the ten péeces come to, at 7 ℔: and what the 20 péeces come to at 8 ℔ the one comes to 70, and the other to 160: both which togither make 230, which is 50 ℔ more than they coſte: Thirdly, as I haue taught you in the Rule of payment, proportionate the firſt ij. prices, vnto the proportion they beare vnto 230: the producte of their two prices, and you ſhall finde $\frac{7}{23}$ for the firſte, and $\frac{16}{23}$ for the latter. Then fourthly, multiply thoſe partes, by their times: and you ſhall haue $\frac{21}{23}$ and $\frac{64}{23}$: both which togither maketh 3 whole moneths, and $\frac{16}{23}$ of a moneth, which is the iuſte time that both theſe payments are to be paid at once.

Then ſay by the firſt part of the Rule of 3 cõpoſed: If 180 ℔ in 3 $\frac{16}{23}$ monthes do gain 50 ℔, what ſhal 100 gaine in 12 monthes?

Wal-multiply and diuide and you ſhal finde 90 $\frac{12}{37}$ ℔. And ſo muche doth he gaine vpon a 100 ℔ in 12 monthes.

A Merchāt hath bought Synamon which coſt him 9 ß the ℔ readie mony: The queſtion is now at what price he ought to ſell the C waight: To wit 112 ℔: to be paide the $\frac{1}{4}$ at 2 monthes, and the reſidue at the end of 3 monthes, ſo that he may gaine after the rate of 9 ℔ vppon a 100 ℔ for 12 monthes.

Anſwer. Seeke firſte by the Rules of payment what tearme both the payments ought to be paid at once, where the $\frac{1}{4}$ multiplied by his terme 2 monthes maketh $\frac{1}{2}$ monthes: Likewiſe the next paimēt which is $\frac{3}{4}$ multiplyed by his tearme 3 monthes maketh $2\frac{1}{4}$ monthes: both which added togither maketh $2\frac{3}{4}$ monthes: which is the time, that both the payments ought to be paide at once. Thē ſay by the Rule of 3: if 12 monthes doe giue me 10 ℔ what wil $2\frac{3}{4}$ monthes giue? Multiplie and diuide and you ſhal finde $2\frac{5}{8}$ ℔. Thē ſaye againe by the rule of 3: If one pound coſt me 9 ß what wil 112 pound coſt: multiplye and diuide and you ſhal finde 50 ℔ 8 ß. Then ſay once againe if 100 ℔ do giue $102\frac{5}{8}$ what wil $50\frac{2}{5}$ ℔ giue? multiplie and diuide & you ſhall finde 20 ℔–18–$8\frac{11}{25}$ d : And for that price ought I to ſell 112 ℔ of Synamon to be paid at two ſeueral payments aforeſaid: To gain therby after the rate of 10 ℔ vpon the 100 ℔ in 12 monthes.

Briefe Rules for our hundreth waight here at London which is after 112 ℔ for the C.

Item who that multiplyeth the pence that one pound waight is worth by 7 : & diuideth the product by 15: ſhal finde how many poũds in money the 112 pounde waighte is worth.

And contrariwiſe, he that multiplieth the poundes that 112 ℔ waight is worth by 15. And diuideth the product by 7 : ſhall finde howe manye pence the pounde waighte is worth.

Example.

At 10 d the pound waight, what is 112 ℔ waight worth?

Anſwer. Multiplye 10 by 7 and thereof commeth 70: the which diuide by 15: and you ſhal finde $4\frac{2}{3}$ ℔. And thus the 112 ℔ is worth 4 ℔ – 13 – 4 d after the rate of 10 d the ℔ aforeſayde.

At 6 ℔ the 112 ℔ waight what is one ℔ worth?

Anſwer. Multiplye 6 ℔ by 15 and thereof commeth 90: the which diuide by 7: And you ſhal finde 12 d $\frac{6}{7}$: So muche is one pounde worth when the 112 ℔ did coſt 6 poundes.

The eight Chapter entreateth of tares and allowances of merchandiſe ſolde by waight, and of loſſes and gaines therin &c.

AT 16 ℔ the 100 ſuttle what ſhall 895 ℔ ſuttle be worth in giuing 4 ℔ waight vpon euerie 100 for treate?

Anſwer. Adde 4 vnto 100, and you ſhall haue 104. Then ſaye by the Rule of three if 104 be worth 16 ℔ what are 895 ℔ worthe? Multiply and diuide, and you ſhal finde 237 ℔ - 13 — 10 $\frac{2}{13}$ d. And ſo much ſhal the 895 ℔ waight be worth.

Item at 3 ß 4 d the pound waight what ſhal 754 $\frac{1}{2}$ be worth in giuing 4 ℔ waight vpon euerie 100 for treate?

Anſwer. Sée firſt by the Rule of three, what the 100 pound is worth, ſaying, if 1 coſt 3 $\frac{1}{3}$ ß what 100? multiplie and diuide and you ſhal finde 16 ℔ $\frac{2}{3}$: Then adde 4 vnto 100 and they are 104: Then ſay again by the rule of three, if 104 be ſolde for 16 $\frac{2}{3}$: for howe muche ſhall 754 $\frac{1}{2}$ be ſolde for? Multiplye and diuide and you ſhal finde 120 ℔ — 18 — 3 $\frac{18}{53}$ d: And for ſo much ſhal the 754 $\frac{1}{2}$ be ſold for, at 3 ß 4 d the pound in giuing 4 vpon the 100.

Item if 100 ℔ be worth 36 ß 8 d what ſhal 860 ℔ be worth in rebating 4 pounde vpon euerie 100: for tare and cloffe?

Anſwer Multiplye 860 by 4 : and thereof commeth 3440:the which diuide by 100 and you ſhal haue 34 $\frac{2}{5}$ ℔ : abate 34 $\frac{2}{5}$ from 860: and there wil remaine 825 $\frac{3}{5}$: Then ſaye by the rule of thrée. If 100 ℔ coſt 36 $\frac{2}{3}$ ß what wil 825 $\frac{3}{5}$ coſt after that rate? Multiplie and diuide:and you ſhal finde 13—15 $\frac{1}{5}$ ß And ſo muche ſhall the 800 coſt in rebating 4 pound vpon euerie 100:for tare and cloſſe.

Item whether doeth he loſe more that giueth 4 ℔ vpon the 100 : or he that rebateth 4 ℔ vpon the 100?

Anſwer. Firſt note that he yͤ giueth 4 ℔ vpō 100, giueth 104 for 100:And he which rebateth 4 ℔ vpon the 100 giueth the 100 for 96: Therefore ſay by the Rule of thrée, if 104 be deliuered for 100 : for how much ſhall the 100 be deliuered? multiplye and diuide and you ſhal finde 96 $\frac{2}{13}$: and he which rebateth 4 in yͤ 100, maketh but 96 of 100, ſo yͭ he looſeth 4 in yͤ 100, And yͤ other which giueth 4 vpon the 100, looſeth but 3 $\frac{11}{13}$ vpon the 100: Thus you may ſée that he which rebateth 4 in the 100 : looſeth more by $\frac{11}{13}$ in the 100 ℔. than the other whiche gaue 4 vpon the 100 : for tare and cloſſe.

If 100 ℔ of any thing coſte me 23 ß 4 d: The queſtion is howe I ſhall ſel the ℔ to gaine after the rate of 10 ℔ vpon the 100 pound.

Anſwer Saie by yͤ rule of 3, if 100 ℔ giue 110 ℔, what ſhall 23 $\frac{1}{3}$ ß giue? multiply ⁊ diuide, and you ſhal finde 1 ℔ $\frac{17}{60}$: Then ſaye againe if 100 ℔ be woꝛth 1 $\frac{17}{60}$ ℔: what is one pound woꝛth? multiplye and diuide, and you ſhal finde 3 d $\frac{6}{75}$: And ſo muche is the pounde woꝛthe in gayning 10 poundes vpon the 100 pounde.

Item a Grocer hath bought C waighte of a commoditie foꝛ 6 ℔ 10 ß: The queſtion is now to know how many poũds ther of he ſhal ſel foꝛ 33 ß 4 d: to gaine 20 ſhillings in the C. waight.

Anſwer Adde 20 ß vnto 6 ℔ 10 ß: and they make 7 ℔ 10 ß. Then ſay if 7 $\frac{1}{2}$ yeeld me 112 ℔, what ſhal 1 $\frac{2}{3}$ ℔ yeeld? multiplie and diuide and you ſhal finde 24 ℔ $\frac{8}{9}$. And ſo many pound ought he to ſel to gaine 20 ſhillings in his C. waight.

If one pounde waight coſt 3 ß 4 d. and I ſel the ſame againe for 4 ß what is gained in 100 pound?

Aunſwer. You may ſaye if $3\frac{1}{3}$ ß giue 4 ß what will 100 ℔ giue? But then when you haue found you muſt ſubtract the product out of 100 ℔, the reſt is your neate gaine: Or elſe to produce the neate gaine in your worke at the firſte: Then ſubtract the iuſt price out of the ouer price, as I taught before in the firſte beginning of loſſe and gain: And your concluſion ſhal be al one, multiply & diuide by which of the two wayes you thinke good, and you ſhal find that he gaineth 16 ℔ 13 ß 4 d. in the 100 pound.

Item if the pound waight which coſt 4 ß be ſold again for 3 ß 4 d: I demaund what is loſt in the 100 ℔ of mony?

Anſwer. Say if 4 ß looſe $\frac{2}{3}$ ß what ſhal 100 ℔ looſe? Multiplie and diuide and you ſhal find 25 ℔: and ſo much is loſt uppon the 100 ℔ of money.

Item if C waight of any commoditie coſt 45 ℔: And the buyer repenting would looſe 5 ℔ in the 100 ℔ of money: I demaũd how the pound may be ſolde: his loſſe to be neither more nor leſſe than after the rate aforeſaide of 5 by the hundreth?

Anſwer. By the Rule of thrée, if 100 ℔ loſe 5 ℔ what ſhal 45 ℔ léeſe? Work and you ſhal finde 2 $\frac{1}{4}$ ℔: which rebated from the principal 45: reſteth 42 ℔ 15 ß: Laſtly ſay if 112 ℔ yéeld but 42-15 ß: what 1 pound? multiply & diuide, and you ſhal find 7 ß 7 d. $\frac{17}{28}$. And ſo much is the pound worth after that loſſe.

A Grocer hath bought 3 péeces of reaſõs: waying 175 $\frac{1}{2}$ ℔ 182 $\frac{3}{4}$ ℔ 191 ℔: tare for eache fraile 2 $\frac{1}{4}$ ℔: at 25 $\frac{1}{2}$ ß the C waight. The queſtion is what they amounte to in money?

I anſwere 6 ℔ 8 ß $\frac{9}{40}$.

A Grocer hath bought 3 ſacks of Almonds waying 267 $\frac{1}{2}$ ℔ tare 2 ℔: 257 $\frac{1}{2}$ ℔ tare 2 $\frac{1}{2}$ ℔ 252 ℔ tare 3 ℔ at 2 ß 10 $\frac{1}{2}$ the pounde what amount they to in money?

I anſwere 110 ℔-11-7 $\frac{1}{8}$ d.

The ninth Chapter treateth of lengthes and breadthes of Arras, and other clothes with other queſtions incident vnto length and breadth.

If a péece of Arras be 7 els and $\frac{3}{4}$ long: & 5 els and $\frac{2}{3}$ broad: how many els ſquare doth the ſame péece containe?

Anſwer. Multiply the length by the bredth that is to ſay $7\frac{3}{4}$ by $5\frac{2}{3}$: And therof wil come $43\frac{11}{12}$ elles: ſo manye elles ſquare doeth the ſame péece containe.

Item more a péece of Arras doeth containe 22 elles ſquare, And yͤ ſame being in length $3\frac{1}{4}$ elles I demaunde how manye elles in bredth the ſame péece doth cõtaine?
Anſwer Diuide 22 elles by $3\frac{1}{4}$ and therof commeth $6\frac{10}{13}$: So many elles doth yͤ ſame containe in bredth

Item more a Merchant hath $3\frac{1}{4}$ elles of Arras, at $1\frac{2}{3}$ elles broade which he wyll chaunge with another man for a péece of Arras, that is $\frac{7}{8}$ el ſquare. The queſtion is howe manye elles of that ſquareneſſe ought the firſt Merchant is to haue.

Aun-

Anſwere. Multiplie the firſt Merchantes peece his length by his breadth, and you ſhall find it containeth $5\frac{5}{12}$ elles, which $5\frac{5}{12}$ elles you ſhall diuide by $\frac{7}{8}$ and you ſhall finde $6\frac{4}{21}$ elles: and ſo many elles of that ſquareneſſe, ought the latter Merchant to giue the firſt.

Item, a ſtudent hath bought $3\frac{1}{2}$ yards of broade Cloth, at 7 quarters broade, to make a Gowne: and ſhould line the ſame throughout with Lambe, at a foote ſquare eche ſkin: the queſtion is now how many ſkinnes he ought to haue?

Anſweare. Seeke firſte the number of yards ſquare that his cloth containeth, which to doe, multiplie $3\frac{1}{2}$ his length, by $1\frac{3}{4}$ hys breadth, and you ſhal find $6\frac{1}{8}$ yeards ſquare: then ſay by the rule of three: if 1 yeard ſquare giue 9 foote, what ſhal $6\frac{1}{8}$? worke, and you ſhall finde $55\frac{1}{8}$ ſkinnes.

Item more, a Lawier hath a rich peece of ſeeling come home which is 24 foote, and three inches long: and 7 foote and $2\frac{1}{2}$ inches high: the Ioiner is to be paide by the yard ſquare: the queſtion is, how many yeards

this

this pèece containeth?

Anſwere. Multiplie his length by hys breadth, that is to wit, 24 $\frac{1}{4}$ foote by 7 $\frac{5}{24}$ foote and you ſhall finde 174 $\frac{77}{96}$ foote ſquare, which 174 you ſhall diuide by 9 (for ſo many foote make a yeard ſquare) and you ſhall finde 19 yeardes, 3 foote, and $\frac{77}{96}$ of a foote: and ſo many yeards doth this pèece holde.

Item, bought a pèece of Hollande Cloth containing 36 elles $\frac{1}{3}$ Flemmiſhe: The queſtion is, how many Elles Engliſhe it makes?

Anſwere. You muſte note, that 5 Elles Flemmiſh do make but 3 Elles Engliſhe: Therfore ſay by the Rule of thrèe: if 5 elles Flēmiſh make but 3 elles engliſh, how many Elles Engliſhe will 36 $\frac{1}{3}$ Elles Flēmiſh make? Multiplie and diuide, and you ſhall finde 21 $\frac{4}{5}$: and ſo many Elles Engliſh doth 36 $\frac{1}{3}$ Elles Flemmiſh containe. The like is to be done of all others.

Item more, I haue bought 242 Elles Flemmiſh, of Arras worke at two Elles broade, Flemmiſh, and I woulde line the ſame with Ell broad Canuas of Engliſh meaſure: The queſtion is, howe manye Elles Engliſh will ſerue my turne?

Anſwere. For as much, as 3 elles Engliſh are worth 5 elles Flemmiſh: therfore put 3 elles Engliſh into his ſquare: in multiplying 3 by himſelfe, whiche maketh 9: Likewiſe multiplie the Engliſh ell which is 5 quarters euery way into himſelf ſquarely, and you ſhal finde 25: Then multiplie 342 whiche is the length of the peece, by 2 whiche is the breadth, and thereof commeth 684: then ſay by the rule of 3, as before: if 25 elles ſquare of Flemmiſhe meaſure be worth 9 elles ſquare of Engliſhe meaſure, what are 684 of Flemmiſh meaſure? multiply, and diuide, and you ſhal find 246 $\frac{6}{25}$ elles Engliſh.

The ſame is alſo wrought by the backer Rule of 3, in ſeeking the ſquares contained in the Flemmiſh ell of 2 elles broade (which are 18): and alſo in ſeeking the ſquares conteined in the Engliſh ell (which are 25:) then ſay by the Rule of 3 backward: if 18 quarters requirs 34^{2} elles, what ſhal 25 quarters giue? Multiply‸, & diuide by yͤ Rule of 3: Reuerſe, & you ſhal finde as before 246 $\frac{6}{25}$ elles Engliſhe.

Item more, at 3 ß 4 d the Flemmiſhe Ell what is the Engliſhe Ell worth after the rate?

Anſwere. Say, if 5 Elles Flemmiſh be worth 3 Elles Engliſhe, what is one Ell Flemmiſh worth? Multiplie, and diuide, and you ſhall finde $\frac{3}{5}$ of an Engliſhe Ell : Then ſay by the Rule of thrée, if $\frac{3}{5}$ of an Ell bée worth 3 $\frac{1}{3}$ ß, what is one ell Engliſh worth? multiplie and diuide, and you ſhall finde 5 ß 6 $\frac{4}{9}$ d?

Item more, at 8 ß 4 d the Flemmiſh El ſquare, what is the Engliſh El worth after that rate.

Anſwere. Say by the aforeſaide reaſon: if 25 Elles Flemmiſh ſquare be worth 9 els ſquare Engliſhe, what is one Ell ſquare Flẽmiſh worth? work, & you ſhal find $\frac{9}{25}$ of a an Ell ſquare Engliſh : Then ſaye, if $\frac{9}{25}$ of an Engliſhe Ell be worth 8 $\frac{1}{3}$ ß, what is 1 ſquare Ell worth? Multiplie and diuide, & you ſhall finde 23 ß — 1 $\frac{7}{9}$ d : and ſo much is the Engliſhe Ell worth.

Item,

Itē, moꝛe at 6 ß 8 d. yͤ Ell ſquare, what ſhall a peece of cloth coſt that is 7 ½ Elles long, and 3 ¼ Elles bꝛoade?

Anſwere. Multiplie the bꝛeadth by the length, and you ſhall finde 24 ⅜ elles ſquare: Thē ſay by the Rule of thꝛee: if 1 Ell ſquare coſts 6 ⅔ ß what 24 ⅜? Multiplie and diuide and you ſhal finde: 8 ℔ — 2 ß — 6 d: and ſo much the ſame peece of cloth ſhall coſte.

Item moꝛe, a Mercer ſolde 3 peeces of ſilke. To wit, 24 ¼, 23 ⅓, and 25 yeards at 9 ¾ ß the yeard: and was glad to receiue in parte of payment againe, a Cloth containing 24 ½ yeardes at 7 ⅔ ß the yeard: The queſtion is now, what the Debitoꝛ is in the Creditoꝛs debt: Woꝛke, and you ſhall finde, he oweth the Mercer 22 ℔ — 3 ß — 2 ¼ d.

The

The tenth Chapter entreateth of the reducing of the Pawnes of Geanes into Engliſhe yeardes.

NOte, that 100 Pawnes doe make 26 yeardes: wherevpon 3 Pawnes $\frac{11}{13}$ do make 1 yeard, and 1 Pawne after that rate and proportion is $\frac{13}{50}$ of a yeard.

In 4563 Pawns of Geanes, how many yeards Engliſhe?

Anſwere. Say by the Rule of thrée, if 100 Pawnes do make 26 yeards, what will 4563 Pawnes make? Multiplie, and diuide, and you ſhall finde 1186 yeards $\frac{19}{50}$: ſo many yeardes doe 4563 Pawnes make.

Otherwiſe, take ſome other number at your pleaſure, as 10 Pawns, which is ẏ $\frac{1}{10}$ part of 100: then to find his proportiō take ẏ $\frac{1}{10}$ parte of 26, which is 2 $\frac{3}{5}$: and then ſaye alſo by the Rule of thrée: if 10 Pawnes giue 2 $\frac{3}{5}$ yeardes, what wil 4563 Pawnes giue? worke, and you ſhall finde 1186 $\frac{19}{50}$ yeardes as before,

More, at 2 ß 6 d the Pawne of Geanes what will the Engliſhe yeard bée worth after that rate?

Anſwere. Say by the Rule of thrée, if $\frac{13}{50}$ of a yeard coſt $2\frac{1}{2}$ ß what one yeard? Multiplie, and diuide, and you ſhall finde 9 ß — $7\frac{5}{13}$ d.

More, if $346\frac{1}{2}$ Pawnes coſte 30 ℔ — 13 ß — 4 d ſterling, what is that ẏ Engliſhe yearde after the rate?

Anſwere. Saye by the Rule of thrée: if $346\frac{1}{2}$ Pawnes coſt $30\frac{2}{3}$ ℔, what are $3\frac{511}{13}$ Pawnes worth (for ſo many Pawnes make a yeard:) Multiplie, and diuide, and you ſhall finde $\frac{3804}{17017}$ partes of a pound, whiche in knowen numbers is worth 2 ß — 9 d — $\frac{20439}{50177}$.

The

The eleauenth Chapiter entreateth of Rules of Loane and Intereſt, with certaine neceſſarie Queſtions and Proofes incident therunto, &c.

ITem, lent my friend 326 ℔ for $5\frac{1}{2}$ monethes ſimply without any Intereſt, vpon condition, to haue the like curteſie againe when I neede: But when I came to borrowe, he could ſpare me but 149 ℔ — 8 ß — 4 d: the queſtion is now how long time, I ought to haue the vſe thereof, to counteruaile my friendſhip before tyme ſhewed him?

Anſweare. Saye by the Backer Rule of three: if 326 giue $5\frac{1}{2}$ monethes, what time will 149 ℔ $\frac{5}{12}$ giue? Multiplie, and diuide, and you ſhall finde 12 monethes: and ſo long time ought I to vſe his mony.

The Proofe.

Item, lent my friend 149 ℔ — 8 ß — 4 d, for 12 monethes: The queſtion is nowe, how howe muche money he ought to lende me againe for 5 $\frac{1}{2}$ monethes to recompence my friendſhippe ſhewed him?

Anſwere. Say by the backer or reuerſe Rule of 3: if 12 monecths giue 149 $\frac{5}{12}$, what ſhall 5 $\frac{1}{2}$ monecths giue? worke: and you ſhal finde 326 ℔: and ſo much ought he to lende me to requite my gentleneſſe or good turne.

Two other Braunches yet more for Proofe out of the ſame Queſtions.

Item, lent my friend 149 ℔—8 ß—4 d—for 12 monethes, to haue the like friendſhippe againe when I néede: And comming to borrowe of him, he very curteouſly tooke me 326 ℔ (for that he coulde well then ſpare the ſame;) The queſtion is nowe, how long I ought to occupie it, not vſurping frienſhippe, but in his due time to reſtore it againe.

Anſwere. Say by the Rule of thrée Reuerſe, if 149 $\frac{5}{12}$ giue 12 monethes, what ſhall 326 ℔ giue? Multiplie, and diuide, and you ſhall finde, that at 5 ½ monethes terme, I ought to reſtore it againe.

Proofe.

Item lent my friend 326 li for 5 ½ monethes: The queſtion is nowe, how many pounds he ought to lend me for 12 moneths to recompence the pleaſure againe.

Anſwere. Worke by the rule of thrée Reuerſe, as you haue done before, and you ſhall finde 149 li — 8 ß — 4 d.

Againe, foure other ſelected queſtions of Loane and Intereſt, all out of one braunch, and eche one alſo a neceſſarie Queſtion, and a particular proofe to other.

ITem, lent my friend 430 li at Intereſt for 3 moneths to receiue after the rate of 8 li in the 100 for 12 monethes: The queſtion is, what the Intereſt commeth to?

Anſwere. Say by the firſte parte of the Rule of 5 numbers forwarde: if 100 ℔ in 12 monethes gaine 8 ℔, what ſhall 430 gaine in 3 moneths? Multiplie the firſte by the ſeconde for your Diuiſor: and the other three the one into the other for your diuided: and you ſhall finde 8 ℔ — 12 ß.

Proofe.

Item a friend of mine receiued of mee 8 ℔ — 12 ß for the Intereſt and Uſe of 430 ℔ for 3 monethes terme: the queſtion is now, what he tooke in the 100 ℔ for 12 moneths after that rate.

Anſwere. Say by the firſt part, or rule of 5 numbers forward: if 430 ℔ in 3 monethes did pay 8 ℔ — 12 ß, what doth 100 ℔ in 12 moneths, take after that rate? worke, and you ſhall finde 8 ℔: and ſo much he tooke vpon the 100 ℔ for 12 monethes.

A third queſtion and Proofe alſo wrought by the backer rule of 5 Numbers.

Item, lente my friende 430 ℔ to receyue for the Intereſt thereof: after the rate of 8 ℔ in the 100 ℔ for 12 monethes: The queſtion is now, how long time my friend ought to haue the vſe therof, that it may be returned with 8 ℔ 12 ß gaines.

Anſwere. Say by the Backer Rule of 5 numbers: if 100 ℔ in 12 monethes doe gaine 8 ℔: how long time ſhall 430 ℔ be a gaining of 8 ℔—12 ß: Multiplie the firſte and the ſecond into the laſte for your diuidend: and the thirde and fourth multiplie togither for for your diuiſor: And then diuide and you ſhal finde 3 moneths: the iuſte time that my friend ought to vſe it, to retourne it with 8 ℔—12 ß gaine.

A fourth deriued queſtion out of this branch which is a Proofe of this laſt, and alſo of the other two going before.

ITem, how much money ought a Merchant to deliuer after 8 ℔ in the 100 for 12 moneths that in 3 moneths I may gaine 8 ℔—12 ß.

Anſwere. You maye alſo if you pleaſe

worke it by the Golden Rule of thrée at twice, firſte ſaying if 3. moneths gaine 8. ℔. what 12 months gaine? you ſhal finde 34 ℔: Then ſay againe, if 8 ℔— become of 100 ℔ what ſhall come of 34 ℔—8 ß? worke, and you ſhall finde: the anſwere to the queſtion, whiche is 430 ℔: and ſo muche ought the Merchant to deliuer.

But moſte briefly it is aunſwered by the Backer Rule of 5 numbers, where I argue thus, ſaying: if 100 ℔ be 12 monethes a gaining of 8 ℔: then but for 3 moneths terme, onely to take 8 ℔—12 ß muſte néedes be a good rounde ſumme: to worke it, ſette your number thus, 100—12—8—3—8 $\frac{3}{5}$: multiplying the firſte into the ſecond: and alſo by 43 the product of the fift for your diuidend: and the third and fourth togither with 5 the Denominator of your Fraction, for your Diuiſor: then diuide, and you ſhal find as before 430 ℔: The true ſolution to your queſtion.

The tvvelfth Chapter treateth of the making of Factors which is taken in twoo ſortes.

THe firſt is when the eſtimation of the Factoꝛ, is taken vppon the ſending of the Merchant: as if the eſtimation of his perſon be $\frac{1}{4}$, it is vnderſtoode, that he ſhall haue $\frac{1}{4}$ of the gaine, and the Merchaunte the other $\frac{3}{4}$.

The other ſorte is when the eſtimation of his making is out of the ſending of the Merchaunt: as if the oꝛder and agreement betweene them were ſuch, that the Merchant ſhall putte in 800 ℔: and the Factoꝛ foꝛ hys making ſhall haue $\frac{1}{4}$: neuertheleſſe he ſhall haue but $\frac{1}{5}$ of gaine oꝛ pꝛofit: foꝛ the $\frac{1}{4}$ of 800 is 200 (foꝛ the eſtimation of his making) which with the 800 ℔ make 1000 ℔: whereof the 200 ℔ is $\frac{1}{5}$.

A Merchaunt dothe putte in 800 ℔

into the handes of hys Factor: vnder ſuche condition, that the ſaide Factor ſhall haue $\frac{1}{4}$: And after certayne time, they finde in profite 124 ℔—6 ß—8 d. I demaunde howe muche the Merchant ſhall haue hereof, and howe muche oughte the Factor to haue?

Anſwere. When the eſtimation of the Factor is out of the ſending of the Merchaunt, it maketh

℔	ß	d	
99	9	4	for the Merchant
24	17	4	for the Factor

But if that his eſtimation be at the ſending of the Merchaunt, then it maketh but

℔	ß	d	
93	5	0	for the Merchant
31	1	8	for the Factor.

For the Merchaunt is then to haue $\frac{3}{4}$ and the Factor $\frac{1}{4}$.

A Merchant doth putte into the handes of his Factor 800 ℔, & the Factor 400 ℔ to haue the $\frac{1}{5}$ of the profite : I demaunde nowe, for howe much his perſon is eſteemed : when the ſame is counted vpon the ſending of the Merchant.

Anſwere. According to the tenor and order before preſcribed in the firſte Rule: That is, if his eſtimate be $\frac{1}{4}$, he ſhall haue the $\frac{1}{4}$ of the gaine. Therefore ſaye by the Rule of 3 direct: If $\frac{1}{4}$ taken putte in 400 ℔ what is yͤ eſtimate, or putting in of $\frac{1}{5}$ taking? Multiplie, and diuide, and you ſhal find 320 ℔: and ſo much is the perſon of the Factor eſtimated.

Otherwiſe.

To finde the eſtimation of the perſon of the Factor, you ſhall conſider, that ſeeyng it was agreede betweene them, that the Factor ſhoulde take the $\frac{1}{5}$: then the Merchaunt ſhall haue the reſidue, which are $\frac{4}{5}$: wherefore the gaine of the Merchant, vnto that of the Factor is in ſuch proportion as 5 vnto 4: Then if you will know the eſtimation of the perſon of the Factor; Say, if 5 giue 4, what will 400 giue? multiply and diuide, and you ſhal find 320 ℔: And ſo much is the perſon of the Factoꝛ eſtéemed to be woꝛth.

Other conditions then theſe afoꝛeſaid, may alſo be betwéene Merchauntes and Factoꝛs without reſpect, either of ſending oꝛ not ſending of the Merchaunt: where moſt commōly the eſtimation of the bodie of the Factoꝛ is in ſuche pꝛopoꝛtion to the ſtocke whiche the Merchaunt layeth in, as the gaine of the ſaide Factoꝛ is vnto the gaine of the Merchaunte. As thus: if a Merchant do deliuer into yͤ hãds of his Factoꝛ 400 ℔ & he to haue half the pꝛofit: The perſon of the ſaid Factoꝛ ſhalbe eſtéemed to bée woꝛth 400 ℔ And if the Factoꝛ do take but $\frac{1}{3}$ of the gaine: he ſhoulde haue but $\frac{1}{2}$ ſo much of the gaine as the Merchant taketh: which muſte haue $\frac{2}{3}$: wherefoꝛe the perſon of the Factoꝛ is eſtéemed but the $\frac{1}{2}$ of that which the Merchaunte layeth in. That is to ſaye 200 ℔.

And if the Factoꝛ did take the $\frac{2}{5}$ of the gain, then the Merchant ſhal take the reſidue whiche are $\frac{3}{5}$: wherefoꝛe the gaine of the Merchaunt vnto the Factoꝛ is then in ſuche pꝛopoꝛtion as 3 vnto 2: whervpon if you wyll then know the eſtimation of the perſon of the Factor. Say if 3 giue 2: what ſhal 400 giue? work and you ſhal find 266 $\frac{2}{3}$ pounds: And ſo much is the perſon of the Factor eſtéemed to be worth.

And if the Merchaunte ſhoulde deliuer vnto his Factor 400 pounde and the Factor woulde lay in 80 : and his perſon to the ende he might haue the $\frac{1}{5}$ of the gaine I demaunde howe muche ſhall his perſon be eſtéemed?

Anſwer. Abate 80 from 400 : and there will remaine 320. And at ſo muche ſhall hys perſon be eſtéemed.

A Merchaunt hath deliuered vnto his Factor 900 ℔ to gouerne in the trade of merchandiſe, vpon condition that he ſhall haue the $\frac{1}{5}$ of the gain, if any thing be gained : and alſo to beare the $\frac{1}{4}$ of the loſſe if any thing be loſt: Now I demaunde howe much his perſon was eſtéemed at?

Anſwer. Séeing that the Factor taketh the $\frac{1}{3}$ of the gaine, his perſon ought to be eſtéemed as muche as $\frac{1}{2}$ of the ſtocke, whiche the Merchant layeth in. That is to ſaye the $\frac{1}{2}$ of 900 pound which is 450: The reaſon is, becauſe $\frac{1}{3}$ of the gaine that the Factor taketh: is the $\frac{1}{2}$ of the $\frac{2}{3}$ of the gain that the Merchant taketh, and ſo the Factor his perſon is eſteemed to be worth 450 pound.

A Merchaunte hathe deliuered vnto hys Factor 600 pound. And the Factor layeth in 250 pound and his perſõ. Now bicauſe he layeth in 250 pound, and his perſon: it is agreed betweene them, that he ſhal take the $\frac{2}{5}$ of the gaine. I demaunde for howe much his perſon was eſteemed?

Anſwer. For as muche as the Factor taketh $\frac{2}{5}$ of the gaine, he taketh $\frac{2}{3}$ of that which the Merchant taketh: for $\frac{2}{5}$ are the $\frac{2}{3}$ of $\frac{3}{5}$. And therefore the Factors laying in, ought to bee 400 pounde, which is $\frac{2}{3}$ of 600 pounde that the Merchant laid in: Then Subtract 250, whiche the Factour did laye in from 400 pounde whiche ſhoulde haue bene his whole ſtocke: And there remaineth 150 pound: For the eſtimation of his perſon.

More, a Merchaunt hathe deliuered vnto hys Factour 840 ℔ vpon condition that the Factor ſhal haue the gaines of 160 ℔ as though he laide in ſo muche readie money: I demaunde what portion of the gain the ſaid Factor ſhal take?

Anſwer. Sée what parte the 160 ℔ (which the Factor layde in) is of 950 whiche is the whole ſtocke of theyr company: And you ſhal finde $\frac{3}{19}$: And ſuch part of the gaine ſhall the Factor take.

But in caſe, that in making theyr couenauntes, it were ſo agréede betwéene them: that the Factour ſhoulde haue the gaine of 160 ℔ of the whole ſtocke whiche the Merchaunt layeth in: That is to ſaye of the 800 ℔: then ſhoulde the Factor take $\frac{1}{5}$ of the gaine: for 160 is $\frac{1}{5}$ of 800 pounde.

The thirtenth Chapter entreateth of Rules of Barter, and exchanging of Merchandiſe, which is diſtinct into 7 Rules, with diuerſe other neceſſarie Queſtions incident therevnto.

1. Rule.

TWo Merchauntes willing to chaunge theyr Merchaundiſe, the one with the other: The one hath 24 broads clothes at 10 ℔ 10 ſ the péece: The other hath Mace, at 12 ſ the pounde. The queſtion is howe manye pounde of Mace, he oughte to giue him for his clothes, to ſaue himſelf harmeleſſe and be no loſer.

Anſwer. Séeke firſt by the Rule of thrée, what yͤ 24 clothes coſt at 10 ℔ 10 ſ the péece. And you ſhall finde 252 ℔, Then to finde the quantitie of Mace: Say againe by the rule of thrée, if 12 ſ buy one pound what ſhall 252 ℔ buy me? Worke and you ſhal finde: 420 ℔ of Mace. And ſo many pounde oughte he to giue for his clothes,

The

The proofe.

Two barter, the one hath 420 ℔ of Mace at 12 ß the pound, to barter or change for broade clothes at 10 ℔ 10 ß the peece, The queſtion is howe many broade clothes he ought to giue for all his mace.

Anſwer. Firſt ſaye if 1 coſt 12 ß what 400? you ſhal finde 5040 ß: then ſay againe if 10 ½ ℔ giue 1 cloth, what ſhall 5040 ß giue? worke and you ſhall finde 24 clothes: youre deſire.

2. Rule.

Two chaunge merchaundiſe for merchandiſe the one hath Pepper at 2 ß 4 d. the pound: to ſel for readie money: But in barter he will haue no leſſe then 3 ß the pound. And the other hath Holland at 5 ß 6 d. the ell readie money. The queſtion is nowe at what price he ought to deliuer the ell in barter to ſaue him ſelfe harmeleſſe.

Anſwer. Saye by the Rule of thrée direct: if $2\frac{1}{3}$ ß readie money giue 3 ß in barter what ſhal $5\frac{1}{2}$ ß giue in barter? you ſhal finde $7\frac{1}{14}$ ß: and at that price oughte the ſeconde Merchaunte to ſell hys Hollande in barter.

The proofe.

Two barter the one hath Holland at 5 ß 6 d in the ell to ſell for readie money: And in barter he will haue $7\frac{1}{14}$ ß: the other hath Pepper at 2 ß 4 d the ℔ to ſel for readie money. The queſtion is now howe he ought to ſel it in barter.

Anſwer. Say by the Rule of thrée direct if $5\frac{1}{2}$ redie mony giue $7\frac{1}{14}$ ß in barter, what ought $2\frac{1}{3}$ ß to take in barter? multiplie and diuide and you ſhal finde 3 ſhillings your deſire.

3. Rule.

Two barter, the one hath cloth of Arras at 30 ſhillings the ell readie money: but in barter hée will haue $35\frac{1}{2}$ ſhillings.

And the other hath white wines, which he deliuered in barter for 16 ℔ for a Tunne. The queſtion is now what his wines coſt the Tunne in readie money.

Anſwer. Say by the rule of thrée direct, if 35 $\frac{1}{2}$ ß in barter giue but 30 readie money: what did 16 ℔ in barter coſt? Work and you ſhal finde 13 ℔ – 10 $\frac{30}{71}$ ß: and ſo much coſt his wines for a Tunne readie money.

The proofe.

Two barter merchandiſe for merchandiſe. The one hath wines white: at 13 ℔ 10 $\frac{30}{71}$ ß the Tunne to ſell for readie mony: But in barter he deliuered it for 16 ℔. The other to make his matche good and ſaue himſelfe harmeleſſe: delivereth Arras at 35 $\frac{1}{2}$ ß the ell: The queſtion is nowe what an ell of his Arras coſte in readye money.

Anſwer. Saye by the Rule of thrée directe if 16 ℔ in barter giue but 13 – 10 $\frac{30}{71}$ ß in redie money: what ſhal 35 $\frac{1}{2}$ ß yéelde in barter?

worke and you ſhall finde: 30 ſhillings your deſire.

4 Rule.

Two barter, the one hath Carſies at 14 ℔ the péece, readie money: But in barter he wil haue 18 ℔. And yet he wil haue the $\frac{1}{3}$ part of his ouerprice in readie money: And the other hath Ginger at 8 groats the puund to ſel for readie money. The queſtion is how he ought to deliuer the Ginger by the ℔ in barter to ſaue himſelfe harmeleſſe and make the barter equal.

Anſwer Item, for the working of thys queſtion and ſuche other the like, you muſte vnderſtande, if the partie ouerſelling his wares require to haue alſo ſome portion in readye money: as $\frac{1}{2}:\frac{2}{3}:\frac{3}{4}$:&c. Then ſhall you rebate the ſame demaunded part whatſoeuer it be from the ouer price And alſo from the iuſte price. And thoſe two numbers that ſhal remaine after the ſubtraction is made, ſhall be the two firſt numbers in the Rule of thrée. And the iuſte price of the ſame merchandiſe ſhall be the thirde number, whiche by the operation of the Rule of thrée directe ſhall yéelde you a true ſolution: howe and at what price, you ſhal ouerſel that your merchandiſe to ſaue your ſelfe harmeleſſe, and make the barter equal.

Example.

Take the $\frac{1}{3}$ (of eightéene) which is the ouerprice of his cloth: which $\frac{1}{3}$ of eightéen: is ſixe, whiche as appeareth here in the margent you muſt ſubtract from 18: there reſteth 12. And alſo abate it from 14 which is the iuſte price of the cloth: and there remaineth 8: which 8 and 12, are the two firſte numbers in the Rule of thrée. Then take 8 groates or $2\frac{2}{3}$ ſhillings for the third nũber. Then ſay by the Rule of thrée directe: if 8 ß giue 12 ß what ſhall $2\frac{2}{3}$ ß giue? Multiply and diuide, and you ſhal finde 4 ß: And for ſo much ſhal the ſecond Merchãt ſel his Ginger, or his commoditie in barter, to ballaunce the ſame equal.

$$\begin{array}{ccc} 14 & \cdot\ \cdot & 18 \\ 6 & & 6 \\ \hline 8 & \cdot\ \cdot & 12 \end{array}$$

The proofe.

Two barter, the one hath Fine Carſies at 14 ℔ the péece readie money : But in barter he will haue 18 ℔. And yet he will haue the ⅓ part of his ouerprice in readie mony: And the other hath Ginger, whiche he hauing cunning ynough to make the barter equal, deliuered for 4 ß the pounde. The queſtion is nowe what his Ginger coſt him in readie mony.

Anſwer. After you haue made the ſubtraction, abating 6, the ⅓ part of 18 : both frõ 18 and 14 : as before was taught you, then wil there remain 8: and 12 for your two firſt numbers in the Rule of thrée. Then ſay, if twelue giue but eighte : what ſhall come of 4 the ouerprice of the pounde of Ginger ? Multiplye and diuide, and you ſhall finde 2 ß 8 d your deſire.

Two Merchaunts barter Merchandiſe foꝛ merchaundiſe, the one hath Denſhire whits at 7 ℔ — 13 ß 4 d. the péece readie money: but in barter he doth them alwaye foꝛ 8 ℔ — 3 ß — 4 d. And yet he wil haue the $\frac{1}{3}$ parte of his ouerpꝛice in readie money. And the other hath Cottens at 3 ℔ the péece readie money. The queſtion is now at what pꝛice he oughte to ſel oꝛ exchange his Cottens in barter to ſaue him ſelfe harmeleſſe, and make the barter equal.

7 — 13 — 4 —	8 — 3 — 4 —
2 — 14 — 5 $\frac{1}{3}$ —	2 — 14 — 5 $\frac{1}{3}$ —
4 — 18 — 10 $\frac{2}{3}$ —	5 — 8 — 10 $\frac{2}{3}$

Anſwer. Firſt ſéeke $\frac{1}{3}$ parte of 8 ℔ — 3 ß — 4 d: whiche is 2 ℔ — 14 — 5 $\frac{1}{3}$ d which rebated from 8 — 3 — 4 d, there reſteth as appeareth by the example abouesaide 5 — 8 — 10 $\frac{2}{3}$ d, which $\frac{1}{3}$ of 8 — 3 — 4 d alſo rebated from 7 — 13 — 4 d there reſteth 4 18 — 10 $\frac{2}{3}$ the two firſte numbers in the Rule of thꝛée. And the 3 ℔ which is the neate pꝛice of the péece of Cotten is the thirde number. Then ſaye by the Rule of thꝛée derect: as was taught befoꝛe: if 4 — 18 — 10 $\frac{2}{3}$ d giue 5-8—10 $\frac{2}{3}$ d. what ſhall 1 ℔ giue? multiple and deuide and you ſhall finde 3 ℔ — 6 ß — 1 $\frac{91}{292}$ d. the iuſte price that he ought to deliuer his Cottens in barter.

5. Rule.

Two Merchauntes wil chaunge Merchaundiſe for Merchandiſe, the one hath Carſies at 40 ß the peece to ſell for ready monie: And in barter hee will ſell them for 56 ß 8 d. and he will gayne after 10 ℔ vpon the 100 ℔: And yet he will haue the $\frac{1}{2}$ of his ouerprice in ready monie: The other hath Flaxe, at 2 d. the pound readie monie: The queſtion is nowe howe he ſhall ſell the pound of his Flaxe in barter.

Anſwer. See firſte at 10 ℔ vppon the 100 ℔ what the 56 $\frac{2}{3}$ ß commeth to, in ſaying by the Rule of three direct, if 100 ℔ giue 110 ℔ what 55 $\frac{2}{3}$? multiplie and deuide and you ſhall finde 3 ℔—2 ß—4 d. of whiche the $\frac{1}{2}$ that he demaundeth in ready monie, is, 1 ℔—11 ß—2 d.: the ſame 31 ß 2 d. abated from 40 ß, and alſo from 56 ß 8 d.:

there will remayne 8 ſhillings 10 pence: and 25 ſhillings 6 pence, for the two firſt numbers in the Rule of three : And 3 pence the price of the ℔ of flax for the thirde number. Thē Multiply and diuide, and you ſhal find, $9\frac{18}{53}$ d. And for ſo much ſhal he ſel the pound of flaxe in barter.

6. Rule.

Two are willing to exchaunge merchaundiſe : the one hathe Norwiche Grograines at 35 ſhillings the peece redie money: And in barter he will haue 30 ſhillings, and he will haue the ¼ parte of his ouer price in readie money : The other hath Norwich Stockings at 40 ſhillings the Doſen to ſell for readie money : But in as much as the firſt Merchaunts Grograins are no better, he woulde deliuer them ſo to ballance the barter, that he may gaine after 10 ℔ in the 100 ℔. The queſtion is now how he ſhall ſell his hoſe the doſen in barter.

Anſwer.

Anſwere. Say, if 100 giue 110, what ſhall 40 ß giue, which is the iuſte price of the dozen of ſtockings: multiplie, and diuide, and you ſhall finde 44 ß: Then take the $\frac{1}{4}$ of 30 ß whiche is 7 ß—6 d: And ſubtracte it from 25 ß, and alſo from 30 ß: And there will remaine 17 ß—6— and 22 ß—6 d, for the two firſte numbers in the Rule of thrée: and 44 ß which is the iuſt price (with his gaine in the dozen of ſtockings) for the thirde number: Then multiplie and diuide, and you ſhall finde 56 ß—6 $\frac{6}{7}$: and for ſo muche he is to ſell his dozen of ſtockings in barter.

7. Rule.

Two Merchantes will chaunge theyr merchandiſe one with the other: the one hath 720 Elles of Cambricke at 5 ß the Ell to ſell for readie money: but in barter he requireth 6 ß—8 d: And yet notwithſtanding hée loſeth by it after 10 ℔ vppon the 100 ℔: whereuppon he requireth the $\frac{1}{2}$ of his ouer price in readye money: And the other Merchant hauing ſkill ynoughe to make the barter equal deliuereth Engliſh Saffron at 30 ß the ℔: The queſtion is now what his Saffron coſte the pounde in readie money.

Anſwer. You muſte firſte ſeeke what is loſt vpon yͤ 100 ℔, which to do you may ſay if you pleaſe, if 100 ℔ loſe 10? what ſhal $6\frac{2}{3}$ ß loſe? worke and you ſhall finde $\frac{2}{3}$ ſhillings (or eight pence) whiche muſte be rebated from 5 ß 8 d: So reſteth 6 ſhillings ſtill: or you may ſay if 100 ℔ giue me but 90 ℔ what ſhal 6 ß 8 d giue? Worke this waye either and you ſhall finde alſo as before, directly in your quotient 6 ß: your deſire. Then are you next to caſt vp what the 720 elles of Cãbrick commeth to at 6 ß 8 d the ell and you ſhall finde 240 pounde: the $\frac{1}{2}$ whereof the Cambricke Merchaunt wil haue in readie money (which is 120 pound) nextly you muſte caſte what the Cambrick cõmeth to after his loſſe in the 100 ℔ whiche as you founde is but 6 ß an ell: and you ſhal find 216 ℔: now muſt you ſubtract (his readie money which is 120 ℔) out of 240 ℔: & alſo out of 216 ℔. And there wil remaine 120 ℔ & 96 ℔ for your two firſt numbers in the Rule of three, and 30 ß the ouerprice of your Saffron for the third nũber.

Then multiply and diuide,and you ſhal find 24 ß : and ſo muche didde hys Saffron coſte in readie money.

Two Merchants barter, the one hath 50 Clothes, to put away for readie mony at 11 ℔ the Cloth: and in barter putteth them awaye for 12 ℔ : taking Hollande Cloth at 20 d the Flemmiſh Ell, whych was worth no more but 18 d : The queſtion is nowe, what Holland payeth for the Cloth: and what he winneth or loſeth by the bargaine?

Aunſweare. 50 Clothes at 11 ℔ the Cloth, commeth to 550 ℔ : and put away at 12 the péece, maketh 600 ℔ : Then to finde what Holland payeth for the Cloth: Say by the Rule of thrée direct, if 20 d buy 1 Ell: what 600 ℔? worke, and you ſhall finde 7200 Elles: Nowe to finde the eſtate of his gaine or loſſe, you muſt ſéeke what his 7200 Elles commeth to at 18 d the Ell: worke by the Rule of Proportion direct, and you ſhal finde 540 ℔, which is not ſo muche as his Clothes were worth in readie mony, by 10 ℔: and ſo much loſt the firſt Merchant by his exchange.

A Venetian hath in London 100 peeces of Silke, to put away for ready mony at 3 ℔ the peece. But in Barter he delyuereth thē for 4 ℔ the peece, taking woolles of a Felmonger at 7 ℔ 10 ß the C. waight which was worth no more but 6 ℔ the C. in ready money: The queſtion is now, what woolles payeth for the Silkes, and whiche of them winneth or looſeth by the barter.

Anſwer. 100 peeces of Silkes at 3 ℔, is 300 ℔: and at 4 ℔ is 400 ℔: Then to find what woolles payeth for the Silke; Saye by the rule of three direct: If 7 ½ ℔ buy me 1 C. waight, what 400 ℔? worke, and finde 53 ⅓ C. waight of wooll: Nowe to finde the eſtate of their gaine or loſſe, caſte vppe his wooll at 6 ℔ the C. (for ſo muche they were worth readie mony) and you ſhal find 320 ℔, which is 20 ℔ more than the Silkes were to be ſolde for in readie money: whereuppon the Venetian gained 20 ℔ by the Barter.

A Merchant hathe 53 ⅓ C. waight of

wooll at 6 ℔ the C. to ſel for ready mony: but in barter he will haue 7 ℔ — 10 ß: and an other doeth barter wyth hym for Silkes which are worth 3 pounds a péece ready money: The queſtion is now, how he ought to deliuer his Silkes the péece in barter? and howe manye payeth for the woolles.

Anſwere. Say by the Rule of Proportion, or the rule of thrée direct: If 6 ℔ for a C. waight readie money yéelde me 7 ℔ — 10 ß what will 3 ℔ yéelde, which is the iuſt price of a péece of Silke in barter? To make the Trucke equall: worke, and find 3 ℔ — 15 ß the price of a péece of Silke in barter: then ſaye, if 3 ℔ — 15 ß require 1 péece of Silke, how many péeces of Silke are bought wyth 400 ℔, whiche is the valewe of the 53 $\frac{1}{3}$ C. waight of wooll at 7 ℔ — 10 ß: worke by the Rule of thrée direct, and you ſhall finde 106 péeces of Silke, and $\frac{2}{3}$ of a péece: and ſo many péeces of Silke payeth for the woolles, and neither partie hath aduantage of other.

Two Merchants wil change merchãdize yͤ one wͭ the other: The one of thẽ hath béere,

at 6 ß 8d

the Barrell to ſell for ready money : but in barter he will ſell the barrell for 8 ß : and yet he wil gaine moreouer after 10 ℔ vpon the 100 ℔ : and the other hath white Spaniſh wooll at 20 ß the Roue to ſell for ready mony : the queſtiō is now, how he ſhal ſell the Roue of wooll in barter?

Anſwere. Saye if $6\frac{2}{3}$ ß whiche is the iuſte price of the barrell of Béere be ſolde in barter for 8 ß, for how much ſhal 20 ß (whiche is the iuſte price of the Roue of wooll) bée ſolde in barter? worke by the Rule of thrée direct, and you ſhall finde 24 ß : Then for becauſe the firſte Merchant will gaine after 10 ℔ vpon the 100 ℔, he maketh of his 100 ℔ — 110 ℔: And therefore ſay by the Rule of thrée, if the ſecond Merchant of 110 ℔ doe make but 100 ℔, howe much ſhall he make of 24 ß? Multiply, and diuide, and you ſhall finde 21 ß — 9 d. $\frac{9}{11}$ of a penny : And for ſo much ſhal he ſel the Roue of wooll in barter.

Two Merchants wil change their cōmodities the one with the other : The one of them hath white Paper at 4 ß the reſme to ſell for ready mony : And in barter he will doe it alway for 5 ß: and yet he wil gaine moreouer, after the rate of 10 ℔ vpon the 100 ℔: and the other hath Mace at 14 ß — 6 d. the pound waight, to ſel in barter: Now I demaund what the poũd did coſte in readie money?

Anſwere. Say, if 5 ß whiche is the ouer price of the paper in barter be come of 4 ß the iuſte price, of howe muche ſhall come 14 ß $\frac{1}{2}$, which is the ſurprice of the pound of Mace in barter? Multiplie, and diuide, and you ſhall find 11 ß — 10 d: Then for becauſe the firſt Merchant of Paper will gaine after the rate of 10 vpon the 100: Say, if 100 do giue 110, what ſhall 11 $\frac{5}{6}$ ß giue? worke, and you ſhal finde 13 ß — 0 d. $\frac{1}{5}$: and ſo muche didde the pounde of Mace coſte in readie money.

The fourteenth Chapter entreateth of Exchaunging of money from one place to an other.

Exchaunge is no other thing then to take or receiue money in one Citie, to render or pay the valewe thereof in an other Citie,

tie, or elſe to giue mony in one place, and receiue the value thereof in an other, at terme of certaine dayes, moneths, or fayres, according to the diuerſitie of the place.

But this practiſe chiefly conſiſteth in the knowledge of the Mony or Coines in diuers places, of which for thy benefite, (after a few examples giuen to the Introduction to thys worke) I will ſet down by certaine notes of diuerſitie of the common and vſuall Coines in moſt places of Chriſtendome for trafique.

And firſt I will beginne at Antwerpe, where they vſe to make their Accomptes by Deniers de gros: that is to ſay by pence Flemiſh, whereof 12 doe make 1 ß Flemmiſh, and 20 ß doe make 1 ℔ de Gros.

Item, a Merchaunte deliuered at Antwerpe 400 poundes Flemmiſhe, to receyue in London, 20 ß ſterling for euerie 23 ß — 4 d. Flemmiſhe: The queſtion is nowe, howe muche ſterling money is to be receyued at London for the ſaide 400 pounds Flemmiſh?

Anſwere. Saye by the Rule of thꝛée, if 23 1/3 Flemmiſhe giue 20 ß ſterling, what 400 poundes Flemmiſhe ? woꝛke, and you ſhall finde 342 ℔ — 17 ß — 1 5/7 d. : and ſo muche ſterling ſhal I receiue in London foꝛ the 400 ℔ Flemmiſhe.

Otherwiſe alſo wꝛought by Rules of Pꝛactiſe in taking the 1/7 of the Flemmiſh money deliuered : and abating the ſame from the pꝛincipall, the reſt is Engliſhe money ſterling, as befoꝛe.

400 · · 0 · · 0
57 · · 2 · · 10 · 2/7
342 · · 17 · · 1 · 5/7 ſterling.

A Merchant at London deliuereth 200 pound ſterling foꝛ Antwerpe at 23 ß 5 d. Flemmiſhe the pounde ſterling : The queſtion is, how muche he muſte receiue at Antwerpe.

Anſwer. Say by the Rule of thꝛée, if 1 ℔ ſterling giue 23 ß — 5 d. Flemiſh, what 200 ℔

200 ℔ ſterling? worke, and you ſhall finde 234 ℔—3 ß 4 d: So many pounds Fleſhmiſh ſhall he receiue at Antwerp for the ſaid 200 ℔ ſterling.

Otherwiſe alſo by Practiſe.

1 · · 23 · · 5 · · 200	
3 ß 4 d	33 · · 6 · · 8
1 d	— · · 16 · · 8
maketh	234 ℔ · · 3 · · 4 ſterling

In Londõ 200 ℔ ſterling is deliuered by exchange, for Antwerpe at 23 ß—9 d Flemmiſhe the ℔ ſterling: The queſtion is, what rate the Flemmiſhe money ought to be retourned to gaine 4 ℔ vppon the 100 ℔ ſterling at London?

Anſwer. Firſt ſay by the rule of 3 direct: if 1 ℔ ſterling giue $23\frac{3}{4}$ Flemmiſhe, what 200 ℔ ſterling? Multiplie, and diuide, and you ſhall finde 237 ℔—10 ß: The whiche to returne to gaine 8 ℔ ſterling in London. Say by the Backer Rule, if 200 ℔ ſterling require the exchāge 23 ß—9 d Flemmiſh, what the exchange to make 208 ℔ ſtyrling? worke by the Rule, and finde 22 ß—10 d $\frac{1}{26}$ d Flemmiſh, the effect in the queſtion required.

If I take vppe money at Antwerpe after 19 ß—4 d Flemmiſh, to pay for the ſame at London 20 ß ſterling: and when the day of payment is come, I am forced to returne yͤ ſame mony againe in Londō to pay my Bill of Exchange: So that for 20 ß whiche I take vppe here at London, I muſte paye 19 ß—6 d at Antwerpe, I demaunde whether I do winne or loſe: and howe muche in or vpon the 100 ℔ of money?

Anſwer. Say by the rule of three: If 19 $\frac{1}{2}$ giue 19 $\frac{1}{3}$ what will 100 ℔ giue? Multiplie, and diuide, and you ſhall finde 99 ℔—2 ß $\frac{106}{117}$ whiche being abated from 100 ℔, there will remaine 9 ß $\frac{11}{117}$: and ſo muche do I loſe vppon the 100 ℔ of money.

If I take vp at *London* 20 ß ſterling to pay at *Antwerpe* 22 ß 4 d: and when the daye of payment is come my Factor is conſtrained to take vp money againe at *Antwerpe*, wherewith to pay the foreſaide ſumme: and there he doeth receiue 23 ß 4 d Flemiſh, forthwith I muſte pay 20 ß at *London*: the queſtion is now, whether I do winne or looſe, and howe muche vpon the 100 ℔ of money after that rate.

Anſwere. Say by the rule of Proportion: if 22 $\frac{1}{3}$ ß giue 23 $\frac{1}{3}$ ß, what will 100 ℔ giue? Multiplie, and diuide, and you ſhall find 104 ℔ — 9 $\frac{111}{201}$ ß: from the which abate 100 ℔, and there will remaine 4 ℔ 9 $\frac{111}{201}$ ß, and ſo much is there gained vpon the 100 ℔ of money.

In *Antwerpe* is deliuered 200 ℔ Flemiſh by exchaunge for *London*, at 20 ß ſterling for euerie 23 ß 4 d Flemmiſhe: The queſtion is, at what rate the ſame is to be returned to gaine 5 ℔ vpõ the 100 ℔ Flemiſh in *Antwerp*.

Anſwere. Firſt ſay by the rule of three: if 23 $\frac{1}{3}$ Flemiſh giue 20 ß, what ſhall 200 ℔

giue? worke, and you ſhall finde 171 ℔ 8 ß 6$\frac{6}{7}$ đ: Then ſay againe by the Rule of thrée direct, if 171 ℔—8 ß—6$\frac{6}{7}$ ſterling giue me 210 ℔ Flemiſhe, what ſhall 20 ß ſterling giue? worke and you ſhall find 24 ß—6 đ Flemiſh. And at the ſame rate ought the ſame to be returned at *Antwerpe* to gaine 10 ℔ vppon the 100 Flemiſh.

A Merchaunt of *Antwerpe* deliuereth 234 ℔—3 ß—4 đ Flemiſh, to receiue at *London* 200 ℔ ſterling: the queſtion is nowe, how the Exchange goeth after this rate.

Anſwere. Say by the Rule of 3 direct, if 200 giue 20, what giueth 234$\frac{1}{6}$? Multiplie, and diuide, and you ſhall find 23 ß 5 đ and for ſo muche goeth the Exchaunge.

Item, the Exchange from *London* into *France*, is not like as it is in *Flanders*, but is deliuered by the French Crowne, whiche is worth 50 ſoulx Turnois the péece.

Whereupon alſo you muſte note, that in *France* they make their accōpts by Franckes, and Deniers Tournois, whereof 12 Deniers maketh 1 Soulx Tournois, and 20 ſoulx maketh 1 ℔ Tournois, whiche they call a Liure or Franc. But the Merchauntes to make their accompts do vſe French crowns, whiche is currant among them for 51 ſoulx Tournois: But by exchange it is otherwiſe, for it is deliuered but for 50 Soulx Tournois the Crowne, or as the taker vp of the money can agrée with the deliuerer. And note that this ▽ Caracter repreſenteth the Crowne by exchaunge, and is euer 50 ſoulx Tournois or French money.

A Merchaunt deliuereth in London 240 ℔ ſterling after 5 ß — 6 d. ſterling the Crowne, to receiue at Paris 50 ſoulx Tournois for euerie Crown, I demaund howe muche Tournois or French mony payeth yͤ billes for the ſaid 240 ℔ ſterling.

Anſwere. Saye by the Rule of thrée, if $5\frac{1}{2}$ ß ſterling giue me 50 ß Tournois, what ſhall 240 ℔ ſterling giue? Reduce yͤ pounds into ſhillings: then multiplie and diuide, and you ſhal finde 2181 Liuers — 16 ſoulx, — 4 Deniers, and $\frac{4}{11}$ Tournois, and ſo much paieth yͤ bils at Paris for yͤ ſaid 240 ℔ ſterling.

A Merchant deliuereth in Roã, or elſewhere in Fraunce, 1430 ℔, or Francks: the whiche Francke or ℔ is 20 ſoulx or pound Tournois, to receiue in London 6 ß 4 d ſterling for euery ▽ of 50 ſoulxe Tournois: The queſtion is howe much ſterling mony I ought to receiue at London for my 1430 pound Turnois?

Anſwere. Say, if $2\frac{1}{2}$ ℔ giue me $6\frac{1}{3}$ what wil 1430 giue me? worke, and you ſhall find 3622 ß $\frac{2}{3}$ ſterling, which maketh 181 ℔ 2 ß — 8 d: and ſo much money is to be receiued at London for the ſaide 1430 Liuers Tournois, after 6 ß — 4 d for euery crowne of 50, ſoulxe.

In London is deliuered 200 ℔ ſterling by exchange for Paris, at 5 ß — 9 d the ▽ of 50 ſoulx Tournois: the queſtion is at what price the ſaide Crowne is to bee returned to gaine 6 ℔ vppon the 100 ℔ ſterling at London.

Anſwere. Firſte ſay by the Rule of three direct, if $5\frac{3}{4}$ ſterling giue 50 ſoulx Turnois, what ſhall 200 ℔ ſterling giue? worke, and you ſhall finde 1739 Franckes, or Liuers, 2 ſoulx $\frac{14}{21}$. Then the which to returne and gaine 6 lb vpon the 100 lb in London: Saye by the Rule of three direct: if 1739 Francks 2 ſoulx $\frac{14}{23}$ yeelde 210 lb, what the ∇ of 50 ſoulx? worke, and finde 6 s — 1 d $\frac{7}{30}$ the effect required in the queſtion.

A Merchant deliuered in *London* 160 lb ſterling, to receiue in *Biskaie* for euerie 5 s — 6 d 1 Duckate of 374 Marueides, the queſtion is, howe many Marueides I ought to receiue at *Biskaie*.

Anſwere. Say, if 5 ½ s ſterling giue 374 Marueides, what ſhall 100 lb ſterling giue? Multiplie, and diuide, and you ſhall finde 217600 Marueides, and ſo many I oughte to receiue at *Biskie* for my 160 lb ſterling.

A Merchant deliuereth in *Bayon*, 20000 Marueides. to receiue in *London* 5 s—8 d ſterling, for euery Duckate of 374 Marueides: the queſtion is now howe muche ſterling money payeth the Billes of Exchange for the ſaide 20000 Marueides.

¶ Anſwere. Say, if 374 Marueides make 1 Duckate, what 20000 Marueides? Multiplie, and diuide, and finde 106 Duckates $\frac{178}{187}$.

Then ſaye againe, if 1 Duckate giue 5 $\frac{2}{3}$ ß: what giueth 106 $\frac{178}{187}$ Duckates? worke, and finde 30 ℔ — 6 ß and $\frac{34}{561}$ ß whiche is worth $\frac{408}{561}$ partes of a penny.

Otherwiſe it is wrought more briefer at one working, as in the laſte queſtion before, in conſidering, that 5 ß — 8 d containeth 1 Duckate, or 374 Marueides. Therefore ſay by the rule of 3: if 374 Marueides giue 5 $\frac{2}{3}$ ß what 40000 Marueides: worke, and you ſhal alſo finde in your quotients: 30 ℔ — 6 ß $\frac{34}{561}$: and ſo many pound ſterling is to be receiued for the 40000. Duckats.

In London 200 ℔ deliuered by Exchange for Vigo 374 Marueids the Duckate, of 5 ß — 10 d ſterling, maketh 256457 $\frac{1}{7}$ Marueides: the whiche is returne and gaine 10 ℔ vpon the 100 ℔ in London: Saye by the rule of thrée directe, if 220 ℔ require 256457 $\frac{1}{7}$ Marueides, what 5 ß — 10 d? worke, and finde 340 Merueides, priſes of euerie

Duckate in returne, which is the effecte in the queſtion required.

Theſe may ſeeme ſufficient for enſtructions.

NOtwithſtanding for thy further aide and benefite hereafter followeth 6 ſpeciall and moſte briefe Rules of Practiſe for Engliſh, French, and Flemmiſh money.

1 how to turne Flemiſh to Engliſh ſterling.
2 how to turn Engliſh ſterling to Flemiſh.
3 teacheth how to turne Flemmiſh to French.
4 how to turne French into Flemmiſh.
5 how to turne ſterling into French.
6 & laſtly, how to turne French into ſterling.

The fifteenth Chapter entreateth of the ſaide 6 Rules of breuitie, and of valuation of Engliſh, Flemiſh, and French money, and how ech of them may eaſilie bee brought to others value.

How brieflie to reduce ℔. ß, and d Flemiſh, into ℔. ß, and d Engliſhe, Sterling.

IT is to be noted, that 7 pound Flemiſh maketh but 6 ℔. ſterling, 7 ß Flemiſhe, maketh 6 ß ſterl. and 7 d flem. 6 d ſterl. So that 7 yéeldeth but 6. Wherin is euidēt, that there is loſt 1/7, (if it may be ſo called) when it is reduced into Engliſh money. Wherefoꝛe to know how much 233 ℔. 12 ß, 4 d, flemiſh maketh Engliſh, you muſt ſubtract from it 1/7, beginning with the pounds, &c. and that whiche reſteth after this ſubtraction is the ſum required: ſo that 233 ℔, 12 ß, 3 d flem. maketh 200 ℔, 5 ß, 8 4/7 d. ſterling.

	Example.			An other Example.			
	℔.	ß.	d.		℔.	ß.	d.
	233	13	4		311	0	0
$\frac{1}{7}$	33	7	7 $\frac{3}{7}$	$\frac{1}{7}$	144	8	6 $\frac{6}{7}$
	200	5	8 $\frac{4}{7}$ ſterl.		264	11	5 $\frac{1}{7}$

To reduce ℔ ß. and d. ſter. into ℔ ß d flem.

Note that a ℔ ſterling, maketh 1 ℔, 3 ß, 4 d flem. that is 1 ℔ $\frac{1}{6}$: 1 ß ſter. maketh 1 ß $\frac{1}{6}$ flem. and 1 d ſter. maketh 1 $\frac{1}{6}$ Flem. So that there is gained (if it may ſo be called) $\frac{1}{6}$ of the ſumme being thus reduced to Flem. For of $\frac{6}{6}$ is made $\frac{7}{6}$, which is 1 whole, and $\frac{1}{6}$. Then to knowe how much 237 ℔ 7 ß 6 d ſter. maketh Flem. ſubtract from your ſter. the $\frac{1}{6}$ of the whole ſumme, and adde it to the ſame ſumme, and it maketh 276 ℔ 18 ß 9 d, whiche is the ſumme required.

Example.				An other example.		
℔.	ß.	d.		℔.	ß.	d.
237	7	6		337		
39	11	3	$\frac{1}{6}$	56	3	4
276	18	9 ſter.		393	3	4

Ye ſhal note, that the equalitie of Flemiſh and French mony is this, that is to ſay, the ℔

Flemiſh, maketh 7 ℔ $\frac{1}{5}$ French or Turnois: 1 ß Flemiſh maketh 7 ß $\frac{1}{5}$ French, & a groat Flemiſh, maketh 7 d. $\frac{1}{5}$ French.

Wherefore to knowe howe muche 143 ℔ 4 ß 9 d. Flemiſh maketh French. Ye muſt multiplie the whole number twice by 6, beginning at d., and ſo forwarde: and the product of your ſecond multiplication, diuide by 5, ſo that worke is finiſhed. Or multiplie the ſaide ſumme by 7, and take out of it $\frac{1}{5}$ adding it to the producte of your multiplication by 7, and that is your number required. So that as well by the one, as by the other, 143 ℔ 4 ß 9 d. Flemiſh, maketh 1031 ℔ 6 ß 2 d. $\frac{2}{5}$ French or Tournois.

Example. An other Example.

℔.	ß.	d.	℔.	ß.	d.
143	4	9 flem.	143	4	9
		6			7
859	8	6	1002	13	3
		6	$\frac{1}{5}$ 28	12	11 $\frac{2}{5}$
$\frac{1}{5}$ 5156	11	0	1031	6	2 $\frac{2}{5}$
1031	6	2 $\frac{2}{5}$ fre.			

Ano-

Another example.	or thus.
143 ℔:flem. 6	143 7
858 6	1001 $\frac{1}{5}$—28 12
$\frac{1}{5}$-5148	1029 ℔ 12 fr.
$\frac{1}{5}$-1200 ℔ $\frac{1}{5}$, or 12 ß fr.	

A briefe Reduction of ℔ ß and d French, into ℔ ß and d Flemiſh.

Multiply 233 ℔,8 ß,4 d,fr.by 5,and diuide the product twice by 6,that is the ſaide number by 6,and the product againe by 6:and the quotient of this ſeconde diuiſion is the thing required. So that 233 ℔, 8 ß, 4 d, fren. maketh 32 ℔,8 ß,4 d, $\frac{5}{9}$ flemiſh.

Example.				Another.			
℔.	ß.	d.		℔.	ß	d.	
$\frac{1}{6}$—233	8	4	fren.	753			fren.
		5		5			
$\frac{1}{6}$—1167	1	8		2765			
$\frac{1}{6}$—194	10	3 $\frac{1}{3}$		$\frac{1}{6}$—627	10		
$\frac{1}{6}$—22	8	4 $\frac{10}{18}$ or $\frac{5}{9}$		$\frac{1}{6}$—104	11	8	

To reduce ℔, ſ and d Sterling, into ℔ ſ and d French or Tournois.

The ℔ ſter. maketh 8 ℔, 8 ſ french, that is to ſay, 8 ℔ $\frac{2}{5}$: the ſ maketh 8 ſ $\frac{2}{5}$, and the peny 8 d $\frac{2}{5}$ frenchs. Wherefore to know what 231 ℔, 13 ſ, 4 d ſter. maketh french, ye muſt multiply your whole ſum by 42, that is by 7, and the product of it by 6, and diuide thys ſecond product by 5, and that is the ſum required.

Otherwiſe multiply the ſum ſter. by 8, and adde twice to the product $\frac{2}{5}$, and it ſhall produce the ſum required. So that both waies, 231 ℔, 13 ſ, 4 d ſter. maketh 1946 ℔ french. As here vnder followeth.

The ſame otherwiſe.

	℔.	ſ.	d.	℔.	ſ.	d.
	231	13	4 ſterl.	231	13	4 ſter.
			6			8
	1390	0	0	1853	6	8
			7	46	6	8
$\frac{1}{5}$	9730	0	0	46	6	8
$\frac{1}{5}$	1946	0	0 fren.	1946	0	0 fr.

An

An other Example.	The ſame.
753 ſter.	753
6	8
4518	6024
7	$\frac{1}{5}$ 150 12
31620	[illegible] 150 12
$\frac{1}{5}$ 6325 4 fren.	6325 4 tre.

To reduce ℔ ß and d. fren. into ℔ ß d. ſter.

To know how much 1256 ℔ 12 ß 6 d. fren. maketh in ſterling money, multiplie the ſum by 5, and diuide the product by 7 and 6 at twice, and the laſt quotient ſhal be the thing required that is to ſay, 1256 ℔ 12 ß 6 d. maketh 149 ℔ 11 ß 11 d. $\frac{4}{7}$ ſterling.

Example.					An other Example.			
	℔.	ß.	d.			℔.	ß.	d.
	1256	12	6 ſter.			2531		fren.
			5			5		
	6283	2	6			12755		
$\frac{1}{6}$	1047	3	9		$\frac{1}{6}$	2109	3	4
$\frac{1}{7}$	149	11	11 $\frac{4}{7}$		$\frac{1}{7}$	301	6	2 $\frac{2}{7}$

Note, that when any money is giuen by exchange at *London* for *Roan* at 71 d. $\frac{1}{2}$: or rather 71 $\frac{1}{7}$, for the crowne of 50 ſ french, there is neyther gaine nor loſſe, for it is one mony for an other, accompting 8 ℔ 8 ſ French for 1 ℔ ſterling. So the Gyuer loſeth the time of payment which is about 15 dayes, & hée that taketh it, hath gaine of the ſame.

They of *Roan* that putte forth, or take money by exchange for *London*, ought to haue like conſideration.

Item, when any man giueth at *London* 64 d. $\frac{1}{3}$, or rather 64 d. $\frac{2}{7}$, to haue at one of the Faires of *Lions* a crowne de Marc, he that ſo giueth his mony, loſeth the time, and he that taketh it, gaineth the ſame: for 62 d. $\frac{2}{7}$ is equall in value to 45 ſ French. He that putteth or taketh money at *Lions* for *London*, ought to conſider the ſame.

Item, when anye deliuer in *Antwerpe* 75 d. to receiue at *Lions* a crown of Marcke, hée that putteth it forth, loſeth the tyme, and hée that taketh it, gaineth the ſame: For 75 groates Flemiſh is equall in valewe to 45 ſ French.

Thus for this time I make an ende of the practiſe of exchaunge and the enſtructions therevnto belonging, and according to my promiſe gratify ſuch as are deſirous to know the common coynes vſed for traffique among Merchauntes in theſe Cities following: Here followeth a briefe declaration of their monies and yͤ recknings and accompts of them.

The ſixteenth Chapter contayneth a declaration of the valuation and diuerſitie of coines of moſt places of Chriſtendome for traffique: And the manner of exchaunge in thoſe places from one citie or towne to an other, whiche knowne is righte neceſſarie for Merchauntes, by meanes whereof they doe finde the gaine or loſſe vpon the exchaunge.

Item for as much as the greateſt diuerſitie of money of exchaunge is at *Lions* Therefore I will beginne duely of the money of that place.

At *Lyons* they vſe Franckes, Soulxes and Deniers Tournois: a Francke maketh 20

Soulz

Soulx, and one Soulx 12 Deniers: But the Merchants to kéepe their bookes of accompts doe vſe French Crownes of the marke at 45 Soulx the péece, and doe diuide it into 20 ß, 1 ß and 12 d.

Item a Marke of golde 65 ▽ of the Marke, whiche ſerueth for exchaunge. And diuide it into 8 ounces. The ounce into 24 pēce or Deniers, the Denier into 24 graines: And ſo the ſum or whole by imagination or geſſe.

Alſo at *Lyons* there are 4 fayres in a yere, at the which they do commonlye exchaunge, whiche are from thrée monthes to thrée monthes.

At *Geanes* they vſe the Soulx: on Ducket maketh ℔.3.

At *Naples* they vſe Duckets, Tarie, and Graines: The Ducket maketh 5 Taris, & one taris 20 graines, but they take 6 Duckets (whiche maketh thirtie taries) for the ounce.

A Ducket maketh 10 Carlins, & a Carlin 10 graines, ſo that 2 Carlins make a Taric, and 100 graines make a Ducket.

At *Rome* they vſe Duckets of the Chamber, one Ducket is worthe 12 *Guylis*, and a *Guli* 10 Soulx.

At *Venice* they vſe Duckets Curraunts at 124 Soulx a peece or 24 Deniers, & one Denier maketh 32 picolis.

At *Palerme* and *Meſſine* they write, after ounce, tary, & graynes, and 1 ounce is worth 6 Duckets, or 30 taris, and 1 tary is twenty graines, and 1 graine 6 picolis, One Ducket is alſo worth 24 Carlins,

At *Milan* they vſe ℔. ß. d. of Duckettes imperials, and ▽ of exchaunge is worthe 4 ℔.

At *Lucques*, *Florence* and *Ancone* they vſe the ▽ of gold: in gold the French Crowne is worth ℔. 7: but at *Buloigne*, ℔. 3. 10 ß

At *Barſelone* they vſe the ſoulx, the Ducket of exchaunge is worth 22 ſoulx.

At *Valence* and *Saragoſſe* they vſe the Liuer, Soulx and Denier, the Frenche crowne of exchange is worth 20 ſoulx, and 1 ſoulx is 12 Deniers.

At the Fayres of *Caſtill* they vſe the Meruaidies, the Ducket is worth 375 Meruaidies.

At *Liſbone* they vſe the Raies, one Ducket of exchaunge is worth 400 rayes.

At *Noremburge*, *Franckford*, and *Auguſte* in *Germanie* they vſe the *Krentzers*, whereof 60 make a Floryn.

At *Antwerpe* they vſe ℔. ß and d. de Gros, and they exchaunge into the Denier de Gros. To wit our Engliſh peny.

At *London* they vſe the 1 ℔ ſterling and 1 d. ſterling and they exchaunge in 1 d. ſterling.

The exchaunge of *Lyons* at ſundrie places.

Item at *Lyons* there is exchaunge in thrée ſorts, at the cities and townes following.

Firſte they deliuer at *Lyons* one Marke to haue or receiue at *Naples* almoſte 41 ½ Duckets: at *Venice*, 70 Duckets corrant, at *Rome* 63 Duckets of the Chamber, *Luques* and *Florence* 65 ▽ of Gold, at *Milan* 82 ▽.

And contrariwiſe at the ſaide Cities aforeſaide they doe giue ſo muche of money to haue a marke at *Lyons*.

Secondlie they giue at *Liſbone* one ▽ of Marke of 45 ſoulx Turnois a pæce to haue at *Gennes*, almoſt 68 Soulx. At *Palerme* and *Meſſine* almoſte 24 Carlins: at *Barſelone* 22 ſoulx at *Valence* or *Saragoſſe* 20 ſoulx. At the fayre at *Caſtili* 350 Meruaides: at *Liſbone* 360 *Rais*: in *Antwerpe* 57 Deniers de Gros: and at *London* 70 đ ſterling.

And contrariwiſe they do giue in the ſaid Cities almoſte as muche of their money to haue a Frēch Crown of the Marke at *Lyons*.

Thirdlye they doe giue at *Lyons* a ▽ of the ſunne to haue almoſt 93 *Krentzers* at *Franckforde*, *Auguſte*, *Norenberge*, or other Cities in *Almaine*.

Alſo at *Lyons* only they do pay the chaunge the $\frac{2}{3}$ in gold, and $\frac{1}{3}$ in mony, or elſe al in money in giuing 1 $\frac{1}{2}$ for the hundreth.

Chaunges at *Naples* and other Townes.

Item at *Naples* they giue or deliuer almoſt 112 Duckets to receiue at *Rome*, 100 Duckets of the chamber at the old value.

Through *Luques* and *Florence* they deliuer 100 Duckets *Carlins* to receiue there almoſt 86 ▽ of gold.

Through *Palerme* & *Meſſine* one ducket of 5 Cary to receiue there almoſt 164 graines.

Throughe *Milan* one Ducket to receyue there almoſt 90 Soulx.

Throughe *Geanes* one Ducket to receyue there almoſt 65 ſoulx. The whole ſum to be paide within 10 dayes after the ſighte of the bil of exchaunge.

Alſo at *Naples* they deliuer one Ducket to receiue in *Antwerpe* almoſt 67 đ. 02 Deniers de Gros. within 2 monthes: At *London* almoſt 60 đ ſterling, in 2 months: At *Barſelone* almoſt 20 Soulx within 2 monthes: At *Valence* almoſt 18 ſoulx wͭin 2 monthes: At *Liſbone* 333 Raies, within 3 monthes: and at the fayre at *Caſtil*, almoſt 340 Meruevdes at the ſame faire.

Chaunge of *Venice* to other places.

At *Venice* they deliuer 100 Duckets curraunte to receiue in *Almayne* almoſte 140. *Florenes* at 60 *Krentzers* the péece.

At *Lucques* and *Florence* almoſt 08 ▽ of gold in 10 dayes.

Likewiſe at *Venice* they deliuer a Ducket curraunt to receiue at *Palerme* and *Meſſine*, almoſte 21 *Carlins*, at *Milan* almoſte 93

Soulx. At *Geanes* almoſt 62 ſoulx, the whole at 10 dayes ende.

Of the *Pair* or *Pari*.

As touching the exchaunge, it is neceſſarie to vnderſtande or knowe the *Pair* which the *Italians* call *Pari* which is no other thing than to make the money of the change of one Citie or towne to or with the money of an other, by meanes whereof they doe finde the gaines or loſſe vpon the exchange.

Example.

Item hauing receiued letters of credite of one of *Antwerpe* that the ▽ of the ſunne is there worth 7 ſoulx. The queſtion is what the ſame is worth at *London* when the *Pair* or exchaunge goeth for 23 ß?

Anſwer. Saye if 23 giue but 20 what giueth 7: worke and finde 6 ß 1 $\frac{1}{23}$ d. : and ſo muche is the ▽ of the ſunne worth at *London*.

The ſeuenteenth Chapter conteineth alſo a declaration of the diuerſitie of the waightes and meaſures of moſte places of Chriſtendome for traffique. At the ende of whiche diſcourſe are two tables, the one for waighte, and the other for meaſure, proportionate and reduced to an equalitie of our Engliſhe meaſure and waighte, by the aide whereof the ingenious may eaſilye by the Rule of three, conuerte the one into the other at their pleaſure &c.

AT *London* and ſo all *Englande* throughe are vſed two kinds of waights and meaſures, As the *Troy* waighte and the *Haberdepoize*: from the *Troy* waighte is deriued the proportion and quantitie of all kinde of drye and liquid meaſures, as Peckes, Buſhels quarters &c. wherwith is bought and ſold all kind of graine and other commodities mette by the Buſhell. And in liquid, Ale, Béere, Wine, Oile, Butter, Honie, &c. vppon theſe groundes and ſtatutes, is Bread made and ſolde by the *Troy* waight. And ſo is golde, ſiluer, pearle, pretious ſtones, and Jewelles. The leaſt quantitie of this *Troy* waight is a graine, 24 of theſe graines make a penny waight, 20 penny waights an ounce: and 12 ounces a pound: 2 ℔ or 2 pintes of this waighte maketh a quarte. And ſo aſcending into bigger quantities, is produced the Meaſures, whereby are ſolde our other naturall ſuſtenaunce, vz. Ale, or Beere, with alſo other neceſſarie commodities, as Butter, Honny, Herrings, Eeles, Sope, &c. Al which laſt before rehearſed, though their Meaſures (wherein they are contained) be framed and deriued from the *Troy* waight: yet are they in traffique, with diuers commodities, as Lead, Tinne, Flaxe, Waxe, with all other commodities both of this Realm, & of other Forraine Countries whatſoeuer, bought & ſolde by the Haberdepoiſe waight, after 16 ounces to the pounde, and 112 ℔ to the C. waight. And vnto euerie C. is allowed 12 ℔ weight at the common beame. From hence is alſo deriued the wey of *Suffolke* Chæſe whyche containeth 32 Cloues, 8 ℔ to a Cloue, and weygheth in all 256 ℔: And alſo the barrell of *Suffolke* Butter is or ſhold be of like waight with the wey of cheeſe : vz. 256 ℔. More 14 of theſe ℔ make a ſtone: And 26 ſtone cõtaineth a ſacke of engliſh Wooll. Forraine Woolles, to wit, *French, Spaniſh* & *Estrich*, is alſo ſolde by the pound or C. waighte, but moſt commonly by the Roue, 25 ℔ to a Roue : other commodities of tale are bought and ſold by the C. fiue ſcore to the C. Except headed ware, to wit, cattel, nailes, & Fiſh, whych are ſold after ſixe ſcore to the C.

Ther is alſo two other ſorts of Meaſures, to wit, the Ell, and the Yeard. By the Ell is vſually mette linnen Cloth, as Canuas, &c. And by the Yeard, Silks, wollen Cloths, &c.

Antwerpe.

At *Antwerpe* are alſo 2 ſorts of waights, their golde & ſiluer waight, & their common waight gold & ſiluer is weighed by ỹ Marke, the Marke is 8 ounces, the ounce 20 Eſterlings, and the Eſterling 32, as our graines, The Goldſmithes diuide that into ſmaller, but not the Merchantes : the proofe of Golde is made by Caractes, whereof 24 maketh a Mark of fine Gold: the Karact is 24 graines: the proofe of the money is made by Deniers, 12 deniers is 1 ℔ fine: that is a Marke of fine ſiluer, the Denier alſo is diuided into 24 graines, and the graine into 4 quarters.

Item, 100 Markes in *Antwerp, Iroy* waighte, maketh at *Lions* 103 Markes, 2 ½ ounces, and 20 graines 23 p̈ : At *Noremburg* 103 Markes, 2 ½ ounces, 2 quints, 3 Deniers: at *Franckford* 105 Markes: at *Auſburge*, 104 Markes, 3 ounces, 1 quint : At *Venice* 103 Markes, 1 ounce, 7 deniers, 18 graines: At *London* 66 ℔.

The Marke of Golde or Siluer at *Antwerp Iroy* waight, which is 8 ounces maketh 7 ½ ounces common waight with which all other merchandize is weighed : So that the *Troy* waight is greater than the common waight by 6 ¼ in the C. By this waighte of *Iroy* they alſo wey Muske, Amber, Perle, &c.

All Silkes are bought at *Antwerpe*, by the *Bruges* Ell, whiche is greater than the common meaſure, by which they retaile by 2 in the C. Their common Ell is ¾ of our yeard, and ⅗ of our Ell.

Lions.

At *Lions* is vſed 3 ſortes of waightes, whereof the firſte is the common Towne waight, with whiche they wey all kinde of Spicerie, and diuers other merchaundize. The ſeconde is called *Geneua* waight, which is 8 in the 100 greater than the common waight, with which they weigh Silkes, &c. The third is French waight, called commõly the Marke waight, and 100 ℔ therof, maketh 106 $\frac{1}{4}$ ℔ *Geneua* and 114 $\frac{3}{4}$ of their common waight, with which French waight is weyed al things that payeth cuſtome or toll.

At *Lions* is alſo vſed two ſortes of Elles or Aulnes: The one wherewith they meaſure groſſe Clothes, as Canuas, and ſuche like. The other is called the French El or Aulne, with which they meaſure all other kinde of merchandize, whereof 7 common Towne Elles maketh 11 ordinarie French Elles.

Roan.

At *Roan* 6 $\frac{1}{2}$ Muydes of Salte, being the meaſure of yͤ place, make an hũdreth at *Armuiden* in *Zeland*, and the C. of *Bronage*, meaſure of *Armuiden* maketh at *Roan*, 11 Muids, 30 Mines maketh a Laſt of Corne, & 16 a laſt of Dates, 100 ℔ waight there maketh at *London* 114 $\frac{1}{4}$: and 109 $\frac{2}{8}$ at *Antwerpe*: And 100 Elles make at *London* 115 $\frac{7}{8}$.

Norem-

Noremburge.

A 100 ℔ waight of *Noremburg*, maketh at *London* $111\frac{3}{4}$ at *Antwerpe* $107\frac{1}{2}$: And 100 Els of *Norēburg* make at *London* $75\frac{3}{5}$ at *Antwerp* $95\frac{3}{5}$ &c.

Liſbone.

The C. waight at *Liſbone* maketh 4 Roues, euerie Roue 32 ℔: So that their C. waight is 128 ℔, and their pound containeth 14 ounces, and 100 ℔ of their waight maketh at *London* $113\frac{1}{8}$.

Their Silke, cloth of Gold and Wollen is meaſured with a meaſure which they call a Cubite, containing about $\frac{3}{4}$ of a Uarre of *Caſtile*: Howbeit, their common Meaſure is called a Uarre which maketh 5 Palmes & containeth $1\frac{1}{4}$ of a Uarre of *Caſtile*: our ell of *London* is equall with the Uarre of *Liſbone.*

All kinde of Merchandiſe brought from *Flaunders*, *Roane*, or *Brittaine*, payeth at *Liſbone*, as a duetie or cuſtome to the King, 20 in the C. whiche they call the tenth in Merchandize, and the other tenth in money.

Note alſo, that all kinde of Merchandize comming to *Liſbone* by land, payeth leſſe in cuſtome than that that commeth by water.

Ci-

Ciuill.

The Roue of *Ciuill* is 30 ℔, 4 Roues make their C. waight which is 120 ℔: The 100 ℔ of *Ciuill* maketh at *London* 102 ℔: Their other common Meaſure is a Varre, whereof 100 maketh at *London* 74 Els: and at *Rome* 40 Canes, &c.

Venice.

At *Venice* be 2 ſortes of Waightes, the one called La Groſſe, the other La Suttle, with the Groſſe is weyed all kinde of greate wares: and with the ſmall, al kinde of Spicerie, and ſuche like: *96* ℔ of groſſe waight there maketh at *London* 100 ℔: And 100 ℔ of Spicerie there without any tare, or alowaunce, make at *London 64*, and with tare *56*.

Their other cõmon meaſure are Braces, whereof 100 make at *London* 55 ½ Elles, at *Antwerpe* 92 ½, &c.

Florence.

At *Florence* the 100 ℔ waight maketh at *Aquila*, for Saffron 110: and 145 ℔ of *Florence* make at *Roan* but 100 ℔: the waight of *Florence*, and that of *Luke* is all one.

Their other meaſures are Braces, wherof 100 maketh at *Antwerp Burges* meaſure 81 $\frac{2}{3}$ Elles, 100 Braces there make at *London* 49 Elles, &c.

Lucque.

The Lucque Sattens are cōmonly ſolde at *Lions* by waight, and 133 $\frac{1}{3}$ ℔, maketh at *Lions* 100 ℔: So that 1 ℔ $\frac{1}{3}$ maketh at *Lions* but 1 pounde.

Their other meaſures are Braces, wherof 100 of them make at *London* 50 Elles, at *Antwerpe* 83 $\frac{1}{3}$ Elles, &c.

Aquila.

At *Aquila* their 100 ℔ maketh at *London* 71 $\frac{1}{4}$, their 136 $\frac{2}{3}$ ℔ of Saffron maketh at *Geneua* but 100: And 11 pounde of *Geneua* maketh 15 pound at *Aquila*.

Valentia.

At *Valentia* be 2 ſorts of waights, a great and a ſmall: The C. waight of great waight containeth 4 Roues: The Roue 36 ℔: So the C. great waight is 144 ℔ : and the C.

waight ſmall containeth but 120 lb: and is alſo parted into 4 Roues whiche is 30 lb to a Roue: By the ſmall is ſolde the Scarlet graine, with all other kinde of Spicerie, and by the great is ſolde Wooll with all ſuch like groſſe wares: The $1\frac{1}{3}$ lb of Silke at *Valentia* maketh at *Lions* 1 lb *Geneua* waight: The Charge of greate Merchaundize at *Valentia*, containeth 432 lb, and in ſmal wares 360 lb: The waight here, and at *Barcellone* is al one: Their 100 lb waight maketh at *London* 78 lb: and at *Antwerpe* 75.

Danſicke.

At *Danſicke* or *Spruce* lande the Rule is, that whoſoeuer buyeth anye Merchaundize there, buyeth it by the Shippound, which is 320 lb: 20 Liſpoundes make a Shippounde, and the Liſpounde containeth 16 pounde: whiche Shippound of *Danſicke* maketh at *Antwerp* $266\frac{2}{3}$ lb: Their 100 lb waight maketh at *London* $86\frac{5}{8}$, &c.

Their other commō meaſures are Elles, whereof 100 make at *London* $72\frac{1}{4}$: And at *Antwerp* $120\frac{1}{2}$ Elles.

Toulouſe.

At *Toulouſe* 6 Cabes of Woad, maketh a Charge, 2 Ceſterns of Coꝛne, and all kinde of graine maketh a Charge: the Ceſter weyeth 160 ℔ waight of that place. Their 100 ℔ in waight maketh at *London* but 91 $\frac{3}{4}$ ℔.

Geanes.

At *Genua* oꝛ *Geanes*, a 100 ℔ of their waight maketh at *London*, 71 $\frac{1}{4}$: And at *Antwerpe*, 68 $\frac{2}{8}$: 100 ℔ waight at *Genua* maketh at *Venice*, to wit, Suttle 106 ℔.

Their other cõmon Meaſures are Palmes, whereof 100 make at *London* 20 $\frac{4}{5}$ Elles: & at *Antwerpe* 34 $\frac{4}{5}$.

The reſt are ſupplyed in 2. Tables which hereafter followeth: whereby the ingenious may gather his deſire.

The agreement of the Waightes of diuerſe Countries, the one with the other, being reduced to an Equalitie, and drawn into a Table, as followeth.

112 pounds waight at *London*, make at

Antwerp	107 5/8
Franckeford	099.
Collen and Auſburge	102 1/4
Noremburge	100 1/8
Roan	098
Paris	102 1/4
Lions	118 1/2
Diepe	100 1/4
Geneua	090 1/8
Towlouſe	122 3/4
Rochel	124 7/8
Marcellis	124 1/4
Ciuill, &c.	109 3/4
Venice groſſe waight	105 3/8
Venice ſuttle waight	166 7/8
Aquila	157 1/4
Vienna	089 3/8
Preſlaw	134 5/8
Leibzig	101 1/4
Danſicke	129 1/4
Lubecke	097 3/8
Barcellon	144 1/2
Liſbone	099
Geanes	157 1/4

The other Table of the agreement of Meaſures of diuers Countries reduced vnto an equalitie, by the aide wherof you may with the vſe of the rule of three conuerte either more or leſſe of anye one meaſure vnto other.

60 Elles or 75 yeards at *London* make at

Antwerpe	100	**Elles**
Norenburg	104 ½	
Franckford	125	
Leibſig	125	
Preſlaw	125	
Danſicke	83	
Vienne in Auſti	87	
Lions in Fraunce	60 $\frac{40}{41}$	**Aulnes**
Paris in Fraunce	57	
Roan in Normandie	52	
Liſbone	60	**Varres**
Ciuill in Spaine	81	
Caſtile in Spaine	81	
Methera Iles	62	
Venice	108	**Braces**
Luques	120	
Florence	122 ½	
Millan	138	
Rome	90	**Canes**
Geanes.	288 $\frac{6}{13}$	**Palmes**

The

The eighteenth Chapter treateth of Sportes and paſtimes, done by Number.

IF you would know the number that any man doth thinke, or imagine in hys minde, as though you coulde diuine. Bid him triple it, or putte twice ſo muche more to it, as it is: which done, aſke him whether it bee euen or odde: if he ſay odde, bid him take 1 to it, to make it euen: and for that 1 keep 1 in your minde: now after he hath taken 1 to it, to make it euen, bidde him giue away halfe, and keepe the other halfe for himſelfe, which when he hath done, bid him triple that halfe: and againe after he hath tripled it, aſke hym whether it be euen or odde: if he ſaye odde: then bid him take 1 to make it euen againe: and for that laſt 1 keepe 2 in your mind, now after he hath made his number euen, bidde him caſte away the one halfe, and kepe the other ſtill: From which halfe that he kepeth, cauſe him ſutlely to put away or giue you 9 out of his nūber, & for eche 9 that he giueth you keepe 4 in your minde: and thereunto ioine the 3 which I bade you keepe, and you ſhal haue your deſire.

Example.

Imagine he thought 7: the triple whereof is 21: and becauſe it is odde, he is to take 1 to make it euen: which firſt 1 giuen is 1 for you to kéepe in minde: Then the halfe of his 22 being caſte away, he reſerueth ſtill 11: which after you haue bidde him triple, it maketh 33: then in giuing of him 1 againe to make it euen: vpon that laſte 1 reſerue 2 in your mind: then his halfe of 34 maketh 17: From whence he can giue you 9 but once: Therefore that yéelding to you 4: and the 3 that you kéepe make 7, your deſire.

An other kinde of Diuination to tell youre frend, how many pence, or ſingle peeces, reckoning them one with an other, hee hath in his purſe, or ſhould thinke in his minde.

Which to doe firſte bid him double the péeces he hath in his purſe, or the number he thinketh: if he participate hys number or ſecreacie vnto ſome one friend that ſetteth by him that can but multiplie, & ad neuer ſo little: if their number be greate, then ſhal they worke as you bidde them ſo much the ſurer.

Nowe after he hath doubled his number, bid him adde thereunto 5 more, which done, bidde him multiply that his number by 5 alſo, whiche done, bidde him tell you the iuſte ſumme of his laſte Multiplication, whiche ſumme the giuer thinking it nothing auaileable, becauſe it is ſo greate aboue his pretended imagination: yet thereby ſhal you preſently wyth the helpe of Subtraction tell his propoſed number.

The Rule is this.

Imagine he thought 17: double 17, and it maketh 34. whereunto if you adde 5, it maketh 39: whiche multiplied by 5, as here is practiſed in ỹ margēt, it yeeldeth 195: which 195 is the ſum deliuered you in the worke: then for a general rule, you ſhal euermore cut off the laſt figure towards your right hād, wᵗ a daſh of your pen, as here is performed, as a figure

17
2
34
5
39
5
19|5
2|
17

nothing auaileable vnto your worke: and then rebate 2 from your firſte figure, after 5 is cutte off: and the reſte ſhall euermore be your deſire, as by this example doth appeare.

If in any company, you are diſpoſed to make them merry by manner of Diuining, in deliuering a Ring vnto any one of them, which after you haue deliuered it vnto them, that you will abſent your ſelfe from them: and they to deuiſe after you are gone, which of them ſhall haue the keeping thereof: And that you at your returne wil tell them what perſon hath it, vpon what hãd, vpõ what finger, & what iointe. Which to do, cauſe yͤ perſons to ſit down al on a rowe, & to keepe likewiſe an order of their fingers: nowe, after you are gone out from them to ſome other place, ſay vnto one of the lookers on, that hee double the nũber of him that hath the Ring, and vnto the double bidde him adde 5: And then cauſe him to multiplie that Addition by 5: And vnto the product bidde him adde the number of the finger, of the perſon that hath the Ring. And laſtly, to ende the worke beyond that number towardes his right hãd, let him ſet downe a figure, ſignifying vppon which of the ioints he hath the Ring, as if it

bee

hée vppon the ſeconde ioint, let him put down 2: Then demaund of him what number hée kéepeth. From the whiche you ſhall abate 250. And you ſhall haue thrée figures remaining at the leaſt: the firſt towards your left hande ſhall ſignifie the number of the perſon which hath the King: the ſeconde or middle number ſhall declare the number of the finger, and the laſte figure towardes your right hande, ſhall betoken the number of the ioint.

Example.

Imagine the ſeuenth perſon is determined to kéepe the King vppon the fifth finger, and the third ioint: firſt double 7, it maketh 14, thereto adde 5, it maketh 19, whiche multiplied by 5 yéeldeth 95: vnto which 95, adde the number of the finger, and it maketh 100: and beyond 100 toward the right hand, I ſet downe 3 the number of the ioint, al maketh 1003, which is the number that is to be deliuered you, from which abating 250, there reſteth 753, whiche prefigureth vnto you the ſeauenth perſon, the fifth finger, and the third ioint.

But note, that when you haue made your ſubtraction, if there doe remaine a 0: in the place of tens, that is to ſay, in the ſeconde place, you muſt then abate 1, from that figure which is in the place of Hundreds, that is to wit from the figure which is next your left hand, and that ſhall be worth 10 tenths, ſignifying the tenth finger: as if there ſhould remaine 803, you muſt ſay, that the ſeuenth perſon vppon his tenth finger, and vppon his third ioint, hath the King.

And after the ſame manner, if a man doe caſte 3 Dice, you may knowe the points of euerie one of them. For if you cauſe hym to double the pointes of one Dye, and to the double to adde 5: and the ſame ſumme to multiplie by 5: and vnto the product adde the pointes of one of the other Dice. And behind the number towards the right hand to putte the figure which ſignifieth the pointes of the laſte Die: and then to aſke what number he kepeth, from which abate 250: and there wil remaine 3 figures, which doe note vnto you the pointes of euerie Die.

Another.

If thrée diuerſe things are to be hidden of thrée diuerſe perſons, and you to diuine whiche of the thrée perſons hathe the thrée diuerſe things doe thus: imagine the thrée things to be repreſented by A.B.C Thẽ ſecondly kéepe wel in your minde whiche of the perſons you meane to be the firſt, ſeconde and third: Then take 24 counters or ſtones, and your thrée things, And giue A. vnto the partie whome you imagine to be your firſte man: and therewithal giue him one of your 24 counters in his hand: And B. vnto your ſeconde man: and therwithal 2 counters: And C vnto your thirde man, and therewithall 3 counters. And leaue the reſte whiche are 18 ſtil among them, which done, ſeperate your ſelf from them, & afterwards bid them chãge the things among them as they ſhall thinke good, whiche done after they are agréed, byd him that hathe ſuche a thing, as before you haue repreſented by A for euery counter that he hath in his hand to take vp as manye moe, And for him that hathe B for euerie one in hys hande to take vppe two: And for hym that hathe C. for euerye one in hys hande to take vp 4: And the reſt of them to leaue ſtill vpon the borde. Theſe thrée things and the thrée perſons being fullye printed in your minde, come to the table, and you ſhall euermore finde one of theſe ſixe numbers 1.2.3.5.6 or 7 If therefore one remaine ſtil vpon the borde, then haue they made no exchaunge, but kéepe them ſtill as they were deliuered vnto them: So that the firſte man hath A the ſecond B and the third C. But if 2 remaine, then the firſte man hathe B. your ſecond man A and your thirde man C: the reſte of the worke and the order therof are here apparant by the table following.

1	1	A	5	1	B
	2	B		2	C
	3	C		3	A
2	1	B	6	1	C
	2	A		2	A
	3	C		3	B
3	1	A	7	1	C
	2	C		2	B
	3	B		3	A

Another Diuination of a number vpon the caſting of two Dice.

Firſt let the caſter caſt both the Dice, and marke wel the number: then let him take vp one of them it maketh no matter which: and looke what number it hath in the bottom, and adde altogither: then caſt the die againe, and keepe in his minde what altogither maketh: then let the Dice ſtande: bring 7 wyth you and therevnto adde the reſt of the pits that you ſee vpõ the vpper ſide of the Dice, and ſo many did the caſter caſt in all.

FINIS.

I. D. To the earneſt Arithmetician.

MY louing friend to Science bent,
Something thou haſt by this booke woonne:
But if thou wilt be excellent,
Another race thou muſt yet runne.

Supplyes thereto but (few do) neede,
And none but ſuch as in our phraſe,
(By Records pen) thou muſt wel reade:
Proceede therefore: Be not ſtunt dwaſe.

The grownd moſt ſure, wheron this race
With ſpeedeful courage muſt be paſt,
Of late hath turnde his Greekiſh face,
By Engliſh tilth, which aye wil laſt.

The famous Greeke of Platos lore,
EVCLIDE *I meane Geometer:*
So true, ſo plaine, ſo fraught with ſtore,
(As in our ſpeach) is yet no where.

A treaſure ſtraunge, that booke wil proue,
With numbers ſkil, matcht in due ſort,
This I thee warne of ſincere loue,
And to proceede do thee exhort.

Plus oultre.

THus gentle Reader I end this my treatiſe, with earneſt requeſt to accept it in good parte. And let my good wil countervaile the baſeneſſe of the ſtile: praying thee alſo to thy better furtherance of perfection, to amende in the Margente of the booke, theſe faultes that haue eſcaped in the Printing, which I haue here in this table of errata following collected.

Vale I. M. *Norwicenſis.*

Faults eſcaped in the Printing, which I deſire thee good Reader to correct with thy penne, for thine owne eaſe and benefit.

E.16.page.line.7 there lacketh 4 lb 2 ſ.
S.12.page.line.26.read as you work into ſ.
Dd.1.page line.5 for $\frac{5}{8}$ read $\frac{1}{6}$.
Gg.2.page.line.8 for $\frac{1}{6}$ read $\frac{1}{6}$.
Gg.6.page.line.11.there lacketh.666 $\frac{2}{3}$.
Ll.10 page.line.10.for $\frac{1}{8}$ read $\frac{1}{3}$.
Mm.1.page.line.13. read for 2 ſ take the $\frac{1}{10}$.
Mm.2.page.line.12.for $\frac{1}{2}$ read $\frac{1}{10}$.
Mm.6.page.line.2.read wherof the $\frac{1}{3}$.
Mm.9.page.line.23. Item for 3 d.take $\frac{1}{8}$.
Mm.15.page.line.2.What 3884 yeardes.

Faultes eſcaped in the Printing.

M m.15.page.line.26.read which 10 d. is $\frac{1}{6}$ of 5 ß.

N n.7.page.line.4 reade 23.

N n.9.page.line.12 mende 48 to 4 ß 4 d.

N n.13 page.line 7.vnder $\frac{2}{4}$ put $\frac{1}{4}$.

O o.8 page, line 13.read ſaying once 3 is 3.

O o.9 page.line.6 in the exãple, ſay if 1 yeard coſt 8 $\frac{1}{4}$ .d.

O o.10.page.foꝛ $\frac{7}{17}$ mende it to $\frac{7}{11}$.

O o.11.page.line.24 foꝛ $\frac{1}{3}$ make it $\frac{2}{3}$.

P p.8 page.line.3 foꝛ 18 ß make 10 ß.

Q q.3.page.line.5 foꝛ the ſeauenth make it the eight.

Q q.4 page.line.13 read whẽ I deliuered it.

Q q.5 page.line.23 foꝛ 9 $\frac{1}{4}$ read 6 $\frac{1}{4}$.

Q q.9 page.line.5.foꝛ $\frac{1}{2}$ make it $\frac{8}{2}$.

Q q.11.page.line.1.put a pꝛicke at 400. to ſeuer him & $\frac{1}{3}$ aſunder, the like do at the end of 600: foꝛ to ſeuer $\frac{1}{2}$:

Q q.16 page.line.11. after the rate of 9 ℔. read 10 ℔.

R r.1 page.line.5.foꝛ 20.18.8 $\frac{13}{25}$: reade 51 ℔. 14.5 $\frac{13}{25}$.

T t.14.page.line.10 put out, by: reade notes of the.

X x.1.page.line.7.read of gold maketh 65.▽

e

.

s
d

ch.

ch.

tes

▽